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A Digital Solution to the Mind/Body Problem

Abstract: We have applied the concepts from the mathematical theory of cellular automata -- as developed to understand the emergence of spacetime at Planck scale -- to consciousness. This gives rise to a digital, spacetime solution to the mind/body problem.

Keywords: Quantum vacuum, Planck scale, cellular automata, consciousness, mind/body problem.

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1. Introduction

Dualist-interconnectionist models for consciousness, from Ancient Greece to Descartes, have disjoint parts connected by a mysterious communication process. Usually no explanation is proposed for this communication process, although the resonance metaphor is sometimes mentioned. In this paper we consider this problem in the context of the mind/body model of Descartes. The intractability of this mind/body problem has been discussed by everyone from Plato on. We are going to apply to it an atomistic mechanism deriving from the theory of the quantum vacuum in modern physics.

We thus bring together the mind/body problem of Descartes and the digital philosophy of Fredkin and others (Hey, 1999) into a joint picture first described by Democritus (Popper, 1998). Our starting point is a cellular dynamical model of the quantum vacuum due to Requardt and Roy (2001) and extended by Abraham and Roy (2007). This is a process, the RRA process, by which the illusion of analog spacetime self-organizes from a digital substructure -- a submicroscopic, corpuscular, cellular dynamical system -- a sort of finite point set on steroids. In this paper we further extend the RRA process from space to spacetime in the domain of terrestrial physics, and then jump up to the mental and spiritual realms, where the constraints of physics no longer apply.

We apply the process twice, once to the mind, and again to the body, to obtain our resolution of the mind/body problem. In our final, composite picture, there is one enormous point set, its size estimated by Wheeler as 10 to the power 88 (Hey,

1999), operating beneath the perceived realities of macroscopic mind, body, and also quantum reality. We are grateful to Dr. Paul A. Lee for his guidance regarding the Ancient Greek tradition.

2. The Mind/Body Problem

The mind/body problem is a perennial thread in philosophy, East and West, so there are many illustrious names on its chronology. We will concentrate on just a few of these, to establish the main milestones of our story, and briefly describe their contributions. For the earliest history, beginning with Homer, see Jaspers (1998, Essay 8).

2.1. Plato, 370 BC.

Plato expanded the teaching of Socrates on the perfection of the soul into a complete system. In this system, morals and justice were based on absolute ideas. Wisdom consists of knowledge of these ideas, and philosophy is the search for wisdom. In fourteen more dialogues, Plato elaborated this unified system.

Plato's theory of soul is set out primarily in six of the dialogues: Phaedo, Republic II, and Phaedrus, of the middle group of dialogues, 387-367 BCE, Timaeus, around 365 BCE, which divides the middle, and last groups, and Philebus and Laws, of the last group, 365-347 BCE. The development of the individual soul is given in the three middle dialogues.

The Phaedo is a long and detailed examination of the individual soul, its immortality, and reincarnation, given by Socrates on the day of his death sentence. The Republic describes Plato’s mathematical curriculum for the Academy: arithmetic, plane geometry, solid geometry, astronomy, and music. At the end [10.614b] is the Tale of Er, which details the reincarnation process of the individual soul, as told by an eye witness. (The numbers in brackets are page numbers of the Stevens translation.) In the Phaedrus, Socrates and Phaedrus discourse on love, and on rhetoric. To understand divine madness, one must learn the nature of the soul. [245c] Soul is always in motion, and is self-moving, and therefore is deathless. [245c,d,e] Then begins the important metaphor of the chariot: two winged horses and a charioteer. [246a -- 248a] This metaphor of the soul is then used to explain divine madness, and the dynamics of reincarnation.

The world soul is developed in the later three dialogues. Regarding the individual soul, the Timeaus explains that as a person becomes a rational creature through education, his human soul moves in a circle in the head (a sphere) of his mortal body. [44] In the Philebus, Socrates introduces the world soul as the source of individual souls. [30a]

In sum, we have from Plato a four-level, hierarchical cosmology, including (from the top):

1. The Good, an integral principle with no spatial extent,
2. The Intellect, or nous, including the Ideas or Forms (pl. eide, sing. eidos),
3. The World Soul (including individual human souls), and
4. The Terrestial Sphere of matter and energy.

Forms exist in the Intellect, and are outside of space and time. Terrestrial objects are instances, or particulars, of Forms. Individual souls are pieces of the World Soul which have instantiated, or incarnated, a Form. When people die, their individual souls reunite with their Forms.

To this Theory of Forms, Plato himself raised an objection, in his dialogue, Parmenides. This problem, later called the third man argument, or TMA, has been the subject of much discussion over the past fifty years. It is somewhat like the Russell paradox of mathematical set theory. That is, if a Form (a class of objects) contains itself as a member, then an unwelcome infinite regress is set up, toward larger and larger collections.

Some have interpreted this objection another way, which we shall call TMA2. This applies when we have two categories which are disjoint -- such as two parallel universes -- and yet which exchange information. A matrix between the two categories -- such as the air between two resonant guitar strings -- must be interpolated, to carry the resonance or intercommunication. For example, in Plato's cosmology, the World Soul intervenes between the Intellect and the Terrestrial Sphere. Or on the individual level, Ficino's Spirit intervenes between the individual soul and the body.

2.2. Kashmiri Shaivism, 1000 CE.

The Indian tradition provides a number of different schemes for levels of consciousness, including five koshas, seven chakras, 36 tattvas, and so on. The five koshas are, from the top down: the bliss body (anandamaya kosha), astral body (vijnanamaya kosha), mental body (manomaya kosha) pranic body (pranamaya kosha), and the food body (annamaya kosha). The bliss body is described as an experience of total transcendence, where only the fundamental vibration of the unconscious system remains. (Saraswati, 1998; p. 54)

The TMA2 problem may be the ultimate cause of the profusion of levels in the Sanskrit literature on consciousness. No matter how many levels, the mystery of the communication between adjacent levels in the hierarchy remains. The vibration metaphor addresses this mystery, but still begs an encompassing matrix or medium to carry information from level to level. The vibration metaphor entered the Indian literature in the Spanda (vibration), Urmi (wave), and Prana (life-force) concepts of the Trika philosophy (Kashmiri Shaivism) due to Vasugupta, his disciple, Kallata, and his student in turn, Abhinava Gupta, tenth century CE. (Probhananda, 2003, 2004; Dyczkowski, 1992; Singh, 1980.)

We may regard the mind/body problem as just the bottom level of a stack of similar problems. We intend that our attack on the M/B problem should eventually be applied throughout the koshas, chakras, or tattwas of a full model of collective consciousness and unconsciousness.

2.3. Descartes, 1632.

Descartes was a dualist, to whom the world consisted of two original substances -- body and mind – between which there was an enormous gulf. Man consists of body and mind, which interact through the pineal gland. His dualist theory, and his mechanical view of nature, dominated philosophy for centuries. His method of thought and his theories have been subjected to devastating criticism, for example, Jaspers (1964). For many historians, the mind/body problem in Western philosophy began with Descartes.

3. Atomism

Like the mind/body problem, atomism is a perennial thread in philosophy, East and West, with many illustrious names on its chronology. Again, we will concentrate on just a few of them.

3.2. Parmenides, 450 BC.

According to Popper (1998), Parmenides -- an important if little known presocratic philosopher of early 5th century BC Greece -- was the creator of atomism (*Atomos*, Greek for indivisible). First of all, he is know for his Two Ways – the Way of True Knowledge (*aletheia*) and the Way of Human Conjecture (*doxa*) – revealed to him by a goddess and described in his only work, *On nature*. The Way of True Knowledge includes the idea that behind the false and illusory world of change perceived by the senses there is an absolute reality that is totally static, a dark sphere of continuous dense matter, called the Being. In our sensory perceptions, we experience a dual world of atoms moving in the void, hence the Way of Human Conjecture.

3.2. Democritus, 400 BC.

Democritus, a student of Parmenides, is widely regarded as the founder of the atomism thread. And it is said that Democritus' ideas were formed to contradict Parmenides. Democritus wrote on math, astronomy, and ethics, and had a great influence on later Greek philosophy, especially Aristotle, and hence, on the whole of the Western Tradition.

Regarding atoms, he believed that material bodies were formed as temporary composites of eternal atoms, like flocks of birds. Atoms are variously shaped and sized. The primary qualities of a material body – its shape, size, and weight -- and its secondary aspects -- smell, taste, etc -- all derive from the size and shape of its

atoms. Atoms move in a "void", which is empty, and yet is not nothing. The soul is made of soul-atoms, which are very small and spherical, and can pass through solid material bodies, like neutrinos.

3.3. Dharmakirti, 650 AD.

It is always a pleasure to follow a thread from Ancient Greece, through trade routes to India, then circuitously to Early Islam, and thence to Europe. In this case we are just guessing. There is a long history of atomism in India. One of the ancient Hindu philosophers, Kanad, discussed the existence of atoms. In fact, the word Kanad is derived from the word Kana, which means atom. Among Buddhist traditions, Vasubandhu and Dharmakirti particularly discussed the existence of atom. Dharmakirti was a student of Dignaga, a Buddhist logician, and then professor at the famed Nalanda University. He introduced in the thread a wondrous novelty, namely, that atoms are not eternal, but rather, flash into and out of existence as points of energy. This seemed somewhat outre until very recently, when the quantum vacuum emerged into physics, as we discuss in this paper.

3.4. Galileo, 1623.

Galileo was famously condemned by the Vatican in 1633, overtly because of supporting Copernicus (that the earth moves) in his book, *Dialogues concerning the two chief world systems*, published in 1632. However, there is a competing (and controversial) theory according to which his real offense was his earlier book, *The Assayer*, of 1623. (Redondi, 1987). This work advocated an atomic theory, according to which (rather like Democritus) the secondary qualities of matter (taste, smell, etc) were determined by the primary qualities (the shapes of atoms comprising the matter). This was of huge concern to the Vatican in that Transubstantiation -- the official dogma of the Church since the Council of Trent (1545-1563) regarding the consecration in the Mass of the Sacraments (turning the bread and wine into the body and blood) -- depended on secondary qualities being independent of primary qualities. (Shea, 1991; p. 181)

3.5. Quantum theory

Shortly following the early death of Descartes, Newton's universal theory of gravitation laid atomism to rest, where it remained for two hundred years. Then it rose from the ashes in a sequence of developments, collectively known as the quantum revolution. Here is a chronology of some of these developments.

1808, John Dalton posed a unique atom for each element.
1897, J. J. Thompson discovered the electron (Nobel prize in 1906).
1900, Max Planck proposed energy quanta, founded quantum theory.
1905, Albert Einstein introduced the photon asa corpuscle.
1927, Dirac, Pauli, Weisskopf, Jordan, Quantum field theory.
1940s, Feynman, Schwinger, Tomonaga, Quantum Electrodynamics (QED).

1966, H. Yukawa, Non-local Field Theory and Quantum Vacuum (QV).

At this point, following QED, we have the theories of QV and the zero-point fluctuation (ZPF) which are basic to the RR model of Requardt and Roy, (2001)l. This view of nature has the vacuum full of activity, in which particles jump out from, and then back into, the vacuum in pairs. In QED, as one calculates the transition amplitudes with respect to the vacuum state, the vacuum as such does not contribute in the calculations. However, Yukawa proposed the concept of non-local field theory where the seat of particles is considered as an extended region or domain in contrast to QED. Now if we take these domains to be quantum theoretical objects, then they are probabilistically connected, and there is no distinction between empty and occupied seats. Effectively, Yukawa introduced a new version of quantum theory of the aether with globular structure.

3.6. Fredkin, 2000.

The cellular automaton (CA) ideas of Stan Ulam and John von Neumann in the 1950s rested in obscurity until the appearance of John Conway‘s Game of Life in the 1970s. Then CA models of nature became a fad, and many successful models for macroscopic physical systems were made, especially in the circle around Feynman in the 1980s (Hey, 1999). However, computer science models of the individual soul, such as we seek, are rare. In this connection we must mention the work of Ed Fredkin. one of the pioneers of the digital philosophy, and the mainstay of the website www.digitalphilosophy.org which explains:

> *Digital Philosophy (DP) is a new way of thinking about the fundamental workings of processes in nature. DP is an atomic theory carried to a logical extreme where all quantities in nature are finite and discrete. This means that, theoretically, any quantity can be represented exactly by an integer. Further, DP implies that nature harbors no infinities, infinitesimals, continuities, or locally determined random variables.*

In "On the Soul" (2000 Draft Paper) Fredkin proposed a computer science definition of the soul, concluding: "The *soul* is an informational entity, which is constructed out of the states and the arrangements of material things."

All these recent developments, which we subsume under the classical heading *atomism*, support the idea that underlying our illusion of continuous space, time, matter, energy, etc (the analog part of the analog/digital dichotomy, and the wave part of the wave/particle duality) is a fundamental layer that is finite, discrete, and intelligent (that is, law-abiding). Sometimes all this is called *the finite nature assumption*. (Fredkin, 1992) This is close to the view of Parmenides described above.

4. The RRA model

In this section we recall the RRA process, as defined in (Abrahan and Roy, to appear). In the next section, we extend it from space to spacetime, and finally, we apply the process to the mind/body problem.

The RRA model is a two-level system. The microscopic level, QX, is a *dynamical cellular network* of nodes and bonds. Inspired by the cellular automata of Ulam and von Neumann, a dynamical cellular network is a directed graph with connections (directed links) which appear, disappear, and change direction, according to dynamical rules.

The macroscopic level, ST, that self-organizes from QX is an another dynamical cellular network, in which the nodes are the cliques (that is, maximal fully connected subgraphs) of a graph, G, of the QX level, bound in a network by superbonds.

The system of RR ends with a metric space. But in a sequel paper (Abraham and Roy, 2006) we have developed a neural network approach which imbeds the ST level into Euclidean spacetime, EST. Thus the ambient space of nature, according to consensual reality, is actually an epiphenomenon of the atomistic and finite QX network, according to the scheme: QX --> ST --> EST. This is the full RA process, which we call condensation. More details may be found in the Appendix.

5. The time dimensions

The discrete, microscopic time parameter, t, used above does not represent macroscopic time. Rather, we propose to obtain macroscopic spacetime through our process of condensation. Macroscopic time, T, exists locally as a function on spacetime, but may pretend that there is a cosmic time function, to simplify the exposition. We propose now to obtain macroscopic spacetime from the condensation process applied repeatedly to the entire, t-dependent QX object.

The condessnation process is regarded as being accomplished in a single instant, and it determines instantaneous states for the macrocosmic system in which space appears to be a continuum. Even so, the network, QX, is changing rapidly by a time-discrete process, with time t. We are going to regard the stepwise increasing network time as an internal process variable, microscopic time, that is distinct from the continuous physical time aspect of the spacetime of general relativity, cosmic time. Thus, we envision two dimensions of time.

We adopt the Cauchy perspective of general relativity, in which the Einstein equation is regarded as a system of quasi-linear, second-order partial differential equations. The Cauchy initial value problem for this system regards the past and present as known, and the future to be determined by integration the system along characteristic directions. The topology of spacetime, along with the geometry

(metric tensor) and the physical parameters (energy, mass, electromagnetic fields, etc.) must evolve according to this Einstein equation. Wormholes and black holes may evolve as caustics of the characteristics of he Einstein equation.

Alternatively, for a mathematically less-challenging exposition, we may suppose, like Einstein, that spacetime is created as a finished system, a complete pseudo-Riemannian geometrical object.

So this is our proposal for the emergence of cosmic time. Constrained by the Einstein equation, cosmic time advances in discrete intervals, that might be multiple steps of microcosmic time, giant steps. With each giant step, yet another condensation occurs, as follows.

We consider a memory device, controlled by the cosmic-time function, T. Between cosmic times T1 (corresponding to network time t1) and T2 (with its t2) the memory device records all of the finite states of QX between network-time t1 and network-time t2, and condenses this finite set of QX states into a spacelike continuum corresponding to the discrete cosmic time T2. One method for the condensation of a finite set of QX states is the sum algorithm. That is, we form a QX sum-state by adding the internal node states of all nodes, and all the bond states of all the bonds, of the set of QX states. In other words, fix a node of QX. Sum up the node-states of that one node for all the QX stats with network time in the interval, (t1, t2], that is an integer. Do likewise for each bond of QX, but round down if this sum is greater than one, and round up if less than minus one.

Thus, spacetime is squeezed from the dynamical cellular network, QX, as toothpaste from a tube. As giant steps are still very small compared with the resolving power of macroscopic science, cosmic time appears to be continuous. The macroscopic system, QX, sparkles with activity on the scale of Planck space and time, while macroscopic spacetime unrolls essentially continuously. The past and present become known, while the future remains yet a mystery.

In summary, our scheme, QX → ST → 3ST is extended to the scheme QX → ST → 4ST, all in the context of the body, that is, the physical world. We now wish to apply this new scheme to the mind/body problem.

6. The mind/body problem resolved

We now consider two QX networks: QX1 (the body level), QX2 (the mind level). Each of them might be the basis for an RRA process, one condensing to the body, or the physical world as we have considered up to this point, the other to a separate world of the mind.

However, we may prefer alternatively to join QX1 and QX2 into a single entwined network, QX*, on which two condensation processes operate. We might compare this approach to John Whitney's concept of digital harmony, in which a single

mathematical algorithm is employed to compose a piece of music, and an abstract animated image, which then seem — when played together – to harmonize, due to deriving from a common archetypal process. But we will proceed now with QX1 and QX2.

After all this preparation, our approach to the perennial conundrum is now simple: we apply the idea of condensation from a QX network twice: once to the body level, as in the RRA model, and again by analogy to the mind level, as in Fredkin (2000). This results in the four-part scheme:

QX2 --> Mind
QX1 --> Body

The mystery connection between the disjoint mind and body systems now becomes an epiphenomenon of the connection between QX1 and QX2 which is not mysterious at all. For the nature of the QX model of RRA is that of a dynamical cellular network, and we may regard QX1 and QX2 as a single, entangled network, as directed links between the two systems will be allowed by our dynamical rules.

7. Conclusion

In sum, then, the mind/body connections are completed in a circuit outside ordinary consensual reality in a submicroscopic atomic realm beyond our senses, but revealed by the progress of modern physics. This realm or matrix, an extension of the quantum vacuum into the realm of consciousness, is a finite, discrete, digital, cosmos, which condenses -- in the human perceptual and cognitive process -- into epiphenomena, the continuum illusion of mind/body, hypostases, koshas, cakras, tattwas, and so on, of the perennial traditions of consciousness studies.

Note that the QX level is a static point set with a dynamic network structure, changing in microscopic time, t. Meanwhile, the macroscopic body and mind have been constructed as complete spacetime worlds, with locally defined macroscopic times, T. This provides a background for psi phenomena such as telepathy and clairvoyance, but also leaves a window of opportunity for free will. Like a zipper closing, the past is zipped (or firmed) up, while the microscopic future is subject to interaction with the macroscopic body and mind, until the zipper closure arrives, and condensation (or collapse) occurs.

The end of our construction is an echo ofa the Two Ways of Parmenides, the atomic QX*, and the ST4 continua of body and mind, playing out in digital harmony.

Appendix. Summary of the RRA process.

The RRA process, of Abraham and Roy (to appear) is not a description of physical reality, but just a mathematical model that captures some aspects of our experience

of physical reality. We will summarize this process in three stages. Full details, examples, and graphics, may be seen in Abraham and Roy.

(A) We begin with a description of our microscopic system, QX.
(B) Then we will go on to extract from it our macroscopic system, ST.
(C) Finally, we describe in summary the embedding of ST into an Euclidean space.

A1. There is a finite, but huge, point set, which is static throughout the process. Let S denote this finite set. Enumerate this set by fixing a bijection from S to N, the cardinality of S. Thus, S is a set of points, {n_0, n_1. ..., n_(N-1)}. These points are called nodes.

A2. At each node and each moment of time there is an internal node-state, which is some number of quanta of information. Thus, we have a set of time-dependent node-states, {s_0, s_1, ..., s_(N-1)}.

A3. There are no bilateral connections. That is, for each pair of nodes, n_i and n_j, there may be a directed link from n_i to n_j, or none. We agree there cannot be a directed link from n_i to n_j if there is one from n_j to n_i. That is,.

A4. There is a global time clock for the system. The time variable, t, is a natural number, and increases by one at regular intervals, called clicks.

A5. The directed links may appear, disappear, or change direction, with each click. They change according to a fixed dynamical rule.

A6. With each click, each node n_i sends one quantum of information to the node n_j if there is a directed link from n_i to n_j.

A7. At each time there is digraph, a directed graph on S, defined by the directed links. Let D(t) denote the state of this digraph at clock time t, an integer. Associated with D(t) is a graph G(t), in which the directions of D(t) are ignored.

This is our microscopic system, QX, exactly as described by Requardt and Roy (2001).Next we will describe the emergence of the macroscopic ST system from QX, QX → ST, following Abraham and Roy (to appear).

B1. For each node, n_i, of D(t) let w_i denote its node-weight, that is, the number of directed links of D(t) that either arrive at, or depart from, n_i. Thus, we have a finite sequence of node-weights, {w_0, w_1, ..., w_(N-1)}.

B2. Next, at each time, t, we may construct, from the digraph D(t), a permutation of the set S of nodes, as follows. We reorder the nodes of S according to their node-weights, in decreasing order. If several nodes have the same node-weight, we retain their original order. Let P(t) denote the permutation of N obtained in this way.

B3. A clique of a permutation is a maximal inverse sequence. Compute the cliques of P(t). This may be done by inspection if N is not too large. Let K(t) denote the set of all cliques of P(t). These cliques, which are simply subsets of {0, 1, ..., n-1} in decreasing order, will be considered the supernodes of our macroscopic system, ST.

B4. If K is a finite set of natural numbers, let the span of K denote the filled-in interval, span(K) = [min(K), max(K)]. We define a superbond between two supernodes, or cliques, if and only if their spans are disjoint. Thus we have a graph ST(t) defined by these supernodes and superbonds.

This is our macroscopic system, ST. Finally we will describe the pseudo-isometric embedding of ST into a Euclidean space, ST ➔ EST, again following Abraham and Roy (to appear).

C1. For every pair of disjoint cliques of K(t), we define their overlap, a measure of the entanglement of the two cliques, by counting points in the intersection and union of the sets spanned by the two cliques. Details and examples may be found in Abraham and Roy (2008). These overlap measurements may be used to define distances: more entanglement corresponding to a smaller distance.

C2. Embed K(t) in a Euclidean space, and relax the embedding to approximate as closely as possible an isometry. That is, the distance between the images of two cliques represents their entanglement.

The process QX → ST → EST may be called *condensation*.

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