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Perhaps for ????

The Hexagrams of the Moon

by

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Dedicated to Dane Rudhyar, 1895-1985

Abstract. The two most prominent phases of the moon are discretised into 8 parts each, and the resulting 64 biphases are correlated with the hexagrams of the *I Ching*. These hexagrams of the moon may be regarded as the chaotic rhythm of the moon's orbit, mapped into an eight-fold unfolding of the lunation cycle of Dane Rudhyar. For mundane purposes, this provides a universal divination symbol for the whole world, which changes about twice per week.

CONTENTS

1. INTRODUCTION

The chaos revolution of the last century has coordinated phase shifts in every area of our intellectual culture. The sciences and arts from astronomy to gastronomy — all were transformed. In the first part of this paper we trace out a thread of chaos theory in astronomy — specifically the celestial mechanics of the earth's moon — from prehistory to the present. In the second part, we develop its significance for the arts of divination, specifically, the *I Ching* and astrology. Along this thread we take guidance from the writings of the late modern astrologer, Dane Rudhyar. we owe many thanks to the astrologer William Sheeran of Dublin for his suggestion to investigate chaotic astrology, and for his patient explanations and other assistance.

2. LUNAR ASTRONOMY

2.1. The simple astronomical cycles of the moon

If only the moon's motion were a simple circle around the earth, in the ecliptic, that is, the plane of our solar system! Then we would still have two lunar cycles: the sidereal and the synodic. We call these the simple cycles of the moon, and describe them in this section under the simplifying assumption of a circular, flat, moon orbit. In the next section we will abandon this assumption.

The *sidereal cycle* is the rotation of the ray from a laser in the center of the earth, through the center of the moon, as it sweeps a circle like a lighthouse beacon, counterclockwise (CCW) when seen from the north, around the zodiac. Even though the laser beacon itself is moving, the light ray (under this simplifying assumption) would complete every circle through the zodiacal constellations in the same time interval, a sidereal month of about 27.3 days.

The *synodic cycle* is the directly visible cycle of phases of the face of the moon, from new moon to full moon and back to new moon again. If the earth stood still, the synodic cycle would still be a little longer than the sidereal, about 29.5 days, because the earth moves around the sun CCW, and at the end of a sidereal month following a new moon, the moon still has to move a bit more to line up with the sun. This is shown in Figure 1.

These are the two simple cycles of the moon. As the synodic cycle is directly visible and is known to everyone, we will adopt it as the primary lunar cycle. We may think of it informally as a circle in the plane of the moon's orbit, but from the point of view of mathematics, it is better to think of it as a cycle in the plane of the two variables: phase (amount of illumination) and rate of change of phase.

2.2. The latitude of the moon

It is time to abandon the simplifying assumptions:

1. the lunar orbit is flat — that is, lying entirely within the ecliptic plane of the solar system, and

2. the lunar orbit is circular.

Actually, we will give them up one at a time. So now, let us admit that the moon's orbit is not flat. It lies in a plane inclined about 5 degrees to the plane of the ecliptic, or more precisely, to the plane of the earth's orbit. These two planes intersect in a line, the *nodal line*. The moon thus spends about half of its circle around the earth north of the ecliptic plane and half to the south.

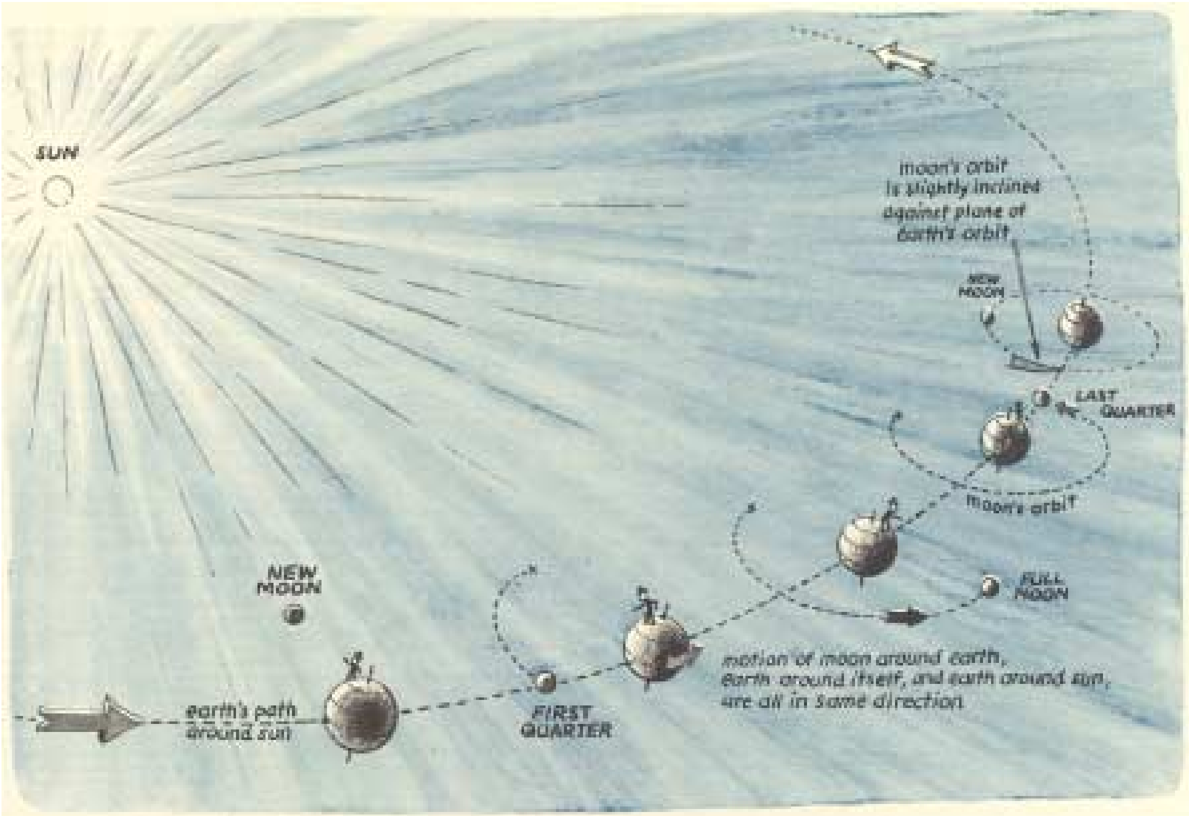


Figure 1. The synodic phases of the moon, from one new moon to the next.
(Rey, 1970, Fig. 28, p. 137).

When passing through the ecliptic plane from south to north, through the nodal line, the transition point is called the *ascending node*. Half a cycle later, the moon passes from north to south of the ecliptic through the *descending node*. The interval between two successive transitions through the descending nodes is called the *nodal month*. All of this is shown in Figure 2.

If the moon circled the earth in the plane of the earth's orbit, we would have an eclipse of the moon every full moon, and an eclipse of the sun every new moon — two eclipses every month! The inclination of the moon's orbit is the reason for the relative rarity of eclipses. We now wish to quantify this inclination.

From the modern point of view, there are two coordinate systems in common use to describe points in the sky. Angular variables relative to the center of the earth and relative to the celestial equator — the projection outward of the earth's equator from the center of the earth — are called *right ascension* and *declination*. These are the usual coordinates of astronomy. See (Rey, 1952, pp. 109-115) for a clear pictorial explanation. Angular variables relative to the center of the earth and relative to the ecliptic — the plane of the earth's orbit — are called *ecliptic longitude* and *ecliptic latitude*. These are the usual coordinates of astrology. (Schultz, 1986, p. 45) The two coordinate systems are equivalent, and transformation between them may be described by trigonometric identities. The two "equators" are great circles which cross, at an angle of 23.5 degrees, in diametrically opposite points, called the *vernal* and *autumnal equinoctial points*.

Ecliptic latitude and longitude will be most useful for our description of the apparent motion of the sun and moon. Longitude is an angular measure around the ecliptic, counterclockwise, beginning from the vernal equinoctial point. The signs and degrees of the zodiac are an equivalent measure of this ecliptic longitude commonly used in astrology. Latitude is an angular measure of altitude above or below the ecliptic, positive to the north.

Thus, the apparent motion of the sun in ecliptic coordinates is an increasing cycle of values of ecliptic longitude, always at zero degrees ecliptic latitude, taking one year to complete 360 degrees. The motion of the moon in these coordinates is an increasing cycle of values of longitude also, taking about one month, but the latitude increases from zero to about five degrees, decreases again to zero, passes though a minimum of about five degrees negative, and then up to zero again. The period of this cycle is about 27.2 days. (Schultz, 1983, p. 219)

2.3. The wandering of moonrise

Sir Norman Lockyer, Astronomer Royal of England at the turn of the 20th century, said that the horizon was the telescope of prehistoric people. (Lockyer, 1906b) The positions on the horizon at which the sun and the moon rose and set were the basis of the archeoastronomy of the early civilizations, and particularly the megalithic people of Britain and Brittany. Stonehenge, in the south of England, and Callanish, in the north of Scotland, may be taken as outstanding examples. (Thom, 1971; Hawkins, 1965)

When the moon is north of the sun in ecliptic latitude, moonrise appears on the eastern horizon north of the sunrise point. And when the moon is south of the sun, moonrise is south of sunrise. The same holds for moonset relative to sunset positions on the western horizon. The solunar rising azimuth — the angle which we observe along the horizon between sunrise and moonrise points —

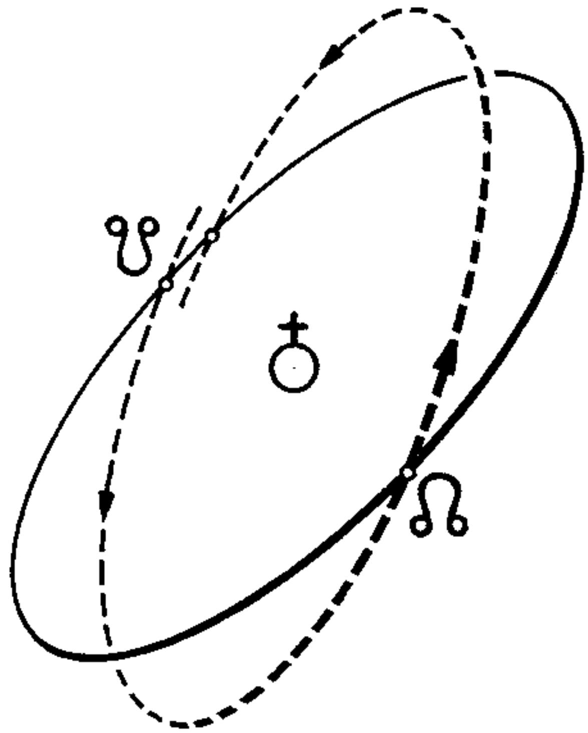


Figure 2. The moon’s orbit (dashed) and the ecliptic (solid), during one nodal month.
Note the movement CW of the descending node during the nodal month, and the displacement of the Sun from the center. Both are exagerated for emphasis in this drawing from Schultz. (Schultz, 1963; Fig. 56, p. 87).

depends on the ecliptic latitude of the moon, as well as our terrestrial latitude — north or south of the earth's equator — according to a complicated trigonometric formula. (Schultz, 1983, p. 215)

But the direct observation is quite simple, provided one can stay awake through the night when necessary, and has a decent view of a fairly level horizon. For example:

1. Early morning: mark the sunrise point.
2. Later during the day or night, mark the point of moonrise.
3. Record the difference angle, the solunar rising azimuth.

Then we would see that the solunar angle increases from zero — when the moon passes through the ascending node, to a maximum (northern) value, then decreases through zero as the moon passes through the descending node, to a minimum (southern) value, and again increases to zero, as the moon completes one cycle, north node to north node, in a nodal month.

If we record the rate of change of the azimuth as well as the azimuth, then we may plot the monthly motion of the moonrise point relative of the sunrise point as a cycle in phase space, the plane of the two coordinates: solunar azimuth angle, and its rate of change. This is the solunar rising cycle, our second important phase of the moon, and one well known to the megalithic people. The amplitude of the solunar motion on the horizon depends on the latitude of the observer, and reaches a maximum in the mid-latitudes.

2.4. The two main phases of the moon as a solenoid on a torus

Above we have described two solunar cycles, each as a circle in a phase plane. The primary, or synodic, cycle is that of the face of the moon, familiar to everyone. The secondary cycle is that of the ecliptic latitude of the moon, familiar to prehistoric astronomers as an oscillation on the position of moonrise relative to the position of sunrise on the horizon. The periods of the two oscillations are both about a month, approximately 29.5 days and 27.2 days. For the sake of discussion now, we may further approximate these periods as 29 days and 27 days. If these crude values were exact, then the two phases would coincide every 27*29=783 days, or about 2 years and a half. Using more exact values for the two periods, a much longer interval is required before the two cycles coincide. An interval known as the *Saros* (Chaldean for repetition) of about 18 years 10 days is defined as 223 synodic months, and is within a few hours of being 242 nodal months. This interval is important in predicting eclipses, but is still not an exact ratio of the two months. See (Schultz, 1983, p. 100) and (Rey, 1952, p. 138). Given the new knowledge of chaos theory, we may safely assume that there is no fraction which exactly expresses the ratio of the two periods. That is, they are most likely irrationally related.

We are now going to develop a graphical strategy for representing the two phases of the moon, in the style of chaos theory. This strategy involves a two-dimensional torus, the basic geometry for two cycles, which we will visualize as cut twice and flattened into a plane rectangle. An extensive course in this "toral arrangement" strategy may be found in (Abraham, 1992, Part 1). So we imagine a plane rectangle ruled with a square grid, each rule of which indicates the passage of one day's worth of phase change. This rectangle is 27 days wide and 29 days high. See Figure 3. We now imagine a sequence of daily measurements of the two phases of the moon, expressed not in angles observed, but rather as angles around the cycles of phase. Thus, the synodic phase is represented as a uniform vertical motion downwards, which after 29 days disappears off the bottom side of the rectangle, and instantly reappears on the top side directly above. Similarly, the nodal phase is represented as a uniform horizontal motion to the right, which disappears off the right edge after 27 days, and instantly reappears on the left, at the same height. A sequence of daily observations, represented according to this strategy, creates a sequence of dots descending along a straight line of slope minus one. That is, each day, we record one step to the right and one step downwards. These are shown in Figure 3, connected as a line. Note that after 27 days, the line jumps from the right edge of the rectangle to the left edge. And after 29 days, the line jumps again, this time from the bottom edge to the top, where it appears two days to the right of the starting point. We quit drawing the trajectory near this point in Figure 3. However, if we were to continue it, it would keep wrapping around the torus-rectangle, arriving back at the starting point in the upper left corner after 783 days.

Finally, we may drop the simplifying assumption, in which we rounded off the two periods to 29 and 27, and replace these with more accurate values, such as 29.530589 days and 27.212220 days. (Schultz, 1983, p. 219) This makes the rectangle a little higher and a little wider. And our observations, plotted as above in the new rectangle, will wind about for a very long time, essentially forever, without closing into a loop. In dynamical systems theory, such a trajectory is called an *irrational solenoid*, or an *almost periodic motion*, and may be regarded as a weak sort of chaos: a prechaotic state.

Please note that our prehistoric heritage of lunar observation of the primary phase is direct: the cycle is seen as a uniformly circular motion, and millions of years old. But our experience of the secondary phase has been ongoing perhaps only for about 5,000 years, and is indirect: only the rising and/or setting azimuths have been observed, using lunar observatories such as Callanish or Stonehenge. Thus the graphs of the observations are quite different. As shown in Figure 4, the primary phase is linear, and the secondary phase azimuth (or latitude, equivalently, which is shown here) is sinusoidal. The data graphed in Figure 4 is shown in tabular format in Table 1.

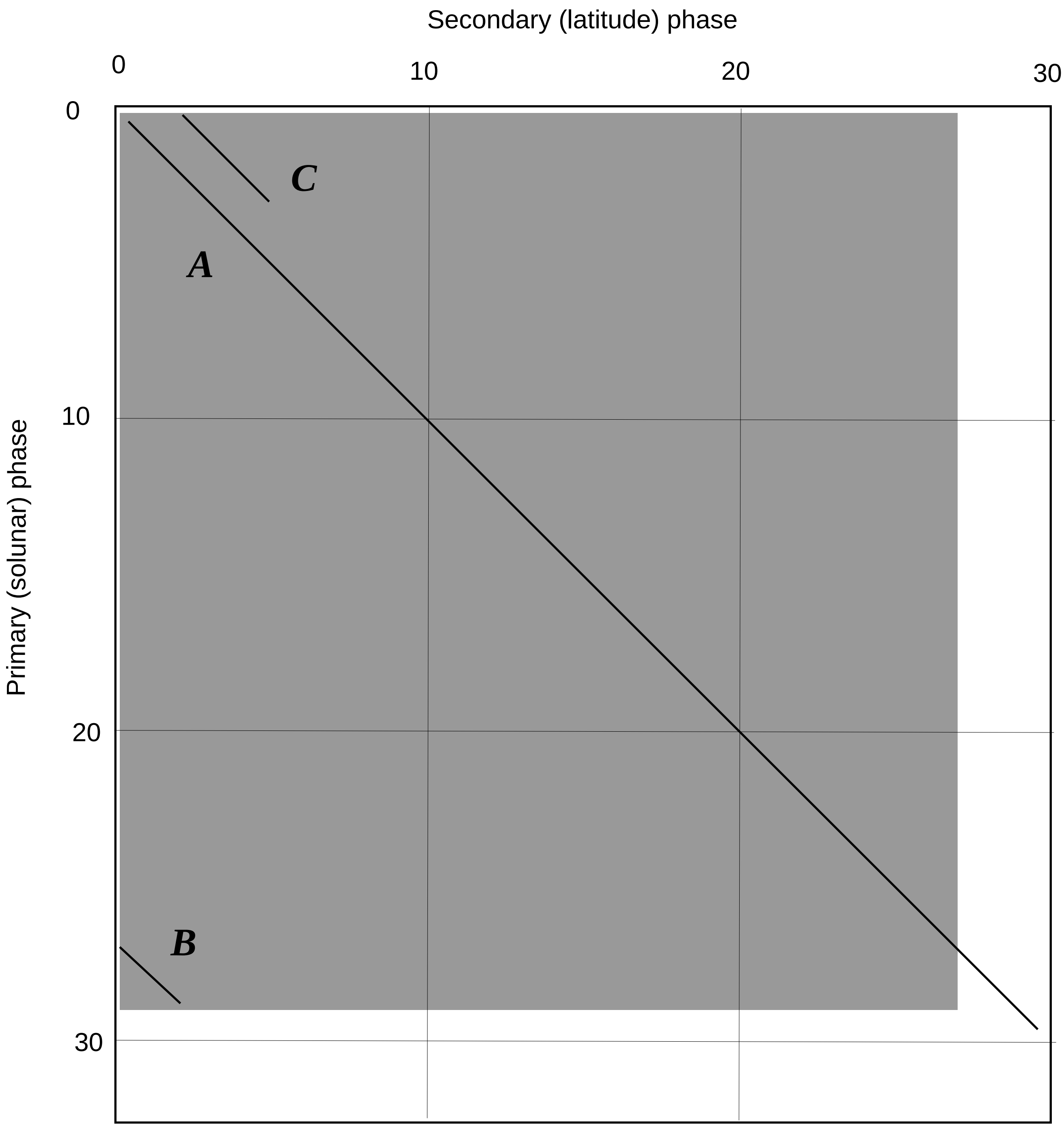
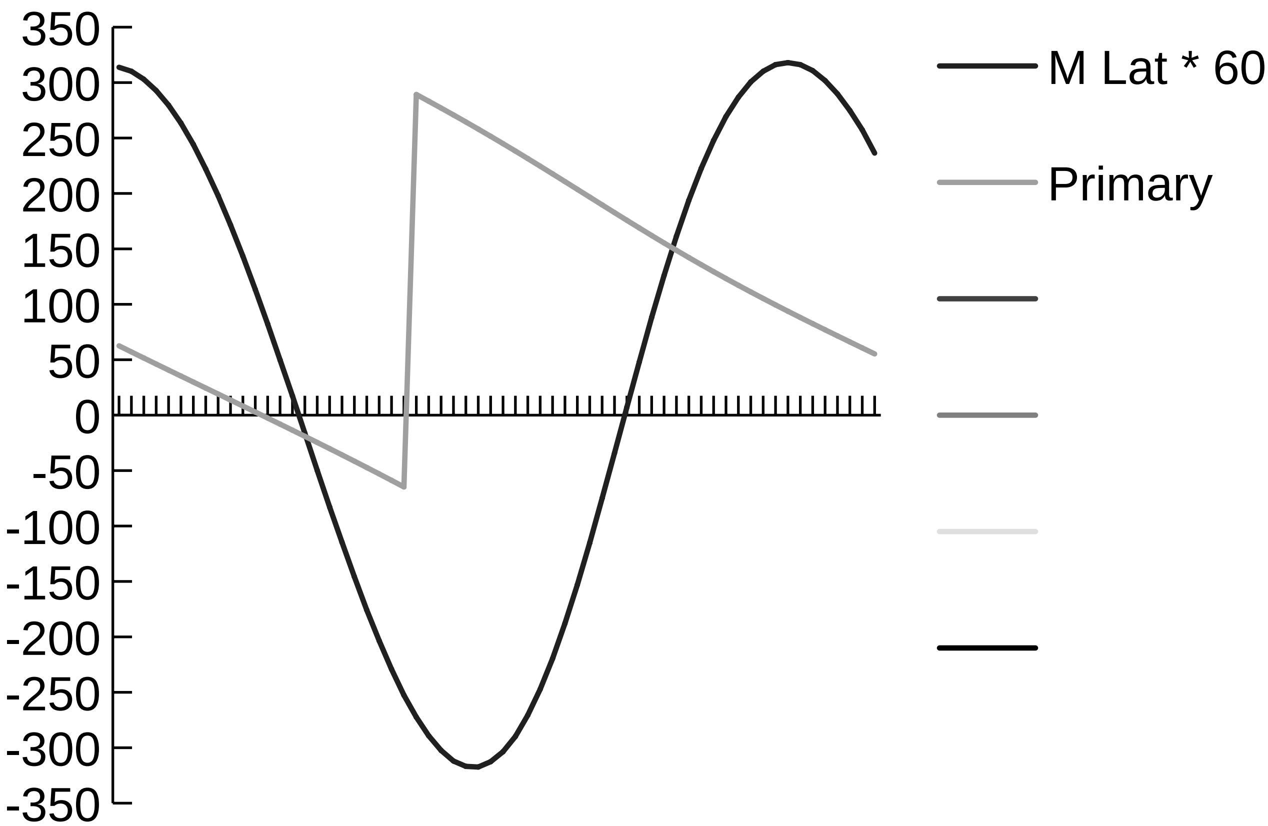


Figure 3. The time-trajectory of the biphase begins at A, proceeds downward and to the right with slope 1 until running off the right edge of the biphase torus, shown here shaded gray. It then immediately reappears at the left edge at B, continues in the same straight line, until running off the bottom edge of the biphase torus. It then reappears at the top edge, at C, and continuous around forever, in a solenoid, eventually filling-in the entire torus almost everywhere.



Latitude (times 60) and Phase vs Time (31 days)

Figure 4. The primary phase versus time is shown on a scale of 360 degrees of circular phase measure. The secondary phase versus time is shown as minutes of azimuth. Two data per day are shown in Table 1 for the month of January, 2000, at Dublin. Many thanks to William Sheeran, astrologer of Dublin, for sending this data.

TABLE 1.

Date	M Long	M lat	S Long	Primary	M Lat * 60
1/01/00 0:00	217.3	5.23	279.86	62.56	313.8
1/01/00 12:00	223.33	5.17	280.37	57.04	310.2
1/02/00 0:00	229.32	5.05	280.88	51.56	303
1/02/00 12:00	235.28	4.88	281.39	46.11	292.8
1/03/00 0:00	241.23	4.66	281.9	40.67	279.6
1/03/00 12:00	247.15	4.39	282.41	35.26	263.4
1/04/00 00:00	253.06	4.07	282.92	29.86	244.2
1/04/00 12:00	258.97	3.7	283.43	24.46	222
1/05/00 00:00	264.88	3.3	283.94	19.06	198
1/05/00 12:00	270.8	2.86	284.45	13.65	171.6
1/06/00 00:00	276.72	2.39	284.96	8.24	143.4
1/06/00 12:00	282.65	1.89	285.47	2.82	113.4
1/07/00 00:00	288.6	1.37	285.98	-2.62	82.2
1/07/00 12:00	294.57	0.83	286.49	-8.08	49.8
1/08/00 00:00	300.57	0.29	287	-13.57	17.4
1/08/00 12:00	306.59	-0.27	287.51	-19.08	-16.2
1/09/00 00:00	312.64	-0.83	288.02	-24.62	-49.8
1/09/00 12:00	318.73	-1.38	288.53	-30.2	-82.8
1/10/00 0:00	324.87	-1.91	289.04	-35.83	-114.6
1/10/00 12:00	331.04	-2.43	289.54	-41.5	-145.8
1/11/00 0:00	337.28	-2.93	290.05	-47.23	-175.8
1/11/00 12:00	343.56	-3.39	290.56	-53	-203.4
1/12/00 0:00	349.91	-3.82	291.07	-58.84	-229.2
1/12/00 12:00	356.33	-4.21	291.58	-64.75	-252.6
1/13/00 0:00	2.82	-4.54	292.09	289.27	-272.4
1/13/00 12:00	9.4	-4.82	292.6	283.2	-289.2
1/14/00 0:00	16.06	-5.04	293.11	277.05	-302.4
1/14/00 12:00	22.8	-5.2	293.62	270.82	-312
1/15/00 0:00	29.64	-5.28	294.13	264.49	-316.8
1/15/00 12:00	36.58	-5.29	294.64	258.06	-317.4
1/16/00 0:00	43.61	-5.21	295.15	251.54	-312.6
1/16/00 12:00	50.73	-5.06	295.66	244.93	-303.6
1/17/00 0:00	57.94	-4.83	296.17	238.23	-289.8
1/17/00 12:00	65.23	-4.51	296.68	231.45	-270.6
1/18/00 0:00	72.6	-4.12	297.19	224.59	-247.2
1/18/00 12:00	80.02	-3.66	297.69	217.67	-219.6
1/19/00 0:00	87.5	-3.13	298.2	210.7	-187.8
1/19/00 12:00	95	-2.55	298.71	203.71	-153
1/20/00 0:00	102.52	-1.92	299.22	196.7	-115.2
1/20/00 12:00	110.04	-1.25	299.73	189.69	-75
1/21/00 0:00	117.53	-0.57	300.24	182.71	-34.2
1/21/00 12:00	124.99	0.12	300.75	175.76	7.2
1/22/00 0:00	132.38	0.8	301.26	168.88	48

1/22/00 12:00	139.71	1.47	301.76	162.05	88.2
1/23/00 0:00	146.94	2.1	302.27	155.33	126
1/23/00 12:00	154.09	2.69	302.78	148.69	161.4
1/24/00 0:00	161.13	3.23	303.29	142.16	193.8
1/24/00 12:00	168.06	3.71	303.8	135.74	222.6
1/25/00 0:00	174.88	4.13	304.31	129.43	247.8
1/25/00 12:00	181.59	4.49	304.81	123.22	269.4
1/26/00 0:00	188.19	4.78	305.32	117.13	286.8
1/26/00 12:00	194.69	5.01	305.83	111.14	300.6
1/27/00 0:00	201.08	5.17	306.34	105.26	310.2
1/27/00 12:00	207.39	5.27	306.85	99.46	316.2
1/28/00 0:00	213.62	5.3	307.36	93.74	318
1/28/00 12:00	219.77	5.27	307.86	88.09	316.2
1/29/00 0:00	225.85	5.18	308.37	82.52	310.8
1/29/00 12:00	231.88	5.03	308.88	77	301.8
1/30/00 0:00	237.87	4.83	309.39	71.52	289.8
1/30/00 12:00	243.82	4.58	309.9	66.08	274.8
1/31/00 0:00	249.75	4.29	310.4	60.65	257.4
1/31/00 12:00	255.66	3.94	310.91	55.25	236.4

Data such as this, extended for the entire year 2000 and plotted similarly, are shown in Figures 4b and 4c.

2.5. More complex astronomical cycles of the moon

In the preceding we described two simple cycles of the moon's motion: the synodic cycle (the phases of the face of the moon) and the rising azimuth cycle (the relative rising points on the horizon). There are other cycles, more complex and subtle, which we may ignore in the present discussion. But one of these which is most relevant here is the rotation of the nodal line, and thus also the two nodes. The nodal line rotates clockwise (CW) around a point near the center of the earth, at a rate of about 1.5 degrees per month, as shown in Figure 2. (Schultz, 1963, p. 85) This is a factor in the determination of the length of the nodal month, the time between successive crossing of the moon through the ascending node. For as the moon moves CCW through 360 degrees of its orbit, beginning at the ascending node, the node itself moves CW. Thus the moon arrives at the node before completing a full cycle of its orbit, so the nodal month as a little shorter than the sidereal month, by about 2 hours and a half.

Besides this rotation of the plane of the moon, there are more complex motions. If we give up the second of the simplifying assumptions at the top of section 2.2, and admit that the moon moves in an ellipse around the earth rather than a circle, we may then ask where are the perigee (closest approach of the moon to the earth) and the apogee (furthest point). The line connecting these two points define the major, or apsidal, axis of the ellipse, which passes through the center of the earth. Further complicating the picture, this major axis rotates around the center of the earth with a period, the apsidal month, rather close to the sidereal and nodal months.

3. LUNAR ASTROLOGY AND THE I CHING

There is an extensive literature on the psychological effects of the primary phases of the moon. See (Lieber, 1978) for a summary. We are going to follow Dane Rudhyar here.

3.1. The eight phases of Rudhyar

For the pedestrian observer of astrology, it seems intuitively that the moon must surely be the most powerful of the celestial influences on terrestrial life. Due to its eerie brightness, its closeness and mass, its tides and menses, and its rapidly changing faces, the role of the moon in our horoscopes and mundane affairs must be relatively enormous, comparable only to the sun among the wanderers in the sky. We observe the phases of the moon without the aid of an ephemeris, and there are ancient traditions regarding them. One thoughtful codification of the phases and traditions is presented in the book, *The Lunation Cycle*, by the astrologer Dane Rudhyar, who speculates that this scheme constituted the earliest astrology, preceding even the zodiacal cycle of the sun. The scheme of Dane Rudhyar is shown in Table 2, abstracted to keywords by Layla Rael Rudhyar in 1984. (Rudhyar, 1967, p. 146)

TABLE 2, Lunar phases of Dane Rudhyar

#, PHASE	LONG	KEY WORD
1. New moon	0 to 45*	Emergence
2. Crescent	45 to 90*	Expansion
3. First quarter	90 to 135*,	Action
4. Gibbous	135 to 180*	Evercoming
5. Full moon	180 to 225*	Fulfillment
6. Disseminating	225* to 270*	Overcoming
7. Last quarter	270 to 315*	Reorienting
8. Balsamic	315 to 360*	Release

Here * denotes degrees.

While this scheme was originally presented as a guide to the psychological significance of the phase of the moon at the birth of a person, we will abstract the key words as simply nominal characteristics of the eight phases.

3.2. The trigram cycles of the *I Ching*

Astrology is much more than a divinatory system, but it does have divinatory applications, collectively known as divinatory astrology. We now wish to speculate about the divinatory significance of the phase of the moon, rather than the predictive role of these phases in the birth chart of an individual. The *I Ching* is a divinatory system alternative to divinatory astrology, and it is based upon eight trigram symbols, each of which has an extensive list of characteristics. We are now going to correlate these eight trigrams with the eight primary phases of the moon. This will not be particularly difficult nor arbitrary, as the trigrams are traditionally connected in cycles in the ancient Chinese literature of the *I Ching*. In fact, there are two such traditional cycles.

First, there is the *Sequence of Earlier Heaven*. (Wilhelm/Baynes, p. 266) This is traditionally attributed to the legendary Fu Xi, before the compilation of the *I Ching* during the term of King Wen (1177-1122 BC) of the Zhou dynasty. (Lynn, 1994, p. 4) There is also the *Sequence of Later Heaven*. This is traditionally attributed to King Wen. (Wilhelm/Baynes, p. 269) Both sequences are labelled according to the four seasons, with time advancing in a clockwise rotation. But while the first sequence is a grouping by pairs of opposites, the second is intended to represent the cycle of the seasons. Thus, we will take this second sequence as the basis for correlation with the phases of the moon. It is shown in Table 3, with the names in the Chinese and English of Baynes (resp., Lynn). The order here is that given in the commentaries, which begin in the Spring.

TABLE 3, Trigram cycle of King Wen

#	TRIGRAM	NAME
1.	::/	Chen, Arousing (Zhen, Quake)
2.	//:	Sun, Gentle (Sun, Compliance)
3.	/:/	Li, Clinging (Li, Cohesion)
4.	:::	K’un, Receptive (Kun, Earth)
5.	://	Tui, Joyous (Dui, Joy)
6.	///	Ch’ien, Creative, (Qian, Pure Yang)
7.	:/:	K’an, Abysmal (Kan, Sink Hole)
8.	/::	Ken, Keeping Still (Gen, Restraint)

3.3. Trigrams and the primary lunar cycle

Now we have the primary cycle of the phases of the face of the moon, with the keywords of Dane Rudhyar, as shown in Table 2, and the Later Heaven cycle of trigrams, with the traditional keywords, as shown in Table 3. It only remains, then, to select the zero point, to correlate this cycle of trigrams with the phases of the moon. In fact, we see a natural correlation between the two tables as given. That is, we will associate the New Moon with the Spring Trigram. The resulting correlation is given in Table 4.

TABLE 4, Trigrams of the primary cycle

#, PHASE	KEY	TRIGRAM
1. New moon	0 to 45*	Emergence, ::/ Chen
2. Crescent,	45 to 90*	Expansion, //: Sun
3. First quarter	90 to 135*	Action, /:/ Li
4. Gibbous	135 to 180*	Evercoming, ::: K’un
5. Full moon	180 to 225*	Fulfillment, :// Tui
6. Disseminating	225* to 270*	Overcoming, /// Ch’ien
7. Last quarter,	270 to 315*	Reorienting, :/: K’an
8. Balsamic	315 to 360*	Release, /:: Ken

Here, * denotes degrees.

3.4. Trigrams and the secondary lunar cycle

We now wish to correlate the trigrams with the secondary cycle of the moon’ß•s ecliptic latitude, or equivalently, the solunar relative azimuth. As we would like this to result in a practical and usable correlation and we do not expect the user to observe moonrise every day, it will be helpful to base our eight-fold division of the nodal month on readily available data. Our choice here is the ecliptic latitude, which might be found in an ephemeris, or on the internet. We choose the passage through the ascending node as the zero point, and estimate the phase from latitude by assuming a circular lunar orbit. That is, a sinusoid in Figure 4. Dividing into 8 segments of equal time, we have 45* segments of phase, and hence the values of 3.8* of latitude in Table 4. This results in the correlation given in Table 4, combining the keywords of Dane Rudhyar and the trigrams of King Wen.

TABLE 5, Trigrams of the secondary cycle

#, DIRECTION, LAT	KEY	TRIGRAM
1. Ascending, 0 to 3.8*	Emergence	::/ Chen
2. Ascending, 3.5 to 5.4*	Expansion	//: Sun
3. Descending, 5.4 to 3.8*	Action	/:/ Li
4. Descending, 3.8 to 0*	Overcoming	::: K'un
5. Descending, 0 to -3.8*	Fulfillment	:// Tui
6. Descending, -3.5* to -5.4*	Overcoming	/// Ch'ien
7. Ascending, -5.4 to -3.8*	Reorienting	:/: K'an
8. Ascending, -3.8 to 0*	Release	/:: Ken

3.5. The hexagrams of lunar biphases

We may now combine the trigrams of the two phases into hexagrams. We choose to do this by superimposing the trigram of the secondary (latitudinal) phase on top of the trigram of the primary (synodic) phase. This is consistent with the tradition of the *I Ching*, which counts the bottom line as the first. For example, suppose the eightfold phase of the primary cycle is #5, Full Moon, which corresponds to trigram ://, Tui, according to Table 4, and that the eightfold phase of the secondary phase is #3, descending from 5 to 3.5 degrees of ecliptic latitude, which corresponds to trigram /:/, Li, according to Table 5. Then we superimpose /:/, Li over ://, Tui, obtaining the hexagram /:/://, K'uei, Opposition. This is #38 of the King Wen sequence, and may be found directly from the table at the end of the Wilhelm/Baynes edition of the *I Ching*. Arranging the result in a table, we put the number of the hexagram, #38, in row #5 and column #3. Continuing in this way, we obtain the results shown in the 8 by 8 matrix of Table 6.

TABLE 6. Hexagrams of the lunar biphases

UPPER LOWER	01	02	03	04	05	06	07	08
01	51	42	21	24	17	25	03	27
02	32	57	50	46	28	44	48	18
03	55	37	30	36	49	13	63	22
04	16	20	35	02	45	12	08	23
05	54	61	38	19	58	10	60	41
06	34	09	14	11	43	01	05	26
07	40	59	64	07	47	06	29	04
08	62	53	56	15	31	33	39	52

Recall that this matrix corresponds to cycles of time, and thus comprises a mapping of the two-dimensional torus into 64 rectangles. The vertical dimension of the matrix represents the primary lunar cycle of about 29.5 days, while the horizontal dimension corresponds to the secondary cycle of about 27.2 days, as shown in Figure 3.

3.6. The hexagrams of calendrical time

The changing phases of the moon trace a relatively straight line descending downward and to the right in Figure 3, and thus also in Table 6. If we have on hand fairly accurate values for the two phases of the moon at any given time, for example 25 December 1999 at 00:30 GMT, then we may predict the sequence of hexagrams of the moons phases by tracing the straight line at the constant slope. As this slope is irrational, our sequence of hexagrams is more or less chaotic, that is, nonperiodic. For example, beginning with the phases for 25 December above, when the full moon passed the ascending node, we would predict the sequence; 54, 61, 09, 14, 64, 07, 15, 31, 17, 25, and so on. The time durations of each hexagram vary widely. A more accurate sequence may be found from the ephemerides and observatories found, for example, on the internet, and marked on a calendar.

3.7. Applications to divination

Such a calendar of the moon's biphases might be regarded as a refinement of an ordinary lunar calendar, which shows only the primary phases of the moon. The refined biphasic calendar might be used as a prognostication device, by reading the commentaries on the lunar hexagram of the current moment from one of the translations of the *I Ching*. And to the degree that these interpretations are found useful, they may then be extended into the future, as a lunar time-based version of the *I Ching* oracle. Chaotic archeoastronomy revived, so to speak.

4. CONCLUSION

In rediscovering the prehistoric understanding of the irrationality of the lunar periods, as no doubt well-known to the archeoastronomers of Callanish or Stonehenge, we have recovered the prehistory of chaos theory. The two lunar periods being very short, the nonperiodicity of the moon's motion would have been much more apparent than the incommensurability of the solar and the solunar cycles. The association with astrology and the *I Ching* are traditional also, but these traditions are rather less ancient. The full significance of the secondary phases of the moon, like the astrology of Pluto, must be delineated through experience as time goes on. Our goal here will be served by bringing the latitude cycle of the moon, so important for megalithic society but nearly forgotten in recent millennia, back into popular consciousness. And implications for stock prices would be most welcome. (Landscheidt, 1988)

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