respect to $\bar{p}$.

**THEOREM 2.** There exist $f, g \in \text{Diff}(S^1)$ such that (1) $fg = gf$, (2) $f = \text{id}$ on an open set, (3) $\bar{f}, \bar{g}$ sufficiently close ($C^2$) to $f, g$ and $\bar{f}\bar{g} = \bar{g}\bar{f} \Longrightarrow \bar{f} = \text{id}$ on an open set.

**Corollary.** (Reeb, Rosenberg, Godbillon). There is a foliation of $T^2 \times I$ with all interior leaves cylinders such that any $C^2$-close foliation has a 'band' of cylindrical leaves.

**THEOREM 3.** On any $M^2$ there exist vector fields $V$, $W$ with $[V, W] = 0$, $V = kW$ on some open set, and for any $\bar{V}, \bar{W}$ sufficiently $C^2$-close with $[\bar{V}, \bar{W}] = 0$ there exists $\bar{k}$ with $\bar{V} = \bar{k}\bar{W}$ on an open set.

The proofs of Theorems 1 and 2 make use of the following

**Proposition.** $f : R \to R$ linear, $f \neq \text{id}$, $g(C^1)$ satisfies $gf = fg \Longrightarrow g$ linear.

**Proof.** $g(\lambda^n x) = \lambda^n g(x) \Longrightarrow \lambda^n g'(\lambda^n x) = \lambda^n g'(x) \Longrightarrow g'(x) = g'(0)$ (all $x$).

For Theorem 3, we need a generalization.

**Question.** Does $f : R^n \to R^n$ linear ($||f|| < 1$), $gf = fg$ ($g \epsilon C^1$) $\Longrightarrow g$ linear?

**Answer.** No, e.g. $f(x_1, x_2) = (\lambda x_1, \lambda^2 x_2)$ ($0 < \lambda < $ ), $g(x_1, x_2) = (a x_1, b x_2 + c x_1^2)$.

**THEOREM 4.** Let $f : R^n \to R^n$ be a linear diffeomorphism, $||f|| < 1$, and let $\bar{\lambda} = \max |\lambda_i|$, $\underline{\lambda} = \min |\lambda_i|$ ($\{\lambda_i\}$ = eigenvalues). Let $m$ = least integer such that $(\bar{\lambda})^m < \underline{\lambda}$. Then $g$ ($C^m$) commutes with $f \Longrightarrow g$ is a polynomial of degree $< m$.

**THEOREM 5.** Let $\mathcal{A} = \{f \epsilon \text{Diff}(S^1) \mid \{g \mid gf = fg\} = \{f^n\}_{n \epsilon Z}\}$. Then $\mathcal{A}$ is open and dense in $\text{Diff}(S^1)$ ($C^2$ topology).

**Conjecture.** The same is true for general compact $M$.

(21) <u>Predictions for the future of differential equations</u>  R.Abraham

Last year at the Berkeley Summer Institute on Global Analysis

there were 20 talks on differential equations, of which 10 were concerned with the "yin-yang problem". Some large (yin) sets of differential equations with generic properties are known, some small (yang) sets which can be classified are known, but in general the two domains have not yet met. They began to approach each other, but

$$\text{simple} \longleftarrow \underset{\text{yang}}{\longrightarrow}) \longrightarrow (\underset{\text{yin}}{\longrightarrow} \text{complicated}$$

recently progress degenerated into a sequence of conjectures and counter-examples without further approach. For guidance on future progress, the I Ching was consulted on the question : will the current program lead to a solution to the yin-yang problem - a set of differential equations both small enough to be classified and large enough to be generic? The prophesy obtained was hexagram 13 (Breakthrough) changing to 49 (Revolution), interpreted as No.



43        49

This year at the Warwick Summer School there were 22 talks of which only 3 were concerned with the yin-yang problem. The I Ching was again consulted (using for the first time, half-crowns). As it had become clear through the esoteric Buddhist principles of Karma and Transcendence that the yes-no question formerly posed was too restrictive, the question asked this time was : How will the subject evolve in the course of the next year? In view of the theories propounded by Professor Thom, it seems that in place of a

steady approach of the two domains we should expect a number of
bifurcations, and fruitful investigations of new domains.
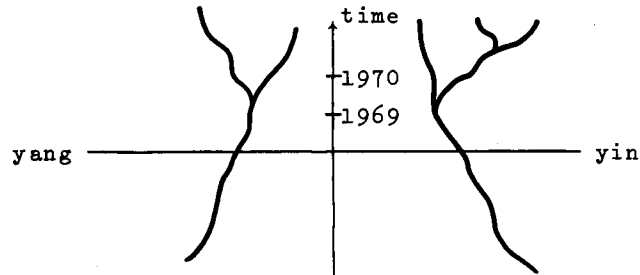


The answer to the question was given by the <u>I Ching</u> as
hexagram 10 (Conduct) transforming to 11 (Peace), a clear indication
of the future dominance of yin.



10                                11
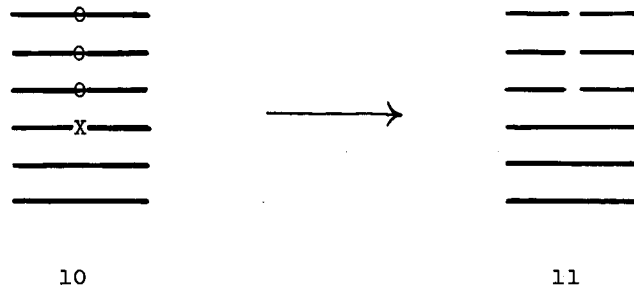
From further study of hexagram 10, the following precepts emerged:

   The superior man discriminates between high and low;

   One should not attempt to exceed one's own strength;

   Overcome danger by going forward in time;

   Resolute conduct and perseverance will lead to success;

   Weigh past conduct : if it is good, then good will follow.

Thus by concentrating on what is important without being too
ambitious, by continuing to work hard and with determination, and
by studying closely and learning from the many counter-examples
of the past, progress will be made towards peace and harmony in
differential equations.  Although the yin and yang aspects of

differential equations will continue to oppose each other as prophesied last year, a harmonious balance between the conflicting forces may be obtained by proper study.