# Emerging 4D Graphics for Math and Science Education

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## Abstract

This is a progress report from an ongoing research project conducted jointly by the Visual Math Institute and the Enter Network. The goal is an adequate visualization system, involving hardware and software, for the teaching of mathematics in schools and colleges, both in the classroom and over the internet. The level of graphics of interest here, 4D (that is, interactive and reactive spatial 3D animation), is particularly suitable for dynamical systems theory, including chaos theory, fractal geometry, and much of mathematical physics.

## 1. Introduction

The teaching of mathematics in the United States, over the past three decades or so, has become less and less successful. A majority of high school graduates are handicapped mathematically, and approach any math task, even simple arithmetic, with dread. This is well known, under the names math anxiety, math avoidance reflex, and so on.

The basic premise of the Visual Math Institute, since its inception over thirty years ago, is that math education is obstructed by the lack of adequate visual images, and that this is a major cause of math anxiety.

For learning arithmetic and plane geometry, static 2D images suffice, and textbooks could have, and should have, more of them. For learning solid geometry, 3D graphics can be very helpful. And for learning chaos theory, which is our primary focus here, real-time animations in stereo 3D are essential. We might refer to this level as 4D graphics. This has become practical only recently, with the development of affordable computer graphic hardware and software, such as Open GL and Java 3D, the implementation of solid modeling in math software such as Matlab, Maple, and Mathematica, and the evolution of fast stereo drivers with 3D graphics cards and state of the art spatial display technology.

Our work stands on the shoulders of giants, such as Tom Banchoff and Charles Schwarz of Brown University, who created Hypercube, one of the first animated math graphic films in the 1970s, and some of the great work that James Blinn did at JPL with computer graphic animation. We must acknowledge the revolutionary work of the late James White, in his Mathwright project, who first combined OpenGL graphics and math text processing into a math authoring system for teachers and students. We expand on his work in the areas of platform independence, program functionality, the ABM paradigm, and next generation display capability"

Meanwhile, the Enter Network has been developing the 4D graphic display hardware that may spearhead a breakthrough for math and science visualization to a new level of functionality and affordability. Inexpensive stereo 3D visualization for math education is now on the horizon, for chaos theory, fractal geometry, and the entire school science program. Most especially, the advent of fast binocular stereo 3D -- on large autostereographic displays that do not require glasses, and on affordable very large high resolution spatial display systems suitable for a lecture and requiring just passive glasses -- creates new opportunities for 4D graphics in a classroom setting. Our project demonstrates this evolution with an exemplary 4D learning module for chaos theory. To see 4D is to believe.

## 2. Our Software Strategy

Computer programming languages have evolved in fits and starts, with a rapid accumulation of new ideas such as the stored program in the 1940s, high level languages, object orientation, and in the 1990s, agent based modeling (ABM). ABM has occasioned a veritable revolution in the social sciences.

A research program applying ABM to science education in schools and colleges, the Center for Connected Learning and Computer-Based Modeling of Northwestern University (CCL) under the direction of Uri Wilensky, has created the NetLogo programming environment for ABM, along with a substantial number of exemplary models in various branches of science. In fact, in addition to these "library modules" distributed with NetLogo, there is a large user community developing models for research, that are posted to the community website.

The advent in March of this year of NetLogo with 3D visualization capability empowers the evolution of our experimental work into actual courseware for schools and colleges. NetLogo 3D is very easy to use, pure Java, and free. It utilizes JOGL for 3D animation, and is superbly platform independent. This recent development has triggered an intensive effort to interface NetLogo 3D with a fast autostereoscopic 3D display, and to develop models for learning chaos theory in NetLogo 3D. The result is a 4D learning module for chaos theory featuring the route to chaos in the Rossler system, a classic of chaos theory. Many similar modules may be rapidly developed by simple modifications of this first example. In fact, learning modules for the physical, biological, and social sciences, already developed by the CCL and other groups, may benefit from 4D display system, as well as the whole field of scientific visualization, and the emerging fields of digital arts and new media.

## 3. Our Hardware Strategy

Our installation will utilize a PC based Spatial media server using nVidia graphics quad buffer processors, driving both an Enter Network 50” Autostereosopic Display System (No glasses) that utilizes a high resolution parallax barrier LCD Display, and an Enter Network large scale stereoscopic rear projection screen system using duel SXGA projectors with passive polarizer glasses allowing groups of observers to view 4D. We are experimenting with a number of unique input devices for user selection and interaction in addition to reactive input technology. Participants can interact with a number wireless 3D controllers that include an, Ultra GT Cordless Optical Mouse features a unique "in-air" motion-sensing capability that allows the user to accurately control the cursor with movements more akin to waving a wand than using a mouse. We can enable some simple body gestures that can be recognized by a reactive machine vision input control system. This system uses a camera focused on the user with software based edge and boundary detection algorithms. The system detects the closest hand gesture point on a plane in front of the observed stereo compression space, for interacting with the spatial 4D visualization on a large format, spatial rear screen projection based display.