Neural Networks for Economic Prediction

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Dedicated to Jay Forrester and Richard Goodwin

CONTENTS

1. Introduction.

Neural networks were introduced by McCulloch and Pitts in 1943, and complex dynamical systems by Forrester in the 1950s. Richard Goodwin pioneered the application of these ideas in economics in the 1960s.

A complex dynamical system is a network of dynamical schemes with state spaces and control of continuous real variables, and either discrete or continuous time. This category includes analog recurrent neural networks as a special case, usually with linear nodes and weakly nonlinear links. Thus a complex dynamical system may be regarded as a radically nonlinear neural network.

The concept of directed learning that has evolved in the neural network context may be applied more generally to any complex dynamical system, to adjust its control parameters so as to approximate given data. Least squares approximation is one approach to directed learning. In this brief report of our joint project we explain these concepts and describe one example, a predictive model for the global economy.

2. Complex dynamical systems.

In this report we will consider discrete time systems only. Thus by a *simple dynamical system* we mean a *state space* together with a *map* from the state space into itself. The state space is a finite-dimensional euclidean space, and the map is a smooth map of the state space to itself. A *dynamical scheme* is a family of simple dynamical systems parameterized by a vector in another euclidean space, called the *control space*. A *complex dynamical system* is a network in which the nodes are dynamical schemes, and the links are smooth (or piecewise smooth) functions from the state space of one scheme to the control space of another, functions having their own control parameters, for example, weights.

There is an extensive literature, and useful software, for the applications of these systems to the various sciences, going back to Jay Forrester, and his model for world population and resources.

3. Analog recurrent neural networks.

The early work on neural networks considered binary nodes only: each neuron had only two states, on and off. By the 1980s, under the influence of new information from studies on biological neurons, analog neurons came to dominate the field. These networks usually involve very simple dynamical schemes as nodes, and very complex networks of connections, an approach known as connectionism.

Recent work in computer science has shown that these networks are more capable than digital computers. See (Siegelmann, 1999) for example. Among the tasks for which ARNNs excel is the prediction of time series data, as in the annual competition of the Santa Fe Institute.

4. Learning by least square approximation.

Given a complex dynamical system, one may fix initial states at each node, and initial values for any free control parameters, and begin iterating the entire system on successive ticks of a master clock. There results a time series of values for the states at the nodes, which may be compared with experimental data from a natural system. If the comparison is a good approximation, we have a mathematical model and computer simulation for the given data. Surprisingly, good approximations may be obtained by an automatic process of learning. One such process is least squares approximation, in which a problem in the calculus of variations is solved by a numerical algorithm to provide the control parameter values to best fit the computed data to the given, experimental data. For the core mathematics of this process, see (Cucker, 2002). For many examples of ARNN applications, see (Grossberg, 1989). The practical software for this process is found in the toolbox of every computational math programming environment: Maple, Gauss, Matlab, Mathematica, etc.

5. Application to the world economic system.

We have developed these ideas in a particular case for simulation and prediction of global economic data from national accounts. We used quadratic polynomials only for our maps, and linear functions for links. These maps were chosen for the richness of their bifurcation diagrams, as studied in (Abraham, 1997) for example. Given two nations and several (say 17) sectors in each nation, this results in a very large total space of control parameters for the entire complex. We used a data reduction scheme introduced by Punzo and Bohm. Least squares solution for the best choice of all these parameters for the available (rather short) time series from the published national accounts of Austria and Italy resulted in a rather encouraging simulation. Solution of the least squares problem is computationally intensive, but running the model with fixed values of the controls is very fast
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6. Conclusion.

Given the rapid evolution of supercomputer hardware and software for desktop machines, the massively parallel model proposed here is feasible for economic systems from a small business to the world economy. All that is required is good data (always a challenge) and time to massage the software and data.

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Punzo and Bohm???

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