The emergence of geometric thinking

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Abstract

Here we seek the earliest occurrences of geometric thinking in prehistory, leading up to the evolved geometry of the late paleolithic painted caves.

CONTENTS

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1. Introduction

As a foundation for our study of the origins and development of the symmetric patterns decorating the walls of medieval Islamic palaces, we have sifted through prehistoric archeology looking for precursors. Except for the amazing art of the hand axe, most of our findings date from the arrival on the stage of our history of anatomically modern humans, some 100,000 years ago.

We propose to interpret the earliest emergence of geometric thinking in the style of phenomenology, that is, in the context of embodied mathematics.. So, we will begin with a brief excursion on this context. And then, as a scaffold on which to attach the artifacts of early geometric thought, we have chosen one of the many scenarios of human prehistory. the one due to Stephen Mithin.[1] Next we describe very briefly the earliest known example of geometric thinking, the hand-axe of Homo erectus, around 1.5 MYA. Then, we present a chronology of the art of Homo erectus from their cave sites, including an examination in more detail of one of these sites (possibly of Homo sapiens), the Blombos cave of southern Africa, bringing us to the approximate date of the *big explosion* of art and religion.

2. Embodied mathematics

The phenomenological tradition in Western philosophy has a long line, but emerged as a major domain with Friedrich Nietzsche (1844-1900), Edmund Husserl (1859-1938), Martin Heidegger (1989-1976), and Maurice Merleau-Ponty (1908-1961).

Merleau-Ponty's books *The Structure of Behavior* and *Phenomenology of Perception*, originally published in French in 1942 and 1945, although still influential, preceded the computer revolution, and the enormous growth of cognitive science.

After a gap of more than half a century, in *The Embodied Mind* of 1993, Varela, Thompson, and Rosch radically updated Merleau-Ponty to the current state of the art of cognitive science, and further, bridged the mind-body problem with Buddhist psychology and meditation practice.

In this book, the authors express the essence of this updated theory in the context of the question, *What is the role of reflection in the analysis of experience?*

[1] See (Mithin, 1996) for the full playscript.

> This question brings us to the methodological heart of the interaction between mindfulness/awareness meditation, phenomenology, and cognitive science. What we are suggesting is a change in the nature of reflection from an abstract, disembodied activity to an embodied (mindful) open-ended reflection. By *embodied*. we mean reflection in which body and mind have been brought together.[2]

A few years later, this updated theory was applied to the cultural emergence of mathematics by George Lakoff and Rafael E. Núñez, in *Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being* (2000). This book includes detailed analysis of some math topics, mostly algebraic and symbolic.

We propose, in this article, to consider the origins of primitive geometry from the perspective of the theory of the embodied mind.

3. The Mithin chronology

The further we peer into the remote past, the more story there is to tell. And the earliest events continue even now to emerge from the soil, under the restless spades of archeology. A recent scenario for the migration out of Africa to the East (Japan and Australia) and late back into Europe – called the single exodus theory – is argued in much detail (including the genetic evidence) in Stephen Oppenheimer's *The Real Eve: Modern Man's Journey Out of Africa*. The journeys of early Homo erectus, 2 MYA, are shown in Figures 1 and 2.[3] Controversies over this scenario abound.[4]

For our present purpose, we may pick up the story about six million years ago (6 MYA). A linear timeline for this epoch is shown in Figure 3. Here, time is increasing upwards, or rather decreasing downwards, as it is experienced by archeologists, digging down from the present through deep strata of the past.

Mithin's Act One, from 6 MYA to 4.5 MYA spans the Miocene/Pliocene boundary.

His Act Two, from 4.5 to 1.8 MYA, is divided into two scenes at 2.5 MYA. The first of these spans the Pliocene/Pleistocene boundary at 2.8 MYA. The second includes the development of stone tool industries.

[2](Varela, Thompson, and Rosch, 1993; p. 27)
[3](Oppenheimer, 2003; pp. 70-71 and 86)
[4]See, for example, (Gamble, 2004), (Gowlett, 1984), and (Renfrew, 2007).

Act Three of Mithin's chronology opens with the appearance of Homo erectus around 1.8 MYA, with their signature tool, the hand axe. This act coincides approximately with the Paleolithic, or Old Stone Age, and the formation of the ice sheets. This act is also divided into two scenes. In the first scene, the Lower Paleolithic, Homo Erectus expands from East Africa into Southeast Asia. And from 500 KYA (500,000 BP), our ancestors occupy Europe. In the second scene, the Middle Paleolithic from 200 KYA, we find the emergence of early Homo Sapiens, along with early language and a highly developed social intelligence. Act Three closes with the emergence of fully modern humans, around 0.1 MYA (100 KYA, or 100,000 BP).

Act Four, the story of our modern ancestors, is divided into three scenes. A linear timeline for this act is shown in Figure 4. In the first scene, continuing the Middle Paleolithic from 100,000 BP, we have the emergence of Homo sapiens sapiens in southern Africa and the Middle East. In the second scene, we have the arrival of humans in Australia by boat from 60,000 BP, and the upper Paleolithic from 40,000 BP, with the arrival of Homo sapiens in Europe, the painted caves and ceremonial centers, up to the Holocene Interglacial around 12.000 BP, and the recession of the glaciers. In the third and final scene, we have the Neolithic, with agriculture, towns, cities, etc.

It is these two scenes of Act Four that form the backdrop for most of our investigations of the emergence of geometric thinking. The only exception is the hand-axe phenomenon of Act Three, to which we now turn.

4. The hand axe

When I was a boy growing up in Vermont, winters were more severe than at present. We used to say Vermont has two seasons, Winter and July. Of the two, I preferred Winter. Thanks to skis and skates, I loved the snow and ice. We used to dig caves in giant snowdrifts, and of course, snowmen made of giant snowballs. And snowball fights were frequent.

There is something highly educational, or developmental, in crafting a good snowball. Size, weight, density, fit to the hand: all self-learned, as our ice-age ancestors did for a million years or more. The expertise and fun of it are carried in the morphic field of our line. And during the rare Gaian fevers, or interglacials, as the ice receded, we used clay as a substitute for snow.

Perhaps this part of the story of the evolution of the hand-axe from earliest appearance in the heyday of Homo erectus, in Mithin's Act Three. 1.5 MYA, in the Olduvai gorge in Tanzania, East Africa, between Dar es Salaam and Lake Victoria. This was the time of the Lower Paleolithic, and the beginning of the ice age. From first appearance, the hand-axe spread over space and time, reaching Europe and East Asia, and was still in use around 40,000 BP.

The hand-axe was a stone tool, the first as far as we know, held in the hand for use as an axe for butchering, digging, shaping wood, throwing, and so on. In fact, it was skillfully shaped to comfortably fit the hand of the user. A fine example, kept in the British Museum, is shown in Figure 5.

Because of the symmetry common to many of them, the hand-axe is associated with the beginning of the artistic sense of our species. And also, because of the symmetry, we are proposing the hand-axe as evidence of the emergence of geometric thinking. While arithmetical thinking – in the form of subtilizing, known to birds, fishes, and mammals including early primates – has no disscernale origin, it seems that geometric thinking appeared on the human stage in the Lower Paleolithic. This emergence is the epitome of embodied mathematics.

Some degree of symmetry would have been possessed by the stone nodules from which the axes were knapped, and this may be regarded as an example of the embodied mathematics of nature. However, the high degree of symmetry found in many of these worked stones indicates a special reverence for aesthetics. This is the source of the controversial *sexy hand-axe hypothesis* of Marek Kohn and Stephen Mithin, according to which the stone sculptures were used not only as tools, but also as ornaments to attract the opposite sex.[5]

Beyond the early mathematics and art of the hand-axe, the further development of geometric thinking becomes manifest primarily in the painted caves of the Homo Sapiens of the Upper Paleolithic.

5. The cave chronology

The earliest hand-axes presently known, found in the Olduvai gorge during Mithin's Act Three around 1.5 MYA, were the product of Homo erectus. The further extant art of Homo erectus include:

[5]See (Mithen, 2005; p. 188).

- 700,000 BP, Petroglyphs of Auditorium Cave, Bhimbetka, (India)
- 700,000 BP, Venus of Berekhat Ram, oldest sculpture (Isreal)
- 500,000 BP, Venus of Tan-Tan (Morocco)

Outside of the hand-axe, we find no other evidence of geometric thinking (symmetry) among these artifacts.

Subsequently, the Neanderthals diffused from around 500,000 BP, and were replaced by modern humans by 100,000 or 50,000 BP.[6] During this transition, as language evolved to its full functionality, we find these sites:

- 100,000 BP, Blombos cave, southern Africa, symmetric markings
- 44,000 BP, Lebombo bone, southern Africa. markings may be a lunar calendar
- 41,000 BP, El Castillo cave, northern Spain, oldest cave paintings
- 40,000 BP, Maros caves on the Indonesia island of Sulewesi, very early cave paintings
- 40,000 BP, Hohle Fels, Venus figurine, Germany
- 40,000 BP, Hohlenstein-Stadel, Lion Man figurine, Germany

These later sites are considered to be the work of Homo sapiens.[7]

- 37,000 BP, Chauvet-Pont-d'Arc cave, southern France,, early cave paintings, symmetries
- 35,000 BP, Vogelherd cave, oldest known animal carving, symmetric markings
- 32,000 BP, Blanchard bone, southwestern France, may be a lunar calendar
- 27,500 BP, Apollo 11 cave, Africa, painted slabs, oldest mobile art
- 22,000 BP, Ishango bone, Africa, may be a lunar calendar
- 20,000 BP, Xianrendong cave pottery, China, oldest ceramic art
- 9500 BP, Siberian wooden statue, 9500 BP, symmetric marks

[6]Approximate dates, some are controversial. When a range of dates is given in the literature I have chosen the oldest.

[7]For a more complete listing, see http://www.visual-arts-cork.com/prehistoric-art-timeline.htm.

Following the hand-axe, the next appearance of symmetry in prehistoric art is found in the Blombos cave site, to which we now turn.

6. The Blombos Cave

On the southern coast of South Africa about two hundred miles east of Cape Town, the Blombos cave was found to contain Middle Paleolithic artifacts dating from about 100,000 BP. These findings include a wide range of tools, beads and other materials, two of which are of special interest to our quest: Still Bay points, and engraved ochre.

Still Bay points

Still Bay (also called Stilbay or Stillbaai) is a town on the South African coast, and also the name given by archeologists, in 1929, to a stone tool industry localized in its vicinity. This industry produced very beautiful bifacial points. More than 500 examples have been found in the cave, and some have migrated as far as Olduvai Gorge, 2300 miles to the north. These are relatively small, about one or two inches in length, compared to the similarly shaped hand-axes of the Lower Paleolithic. It seems they were hafted, and used on spears or knives. In general they are two-pointed, and thus exhibit more symmetry than hand-axes. Some are shown in Figure 6.[8]

Engraved ochre

Ochre (also spelled ocher) is a natural earth mineral containing iron oxide, from which derives its color: from yellow to orange, red, and brown. From early times it has been use as pigment for painting. It is widespread in African Paleolithic sites, and thousands of pieces have been recovered from the Blombos cave. Some of these, dating from 100,000 BP, are engraved with patterns exhibiting striking symmetries. One such is shown in Figure 7.[9] The paleolithic use of red ochre in burials is well documented, and use as a display during menstrual rituals has been proposed.[10]

[8] From en.wikipedia.org/wiki/Blombos_Cave.

[9] Also from en.wikipedia.org/wiki/Blombos_Cave.

[10] (Watts, 2002) and (William Irwin Thompson, 2013; pp. 58-59)

For our story, the significance of this artifact is its engraving, traced in whtte in Figure 7. This drawing shows a symmetry under horizontal translation, and both horizontal and vertical reflections. It has frieze symmetry, and is found elsewhere in the archeological record, from many cultures and times. An example from a peat bog in Siberia, 9500 BP, is shown in Figure 8. It is the oldest (and tallest, at 16 feet) wooden statue.[11]

7. Conclusion

This concludes our search for early geometric thinking in the ice-age records. Following the very early example of the Lower Paleolithic hand-axe of Homeo erectus, 1,500,000 BP, there followed an enormous lapse before the next example appears, the incised ochre of Middle Paleolithic Homo sapiens, 100,000 BP. Both occurred in Africa. This evolution paved the way for the amazing symmetries of the cave paintings of Europe, from 37,000 BP onward. We may regard the Blombos cave as a marker for the beginnings of the "big bang", or "symbolic explosion", remarked by many observers as a major cultural bifurcation, in which human art, song, dance, religion, and perhaps psychedelic use effloresced.[12]

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[11] See siberiantimes.com/science/casestudy/features/is-this-the-worlds-oldest-secret-code/.

[12] See, for example, (Pfeiffer, 1982), (Knight, 1995), (Spivey, 2005), and (Mithin, 2007).

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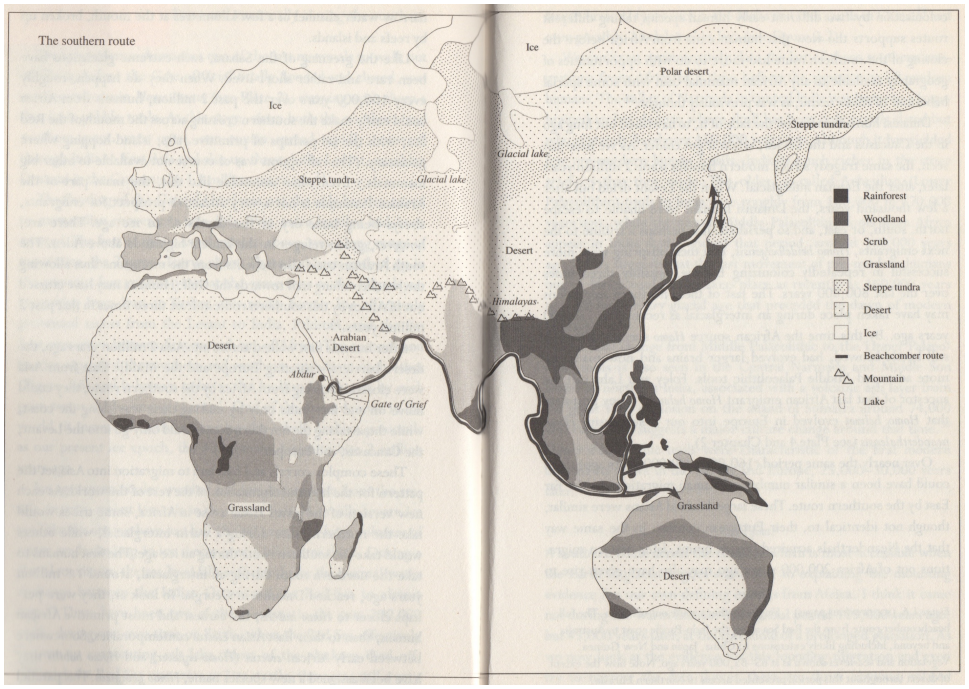

Figure 1: Out of Africa, the southern route.

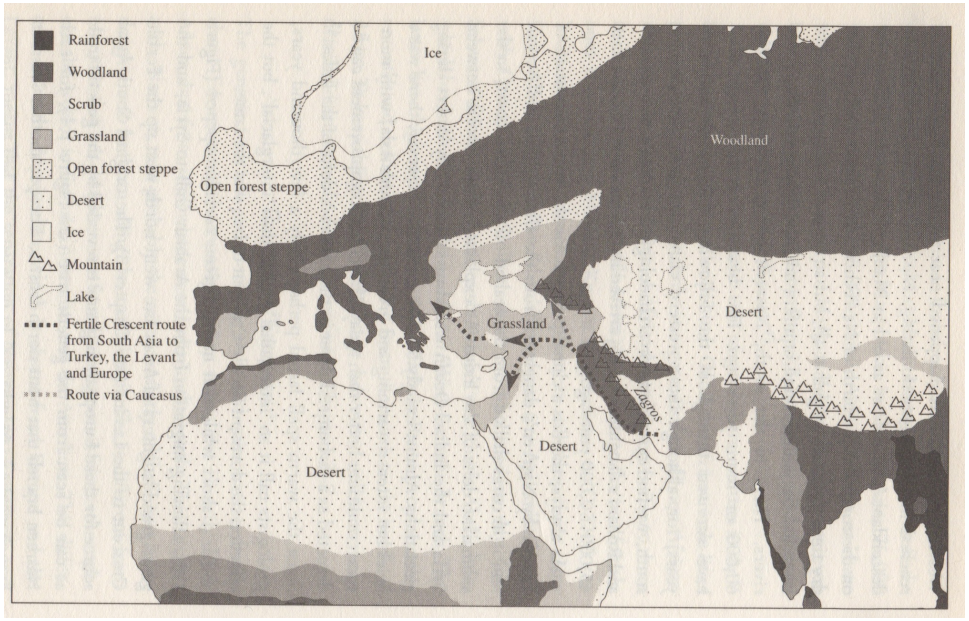

Figure 2: Out of Africa, the Fertile Crescent route.

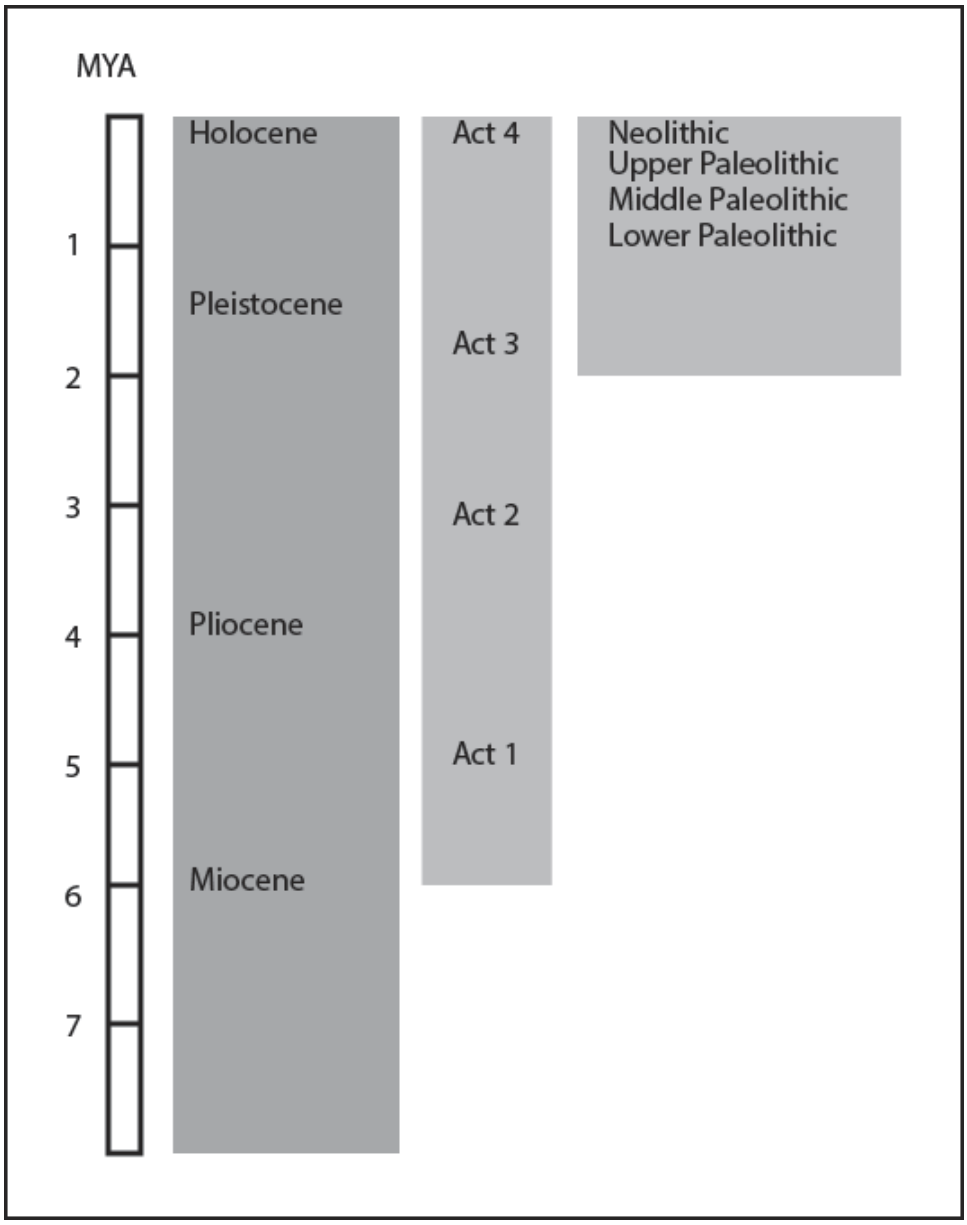

Figure 3: Timeline of Mithin's framework for prehistory, in four acts, from 8 million years BP to the present.

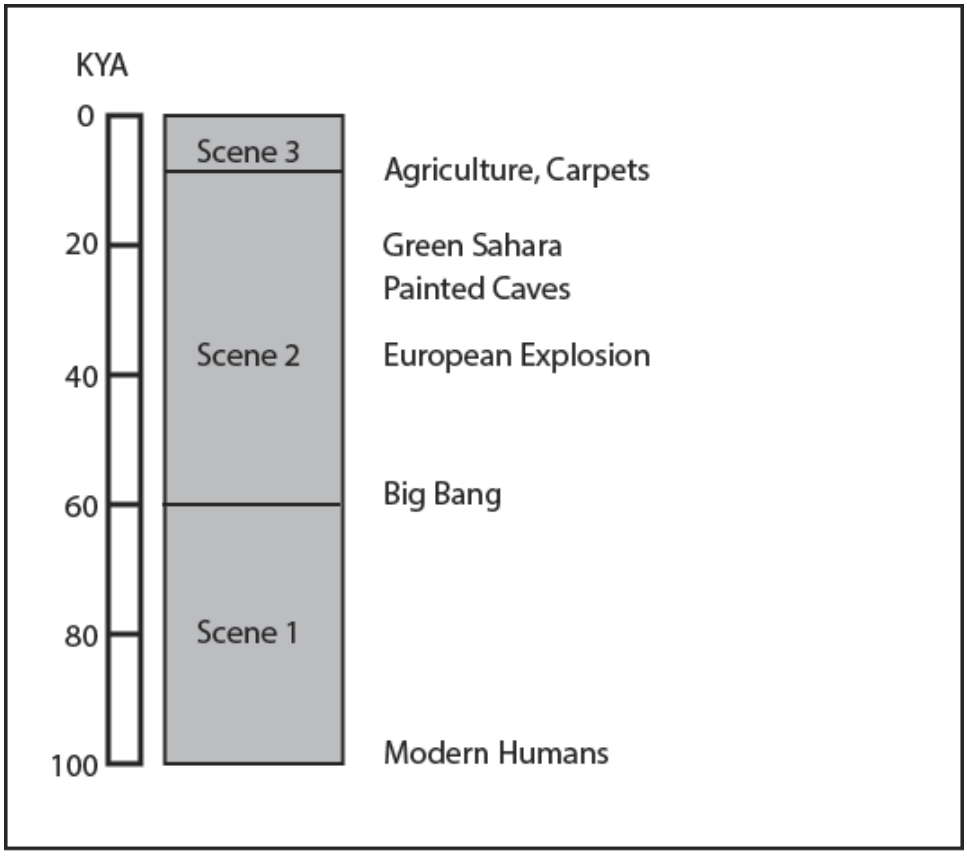

Figure 4: Timeline of Mithin's framework for prehistory, Act 4, from 100 thousand years BP to the present

Figure 5: Hand-axe, Lower Paleolithic, 1.2 MYA, Olduvai Gorge, British Museum.

Figure 6: Still Bay Points, 80,000 BP, from Blombos Cave.

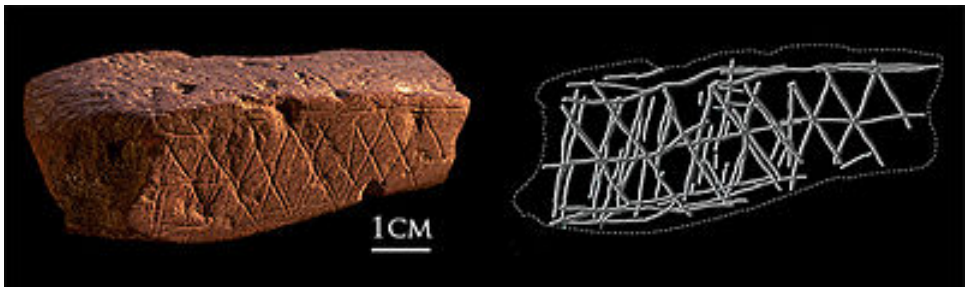

Figure 7: Engraved ochre, 100,000 BP, from Blombos Cave.

Figure 8: Engraved wooden idol, 9500 BP, Siberia.