Mathematics and Mysticism

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23 July 2015

Abstract

Is there a world of mathematics above and beyond ordinary reality, as Plato proposed? Or is mathematics a cultural construct? In this short article we speculate on the place of mathematical reality from the perspective of the mystical cosmologies of the ancient traditions of meditation, psychedelics, and divination.

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1. Introduction

There is a long tradition of speculation regarding the origins of mathematics. This question is similar to those of the acquisition of language, art, religion, and so forth. For Plato and his followers there was a parallel universe in which mathematics resides, now and forever. But for the philosophical phenomenologists of the 19th and 20th centuries, mathematics is a cultural artifact, in coevolution with the human mind and culture. The dialectic of these two opposing views is essentially unresolvable, and devolves into a question of faith: where does mathematics come from?

The neurophenomenological position proposed most notably by George Lakoff and Raphael Nuñez (2000) is based on a detailed study of a few topics from classical mathematics. But since the computer graphic revolution of the 1960s, new branches of contemporary mathematics – such as chaos theory and fractal geometry – have come to light. Fractal structures — having a long and unquestioned existence before being introduced into the visual realm and hermeneutics of human apprehension and discussion — have complicated the arguments.

In this 21st century, a similar conundrum has emerged following the psychedelic revolution: what is the location and origin of the psychedelic state of consciousness? And taking these two questions in hand, what is the relationship of these two parallel universes? We are going to examine these questions now from the perspective of Neoplatonic cosmology, and from the viewpoint of personal experience with this cosmological model. Specifically, we will discuss mathematical research practice in three related practices: meditation, psychedelics, and divination.

It is a pleasure to thank Plamen Simeonov and Otto Rössler for instigating this article, and Frederick Abraham, Frank Galuszka, Paul Lee, Anjani O'Connell, Ray Gwyn Smith, Rick Strassman, and Christina Waters, for useful comments along the way.

2. Mysticism

Mathematics is a field that depends words, signs, and symbols with precise definitions. Yet mysticism is a field that is rife with words of vague definition. Let us begin with glossary arranged not alphabetically, but rather in hierarchies.

2.1 Definitions

The very word, mysticism, is mysterious; that is, it has a vey loose definition. For example:

- *Mystery*: A religious truth that one can know only by revelation and cannot fully understand. Also, a secret religious rite believed (as in Eleusinian and Mithraic cults) to impart enduring bliss to the initiate, or, a cult devoted to such rites
- *Mysterious*: Of, relating to, or constituting mystery.[1]
- *Mysticism*: The experience of mystical union or direct communion with ultimate reality reported by mystics.[2] A religious practice based on the belief that knowledge of spiritual truth can be gained by praying or thinking deeply.[3]

If you chase down the definition of mystic, you may be caught in an infinite loop.

Here I am going to simplify this ambiguity, by claiming my own limited definition of mysticism. I shall consider only three types of mystical experience, those with which I have my own considerable experience: meditation, psychedelics, and divination.

But first, let us have a look at the usage of the term by scientists.

2.2 Science and mysticism

An excellent study of this connection was published in 2001 by Ken Wilber. It is an edited compilation of the most relevant writings of eight eminent physicists of the quantum revolution of the early 20th century, together with well-informed critical comments and evaluations. Included are:

- Max Planck, 1858–1947,
- James Jeans, 1877–1946,
- Albert Einstein, 1879–1955,
- Arthur Eddington, 1882–1944,

[1] from: http://www.merriam-webster.com/dictionary.
[2] from: http://www.merriam-webster.com/dictionary.
[3] From http://www.merriam-webster.com/dictionary/mysticism.

- Erwin Schroedinger, 1887–1961,
- Louis de Broglie, 1892–1987,
- Wolfgang Pauli, 1900-1958, and
- Werner Heisenberg, 1901–1976.

Omitted from the volume, but perhaps of interest, was,

- Paul Dirac, 1902–1984).

Within his commentaries, Wilbur gives these credits:

- Eddington, most eloquent, accomplished philosopher, penetrating mystic,
- Schroedinger, keenest mystical insight,
- Pauli, smartest, and
- Heisenberg, most philosophic,

In my own pantheon, Einstein leads the pack in all categories.

According to Einstein:

> The most beautiful emotion we can experience is the mystical. It is the power of all true art and science. He to whom this emotion is a stranger, who can no longer wonder and stand rapt in awe, is as good as dead. To know that what is impenetrable to us really exists, manifesting itself as the highest wisdom and the most radiant beauty, which our dull faculties can comprehend only in their most primitive forms- this knowledge, this feeling, is at the center of true religiousness. In this sense, and in this sense only, I belong to the rank of devoutly religious men.[4]

From Eddington, Wilber includes three pieces. Within the first (1929) we find,

> . . . those who in the search for truth start from consciousness as a seat of self-knowledge with interests and responsibilities not confined to the material plane are just as much facing the hard facts of experience as those who start from consciousness as a device for reading the indications of spectroscopes and micrometers.[5]

and

[4]First published in 1931. Quoted in (Frank, 1947; ch. 12, sct. 5).
[5](Wilber, 2001; p. 192)

> Surely then that mental and spiritual nature of ourselves, known in our minds by an intimate contact transcending the methods of physics, supplies just that interpretation of the symbols which science is admittedly unable to give.[6]

And from the third (also 1929) we find,

> A defense of the mystic might run something like this. We have acknowledged that the entities of physics can from their very nature form only a partial aspect of the reality. How are we to deal with the other part? It cannot be said that that other part concerns us less than the physical entities. Feelings, purpose, values, make up our consciousness as much as sense impressions. We follow up the sense impressions and find that they lead into an external world discussed by science; we follow up the other elements of our being and find that they lead not into a world of space and time, but surely somewhere.[7]

Finally, from Schroedinger, Wilber includes four pieces. Within the second (1958) we find,

> But, of course, here we knock against the arithmetical paradox; there appears to be a great multitude of conscious egos, the world, however, is only one. ...
>
> There two ways out of our number paradox, both appearing rather lunatic from the point of view of present scientific thought (based on ancient Greek thought and thus thoroughly "Western"). One way out is the multiplication of the world in Leibniz's fearful doctrine of monads: every monad to be a world by itself, no communication between them; ...
>
> There is obviously only one alternative, namely the unification of minds or consciousnesses. Their multiplicity is only apparent, in truth, there is only one mind. This the doctrine of the Upanishads.[8]

In these writings, we find the acknowledgment of two-level map of consciousness: the matter and energy universe of physics, and all the rest.

In this article, following this line, I will consider mathematics as a mystical practice, and compare it to three other categories of mystical practice. In the order in which

[6](Wilber, 2001; p. 197)
[7](Wilber, 2001; p. 209)
[8](Wilber, 2001; pp. 86-87)

I have experienced them in my own life, these are: meditation, psychedelics, and divination. These correlate with three consecutive epochs of my history: early work (pure math, 1960-1967), transition (applied math, 1967-1974), and mature work (computational math, computer graphics, modeling and simulation, 1974-present). The associated mystical experiences may be interpreted in the cosmologies, or maps of consciousness, of various philosophical traditions, such as Yogic, Platonic, Neoplatonic, and so on.

3. Cosmologies

The cosmologies of interest to us here are more expansive than the mind/body context of the cognitive sciences of our time. Especially, the concepts of soul and spirit are crucial, and we begin with a brief chronology of their long history.[9] Dates are approximate.

Prehistoric

- 100,000 BCE, Blombos Cave, southern Africa, symmetric markings. A splendid example is shown in Figure 1.[10] A similar image from the late Ice Age in Siberia is shown in Figure 2.
- 37,000 BCE, Chauvet-Pont-d'Arc cave, southern France, early cave paintings, symmetries.

And from the Ice Age, many later paleolithic painted caves have been discovered.[11] These caves are widely thought to have been the sites of shamanic rituals, involving entheogenic substances in some cases.[12]

The cosmology of shamanism comprises three layers: the Sky, or Upperworld, ordinary reality, and the Cave, or Underworld. Here one level is added to the cosmology of the atomic physicists discussed above.

The work of the shaman involves regular travels between the middle layer, called the Ordinary State of Consciousness (OSC) by contemporary shaman Michael Harner,

[9] This is extracted from (Abraham and Roy, 2010; Chs. 1 and 2). See also (Abraham, 2005).
[10] (Watts, 2002)
[11] For a longer list, see (Abraham, 2011) and (Abraham, 2014).
[12] (Clottes and Lewis-Williams, 1996/1998)

and the other two, called by him the Shamanic State of Consciousness (SSC).[13]

More than half of the images in the epi-paleolithic painted caves are abstract signs, which may be based on shamanic journeys.[14] The representational images are widely interpreted as icons for hunting magic. But some of them may also be vestiges of shamanic journeys, in which the shaman encountered animal spirits.[15] Such an image, from the Chauvet cave, is shown in Figure 3.

Ancient

- 2500 BCE, Ancient Egypt. The concepts of *soul* and *spirit* emerged.
- 1400 BCE, Ancient Hebrews. The parallel concepts of *nephesh* and *ruach* introduced into Hebrew history in Genesis.
- 800 BCE, Ancient Greece. The parallel concepts of *psyche* and *thymus* introduced into Greek history by Homer and Hesiod.
- 520 BCE, Pythagoras, advanced Greek cosmology and philosophy, based on three principles: Ideas, Souls, and Harmony.
- 400 BCE, Socrates introduced Rational Consciousness, and the World Soul.
- 390 BCE, Plato synthesized Pythagoras and Socrates. Developed a cosmology based on a single principle: The One or Good.
- 250 CE, Plotinus developed Plato's cosmology into the Neoplatonic cosmology of three hypostases: the One, Intelligence, and the World Soul, plus Nature. The three hypostases acted upon Nature through an intermediate force, the Logos.
- 450 CE, Proclus further developed the Neoplatonic cosmology into five parts: the One, Intelligence, Soul, World Soul, and Nature.

[13] See (Harner, 1990; p. xxii).
[14] (Abraham, 2011)
[15] As proposed by Michael Harner. See(Harner, 2013; pp. 217–218).

Medieval

- 1000 CE, India. Composition of the *Tantraloka* by Abhinavagupta, the apex of Kashmiri Shaivist philosophy. This included the 36 tattvas, as well as a vibratory field, *spanda*, connecting them.[16] This is reminiscent of the Logos of Plotinus.

Renaissance

- 1400 CE, Ficino revived the Neoplatonic cosmology in the Renaissance, and substituted Spirit for the World Soul.. Thus: the One, Intelligence, Soul, Spirit, and Nature. The Intelligence, or Cosmic Mind (*nous*), contains Plato's Ideas. The Soul (*psyche*) has three parts: rational, sensitive, and vegetative, and gives rise to the World Soul and individual minds. both human and angelic. Reason communicates between Intelligence and Soul, Spirit between the World Soul and Nature.

Modern

Following Ficino, modern science dealt a lethal blow to the higher worlds of the ancient cosmology. But calls for a spiritual renewal are now multiplying, as books of Rupert Sheldrake and Ervin Laszlo, among others, testify. We continue our chronology with a selection of new works.

- 1959, Merleau-Ponty. In his final essay, *The Intertwining – The Chiasm*, Maurice Merleau-Ponty revived a part of Neoplatonism within the cloak of phenomenology. The fourth and final chapter of this essay (about 6 pages long) has been influential on the frontiers of cognitive science. In it, he introduces a medium, a continuous field called *Flesh*. Between the Mind and the Body, it provides a reciprocal connection for perception.

  > It is this Visibility, this generality of the Sensible in itself, this anonymity innate to Myself that we have previously called flesh, and one knows there is no name in traditional philosophy to designate it.[17]

[16] See (Abraham and Roy, 2010; ch. 2) for more details on this.

[17] Merleau-Ponty, 1968; Part Four, p. 139

Despite this disclaimer of Merleau-Ponty, his Flesh reminds us of the Logos of Plotinus, the Spanda of Abhinavagupta, and the Spirit of Ficino.

- 1981, Sheldrake. In his first book, *A New Science of Life: The Hypothesis of Formative Causation* of 1981, Rupert Sheldrake begins with a consideration of unsolved problems of biology, in the areas of behavior, evolution, the origin of life, parapsychology, and so on.[18] He delineates three levels of wholism: mechanism, vitalism, and organicism. We may relate these, respectively, to the levels — Nature, Spirit, and the World Soul — of Ficino. Sheldrake poses the existence of non-energetic fields, called morphogenetic fields, that direct the emergence of form in complex systems of all kinds. Although non-energetic, these fields may have measurable effects on energetic systems. He describes the effect of a morphogenetic field on an energetic system metaphorically as morphic resonance. His hypothesis of formative causation proposes that these fields evolve from unknown seeds called morphogenetic germs. Then they evolve their structures from previous similar systems; the past intervenes in the present; morphogenetic fields have memory. In terms of the premodern cosmologies described above, we may locate Sheldrake's morphogenetic fields in the World Soul.

- 2013, Harner. In his first book, *The Way of the Shaman* of 1980, Michael Harner presented excepts from first-person accounts of contemporary shamanic journeys, especially to the Lower World. Among the various goals of these trips, such as divination, spiritual exploration, and so on, the primary emphasis in this work is the healing function of the shaman. In his second book, *Cave and Cosmos: Shamanic Encouters with another Reality* of 2013, more attention is devoted to the Upper World. Here distinct layers or levels may be distinguished, as many as 49 skies are distinguished by Siberian shamans. And barriers may be encountered. This stucture is similar to the tattvas and maya of Kashmiri Shaivism.[19]

- 2014, Strassman. In his first book, *DMT: the Spirit Molecule* of 2001, Rick Strassman reported on his officially sanctioned research project involving DMT administered to human volunteers in a hospital setting from 1990 to 1995. After reflecting on his findings for a decade, he followed with a second report, *DMT and the Soul of Prophecy*, seeking more deeply into the question of the cosmological location of the DMT universe. As this search, as well as the

[18]The revised and expanded edition is called *Morphic Resonance*, 2009.

[19]See (Abraham and Roy, 2010) for a brief description of this cosmology.

DMT experience itself, may be compared to the question of the cosmological location of the universe of mathematics – addressed by Lakoff and Nuñez on their book of 2000, *Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being* – we will review this recent book of Strassman in some detail in the next section.

4. Math and meditation

My fascination with math began in 1951, after my fifteenth birthday. Confined to bed to recover from tuberculosis, my mother had introduced me to the amateur radio hobby, and in this context I discovered the math of electrical engineering. Besides my technical hobbies and lots of reading, I drifted innocently into an informal mediation practice. Thus, math and meditation entered my life at the same time. Eight years later, the two came together in my graduate research work for the doctorate in mathematics at the University of Michigan.

After two decades of informal meditation, I finally signed up for a formal course. This was a weekend workshop of Transcendental Meditation (TM). I learned a style of concentration meditation based on the repetition of a Sanskrit syllable. After practicing this daily for two years, I met Kyaw Thein, a math professor from the University of Rangoon, who joined me in Santa Cruz for one year as a post-doctoral research student in chaos theory. Also a meditator, like many Burmese people at that time, he brought me into contact with a group of Burmese Buddhist monks in Daly City, California. These monks gave me a copy of the *Abhidhamma*, a text of Buddhist philosophy written (according to tradition) by the Buddha himself.[20]

4.1. Types of meditation

From my reading of the *Abhidhamma* I gathered that the Buddha was a meditation teacher. Apparently there were (and still are) two styles of meditation, the *concentration* or *stabilizing* style (that I had been practicing daily) and another, called *insight (vipassana)* which was regarded as superior. With some difficulty I converted my daily practice to insight meditation, which I continue to practice today.

[20]For details of this philosophy, see (Narada, 1956).

In concentration practice, the repetition of the seed syllable helps to concentrate attention to a point, as it were. Distracting thoughts are suppressed. But in insight practice, attention is more loosely focused, and the arising of a thought is studied, rather than suppressed.

A variety of insight practice called *analytical* meditation is promoted in Tibetan Buddhism, in which a chosen topic – such as peace or compassion – is set up as a target for the loose focus of attention. A combination of stabilizing and analytical techniques has been explained in detail by the Dalai Lama.[21]

This sort of controlled observation of the interior thought process is very similar to the creative process of math research.

4.2. Math as meditation

Not only is math research very similar to analytical meditation, but in my experience, was identical. A research problem may persist as a meditation target for days, weeks, or even years. Daily, or frequently, the meditation is repeated, in which the focal area is explored, thoughts arise, are analyzed, and saved or discarded. Between meditations, some journal writing is practiced to commit the discoveries in words, symbols, and images, for safekeeping. This writing and parallel discussions with colleagues and friends occurs in ordinary reality, rather than the alternate realities of meditation, dreaming, and sleep.

Further, regular math meditation seems to develop the mental capacity for math performance in ordinary reality. This recalls the visualization techniques used by athletes and musicians to improve their skills.

And it is this process of discovery in an alternate state that encourages the Platonic interpretation of mathematical reality. The development of this special form of meditation practice, beginning from prehistoric stargazing, or even prelinguistic music making, may be related to the origins of primitive religion in shamanic practices in caves.

My association of math and meditation is just an hypothesis at this point. However, there have been many neurophysiological studies of meditators, and perhaps some of math research. And one day, evidence may substantiate my theory.

[21]Dalai Lama, 1995); final for pages)

4.3. Meditation and the origins of math

Just as meditation facilitates mathematical discovery today, it may have facilitated it throughout history and prehistory. Although we may not have historical support for explicit meditation practice on the part of the well-known mathematicians of earlier historical times, we may nevertheless speculate that their work on the frontiers of math research included sessions that we would Identify as meditation today.

So what about the prehistory of mathematics? Archeomathematics comprised primarily simple arithmetic and plane, spherical, and solid geometry, developed in the contexts of business accounting, cartography, and astronomy/astrology. The mental arithmetic and mapping involved then required the same skills of analytical meditation as today. Thus we imagine that the earliest developments of math prehistory co-evolved with the meditative rituals of shamanism, our earliest religion. Language, the arts, math, the sciences, and religion – all coevolved as a cultural complex system, from earliest times (perhaps 100,000 years ago) up to the dawn of history. Their prehistorical artifacts survive in archeological sites all around the globe. Then, in Sumer around 3500 BCE, they broke forth, fully formed, into the historical record.

What I am proposing here is that the intentional practice of meditation was a key catalyst of this creative cultural explosion.

5. Math and psychedelics

Like meditation, psychedelics have played a role in our cultural evolution since earliest times.

5.1. The psychedelic hypothesis

First we must reorient our view of the psychedelic revolution. Recognition of the positive contributions of entheogenics has a long but little-known history. Let us begin with 1992.

1992, McKenna

A radical theory of human evolution was advanced by Terence McKenna in his book of 1992. In fact, it was subtitled, *A Radical History of Plants, Drugs, and Human Evolution.* When he gave me an early copy of the book, it was inscribed: *Here is my best effort toward saving the shamanic agenda.* Entheogenic plants, in fact, were credited by him as triggers for the major bifurcations of human evolution.[22]

> My contention is that mutation-causing, psychoactive chemical compounds in the early human diet directly influenced the rapid reorganization of the brain's information-processing capacities. Alkaloids in plants, specifically the hallucinogenic compounds such as psilocybin, dimethyltriptamine (DMT), and harmaline, could be the chemical factors in the protohuman diet that catalyzed the emergence of human self-reflection. The action of hallucinogens present in many common plants enhanced our information-processing activity, or environmental sensitivity, and thus contributed to the sudden expansion of the human brain size. At a later stage in the same process, hallucinogens acted as catalysts in the development of imagination, fueling the creation of internal stratagems and hopes that may well have synergised the emergence of language and religion.[23]

In fact, a key point in his argument is the capacity of psilocybin to enhance linguistic and symbolic thinking.

> Of course, imagining these states of higher self-reflection is not easy. For when we seek to do this we are acting as if we expect language to somehow encompass that which is, at present, beyond language, or trans-linguistic. Psilocybin, the hallucinogen unique to mushrooms, is an effective tool in this situation. Psilocybin's main synergistic effect seems ultimately to be in the domain of language. It excites vocalization; it empowers articulation; it transmutes language into something visually

[22]McKenna was not the first to advance the entheogenic hypothesis. Predecessors include Mircea Eliade (1951), Gordon Wasson (1971), Henry Munn (1973), and Michael Harner (1973). Sequels include (Abraham, 1994) and (Clottes and Lewis-Williams, 1996). Related theories on the origin of religion and art are presented in (Pfeiffer, 1982), (Spivey, 2005), and (Renfrew, 2007). Further, the early research on psychedelic psychotherapy should be noted here. For the early work at the Maryland Psychiatric Research Center in the 1970s and its continued evolution to this day, see (Wolfson and Yensen, 2014).

[23](McKenna, 1992; p. 24)

beheld.[24]

McKenna proposed that mushrooms in the diet of Homo sapiens was instrumental in the emergence of human language. As McKenna noted, this theory was anticipated by Henry Munn in an article of 1973. Many of McKenna's far-reaching proposals have meanwhile been validated in further research, including the discoveries of Rick Strassman.

2001, Strassman

Rick Strassman is a contemporary psychiatrist living in New Mexico. In 1990 he began a research project on the effects of DMT.[25] Over a period of five years he administered over 400 sessions involving 60 human volunteers in a hospital setting. After each session he interviewed the subject, asking questions based on Buddhist psychology. His research report of this project, published as a book in 2001, included quotations from many of these interviews, or case reports.

In these case reports some common themes emerged. The subjects reported visions, auditions, and encounters with "beings" belonging to an alternate reality, which seemed as real as ordinary reality. These themes characterized my own experiences with DMT in 1969.[26]

Although a DMT trip involves the injection or inhalation of a high dose of laboratory (that is, exogenous) DMT, low levels of endogenous DMT abound in nature. Strassman proposes that:

> ...spontaneously occurring "psychedelic" experiences are mediated by elevated levels of *endogenous* DMT. In Chapter 4, The Psychedelic Pineal, I presented a series of biological scenarios in which the pineal may synthesize DMT, and I speculated about the metaphysical and spiritual implications of these possibilities.
>
> How then might this spirit molecule, whether produced from inside through these presumed biological pathways, or taken from outside as in our studies, modify our perceptions so radically?[27]

[24](McKenna, 1992; p. 42)

[25]As the Strassman project got underway, the DMT influenced books (McKenna 1992) and (Abraham, 1994) already existed in manuscript form.

[26]See (Abraham, 2006) and (2008).

[27](Strassman, 2001; p. 311)

This book ends with a proposal for further research.

2014, Strassman

In the years following the end of his DMT research project in 1995, Strassman became aware of the limitations of the models of consciousness, or cosmologies, that had guided his project initially.

> The previous chapter ended with my concluding that the psychological, biophysical, Buddhist, or shamanic models I had brought to my research, or studied soon thereafter, did not satisfactorily account for all aspects of the DMT experience. They rejected the possibility that the contents of the DMT experience were objectively real, did not sufficiently attend to the highly interactive nature of the state, suffered from ethical shortcomings, or lacked cultural compatibility.[28]

For example, the goal of Zen meditation is a sort of enlightenment that was not supported by the case reports of the DMT subjects. As he expanded his search for models of the DMT experience, he came upon that of the Hebrew Bible. It comprises twenty-four books in three sections – Torah, Prophets, and Writings. Prophets contains the books of the three canonical prophets – Isaiah, Jeremiah, Ezekiel – as well as twelve minor prophets. The books of the Prophets report on their prophetic experiences, and it is in these experiences that Strassman finds parallels of the DMT experience. His study of the parallel worlds of DMT and prophecy came together in this book of 2014 entitled *DMT and the Soul of Prophecy.*[29]

Strassman's interpretation of prophecy is greatly influenced by medieval Jewish philosophy. Between 900 and 1400 CE their flourished an extensive commentary on the Hebrew Bible by these medieval Jewish philosophers, or MJPs. Some twenty of these are detailed by historians such as Isaac Husik (1916). Strassman refers primarily to six. These are, in chronological order:

- Saadiah ben Joseph al-Fayyumi, Aristotelian (892–942)
- Solomon ben Isaac, or Rashi (1040–1104)

[28](Strassman, 2014; p. 65)

[29]Here the word *soul* refers to a partition of prophecy into two domains, body and soul. For Strassman, the body of prophecy is its phenomenological appearance or contents while its soul refers to its essential informational/message content. See (Strassman, 2014; p. 282).

- Judah Halavi (1085–1140)
- Abraham ben Ezra, Neoplatonist (1089–1167)
- Moses ben Maimon, or Maimonides (1135–1204)
- Moses ben Nachman, or Nachmanides, Aristotelian (1194–1270)

He includes Baruch Spinoza (1632–1677) in this group. Another interesting MJP is Solomon ibn Gabirol (1021 – 1058), also known as Avicebron.[30] He continued the project of Philo to reconcile Neoplatonism and the Hebrew Bible.

The main contribution of these philosophers to Strassman's *metaphysics of prophecy* is a dichotomy of the individual mind into two faculties, the imaginative and the rational (or intellectual). In his summary of this medieval cosmology, Strassman describes God's influence on humans as progressing via emanations, through intermediaries (angels or spirits), into the individual mind, through these two faculties. And here we may recognize a parallel with the later (Renaissance) cosmology of Ficino.

> The subjective experience of divine influence occurs in the mind, in particular through the operation of two mental faculties, the rational and the imaginative. The *imaginative faculty* is not what we usually consider the imagination, that which "makes things up," but rather it is the mental mechanism responsible for mediating perceptions, emotions, and somatic experiences. In more modern terms, we might call it the circuitry of the mind-brain complex mediating those functions. It also reproduces sensory experience using images of perceptions and sensations whose immediate causes no longer exist; that is the imaginative faculty possesses memory. Finally, the imagination combines these images in novel ways; for example, we might have seen a horse and a human in the real world, and then we imagine a centaur.[31]

Here more modern terms perhaps refers to neurophenomenology, as in the works of Francisco Varela, Evan Thompson, and Eleanor Rosch.

Progressing from the medieval metaphysics of prophecy, Strassman builds his own, *theoneurological model of prophecy*. He then adapts this as a metaphysical model of the DMT experience, explaining:

[30] See Ch. 5 in (Husik, 2014; pp. 58 – 79).
[31] (Strassman, 2014; pp. 244–45)

> I believe I am justified in this attempt because the similarities between the prophetic and the DMT states suggest common metaphysical mechanisms underlying them. Once we appreciate how their shared metaphysical and biological processes may occasion their shared phenomenology, we may then consider how their differences, especially regarding their informational content, reflect different metaphysical processes.[32]

Among the similarities between the states, we have the impression of reality, the visual, auditory, and sensory images, and the sensation of the sacred. Among the differences, much detailed by Strassman, are the informational content transmitted in the states. The prophets bring back from their journeys into the state of prophecy extensive verbiage, as reported in the Hebrew Bible (Old Testament), regarding God's attributes, activities, and commandments. According to Strassman, based on his interviews with his sixty human subjects, the content returned from the DMT state by his volunteers (perhaps apprehended by sensitivity to emanations, but not translated into the rational realm) was meager in comparison with the verbal contents of returnees from the prophetic state.

Specifically, it is the verbal content retrieved by the prophets that outpaced the DMT reports. However, if psylocybin had been studied in place of DMT, the results might have been much different. As Henry Munn reports, the Mazatec shamans ate mushrooms, then spoke for hours.[33]

And this is my main point of contention with Strassman. In the reminder of this article I will argue that a great deal of creative work in computer science and computational mathematics has been inspired by the psychedelic state. We should keep in mind the distinction between visual and verbal content retrieved from psychedelic voyages. DMT favors visions, while psilocybin favors prophesies. We will return to this dichotomy in a later section on divination.

However, any positive impact on our quality of life deriving from the revolutions of the 1960s, or the technical developments of the 1970s, may be difficult to discern. There are people who believe that improved civil rights, gender equality, organic farming, and many other features of life in the 21st century (that we now take for granted) derive in fact of the psychedelic tsunami of the 1960s.

[32](Strassman, 2014; p. 257)

[33]See (Munn, 1973; p. 92).

5.2. Psychedelics and computer graphics

True, the DMT experience did not result in linguistic descriptions of divine will and moral codes. However, the nonverbal information — especially in visual form — was very extensive in my own experiences of 1969. In fact, my career in mathematics took a sharp turn in a new direction, which persists to this day, as I describe in the next section. Here I tell the story of the influence of LSD on the computer graphic revolution of the 1970s.[34]

In August of 1991 my peace was disturbed by numerous phone calls from journalists demanding my response to an article in the San Francisco Examiner.[35] A computer columnist had received a copy of an article appearing in the *Gentleman's Quarterly*, or GQ, in which Timothy Leary was quoted as saying, *The Japanese go to Burma for teak, and they go to California for novelty and creativity. Everybody knows that California has this resource thanks to psychedelics.*

This columnist didn't believe what was asserted by Timothy Leary and others in the GQ article, that the computer revolution and the computer graphic innovations of California had been built upon an entheogenic foundation. She set out to prove this story false. She went to SIGGRAPH, the largest gathering of computer graphic professionals in the world, where annually somewhere in the United States 30,000 people who are vitally involved in the computer revolution gather. She thought she would set this heresy to rest by conducting a sample survey, beginning her interviews at the airport the minute she stepped off the plane. By the time she got back to her desk in San Francisco she'd talked to 180 important professionals of the computer graphic field, all of whom answered *yes* to the question, *Do you take psychedelics, and is this important in your work?*

Here I rest my case. The visual content of entheogenic journeys has made a huge impact on our cultural evolution in the stimulus of the computer graphic revolution, and in the visual arts as well.

5.3. Psychedelics and chaos theory

The 1960s brought together a number of related revolutionary movements: rock and roll, light shows, the hip subculture, the civil rights movement, resistance to the war

[34]More details may be found in my earlier publications on this subject, of 1998/2005, 2006, and 2008.

[35]Article by Denise Caruso, in the issue of Sunday, (August 4, 1991).

in Viet Nam, activation for academic freedom in universities, and especially, underlying them all, the psychedelic revolution. And then in the 1970s, new branches of mathematics — chaos theory and fractal geometry for example — burst forward. I cannot prove the connection between entheogenics and the chaos revolution. However, in my own experiences this connection is indisputable. Here are three personal case reports, regarding marijuana, DMT, and psilocybin.

The case for marijuana, 1967

From September, 1964, to June 1968, I was an Assistant Professor of Mathematics at Princeton University. I lived with my family in faculty housing on the edge of the campus. My teaching duties included a four-year sequence of math courses for honors students. I had the same group of about 15 students for all this time. They were a fabulous group and I got to know them well. Most of them became mathematicians or computer scientists of note. Through some of them I was introduced to marijuana in 1966, and LSD in 1967.

One evening in the Fall of 1966 I walked from home to the undergrad dormitories to visit with them. Along the way I passed my office, which was on the ground floor of Eno Hall. Reaching the room of some of these friends, I found a joint was passing around, and without thinking I took a few tokes. Soon after, I wended my way home, and along the way, a surprisingly strong trip came over me.

Passing my office again, I heard the telephone ringing within. I rushed into my office to find Steve Smale calling from Berkeley. He was among my closest friends at that time, and a leader in the field of dynamical systems theory (later to be given the pop name, *chaos theory*. He wanted to ask my opinion on a new conjecture that had just occurred to him. It concerned an arcane property of abstract dynamical systems called *omega stability*. His new conjecture was that this obscure property was actually generic, that is, typical, or satisfied by almost all dynamical systems.

As soon as had I heard his conjecture over the phone I exclaimed that no, it was not generic. He challenged me to provide a counter example, and I did. Without any hesitation I described a vision in my mind, in which two surfaces in four dimensional space intersected in a robust way. This result was published in a joint paper in 1970.

I do not believe that without the stimulus of marijuana, I could have constructed the image so quickly, nor have described it successfully in words.

The case for DMT, 1969

My experiences with LSD in 1967 and with DMT in 1969 led immediately to my new specialization in experimental and computation mathematics, and a laboratory for the exploration of chaos and fractals in vibratory fields.[36]

With the arrival of the computer graphic revolution, it became possible to share simulations of abstract animations similar to my DMT visions. Following visits to India to study Sanskrit literature and philosophy, I could relate the visions, the mathematics, and the shamanic literature. My work with the Massively Parallel Processor at the NASA Goddard Space Flight Center in 1989 brought all this together in a convincing and amazing way.[37] This technical work resulted in an artistic performance of real-time abstract animation and music at the Cathedral Church of St. John the Divine in New York City in October of 1992.

Despite the display of multidimensional spatiotemporal patterns, it is not possible to make a convincing case for the evolutionary contribution of psychedelic exploration to an outsider of these worlds. The problem is that the information brought back from the DMT state is essentially nonverbal. Even more, it is trans-visual. Or sometimes, trans-musical, in some general sense. In fact, it is mathematical. The alternate reality visited with psilocybin, as McKenna has argued, is more suited to the return of verbal information. In this, it is similar to the phenomenology of the highly verbal prophetic state.

Understanding mathematics as a cognitive system of patterns, we arrive at the question of the apprehension (or cognition) of mathematical information.[38] Following Plato, we may choose to interpret a mathematical revelation as the discovery of pre-existing patterns, or ideas, in the Intellectual sphere. The role of psychedelics, then, is to help the explorer to pierce the barriers between levels of the Upper World. But how can the material psychedelic molecules effect the permeability of some upper levels of the mind-brain complex, opening windows into the world of Plato's ideas?

In 2001, Rick Strassman proposed a far-reaching theory of the action of DMT.[39] Recalling this in 2014, he wrote,

[36]Described in detail in (Abraham, 2008).

[37]First published in (Abraham, Corliss and Dorband, 1991).

[38]Like Strassman, I prefer *apprehension* to *perception*. See (Strassman, 2014; p. 48), where *to apprehend* means to mentally grasp.

[39](Strassman, 2001; ch. 21)

> In *DMT, The Spirit Molecule*, I hypothesized that DMT elicits its effects by chemically modifying the receiving characteristics of the brain-mind complex.[40]

This theory acknowledges the existence of a hierarchy of cosmological levels, within which the brain and the mind may be situated, as reported from shamanic experiences over the millennia, as parallel levels of reality. Strassman utilizes a common metaphor, that of the brain as a sort of TV receiver, to model the interaction across levels.

A mathematical theory of this interaction across levels based on biophysics and nonlinear dynamics was proposed in 1979 by G. B. Ermentrout and J. D. Cowan. In this theory, the patterns of interconnections in a two-dimensional network of excitatory and inhibitory neurons in the visual cortex can mimic the onset of *form constants*, that is, hallucinations, or doubly-periodic spatial patterns perceived as eidetic visions. An early simulation of this sort of complex dynamical system was reported in (Abraham Corliss, and Dorband, 1991). One image from this supercomputer simulation is shown in Figure 4.

The case for psilocybin

As noted above, the psilocybin experience combines harmonies of visions, auditions, and symbolization. My journeys in the magic mushroom realm began soon after the appearance in 1976 of the text written by the McKenna brothers, *Psilocybin, the Magic Mushroom Grower's Guide*. This followed very shortly after the commputer graphic revolution. In fact, it was in 1974 that computer graphic technology became available in my university, the University of California in Santa Cruz, or UCSC. This was also the tipping point of the chaos revolution, in which many scientists became aware of the crucial implications of chaos theory for their work. I then began using computer graphics in my research on chaotic dynamical systems in two dimensions.

Inspired by my journeys with DMT and psilocybin, I became aware of the potential of computer graphics combined with text and symbols as a pedagogic medium for mathematics, a strategy I called the *dynapic technique*. Soon this diffused into my teaching, and the Visual Math Project was born, as a UCSC research group, funded by the State of California.

[40]See (Strassman, 2014; p. 256). With this telling phrase, *the mind-brain complex*, he shows the influence of the recently evolving theories of complexity.

I presented an early example of a dynapic video at an international conference on chaos theory for scientists in Chamonix, France, in the summer of 1981. This experiment lead directly to my four-volume book set on *Dynamics, the Geometry of Behavior*, written with Christopher Shaw, which has been used by thousands of scientists to get a start in learning chaos theory.

6. Math and divination

In the fall of 1967 I was beginning my fourth and final year as Assistant Professor of Mathematics at Princeton University. I had finished my first three books and started my experiment with LSD. Around this time, I m not sure exactly when, I became enamored with the I Ching, a Chinese classic of 4096 oracles called *changes*. The inquirer was to pose an inquiry, then choose one of the changes – by an elaborate ritual involving a stack of yarrow stalks – as the response. Experimentally, I began to consult the book daily with questions regarding my family and academic life.

At this time I had a sizable following of undergrad and grad students due to my research work on global analysis. A major workshop on this new branch of pure math had been organized by the American Mathematical Society in Berkeley for the entire summer of 1968. We all wanted to go as a group, but did not have the funds it would require. Based on multiple consultations of the I Ching, we devised a scheme. We would propose to a publisher a series of several new undergraduate texts, comprising a completely new math curriculum based on global analysis, called the Eagle Flying Math Series.

We obtained a contract with a prepayment large enough for twenty of us to travel to Berkeley, rent an office building for the summer, and write all the books. And so we did. The oracle came through for us, and I have continued to consult it (and astrology as well) to this day. And so began my relationship with divination.

6.1. A glossary of divination

Like mysticism, another difficult word to define is *divination*, along with its related vocabulary. Here is a brief guide to the usage we will follow. Following Richard Stoneman,

> The word 'divination' demands comment. The English word is from the Old French *devin*, i.e. a divine, a theologian but also a soothsayer, which in turn is from the Latin use of *divinus* (normally as an adjective, but also as a noun), which can define a person as inspired by a god, able to foretell the future ... Divination in common usage, and in this book, refers to techniques of ascertaining the will of the gods and, sometimes, predicting the future.[41]

We will take this word, divination, as our key word in this section. It implies a contact with the divine world, the world of the gods. A diviner is one who tries to ascertain the will of the gods, usually revealed in an omen, or oracle. *Manticism*, meaning the arts of divination and prophesy, might be an alternative.

An *omen* is a warning, and usually takes the logical form: if P, then Q, and the warning may be averted. An omen might be either a vision, a verbal message, written or voiced, or some natural phenomenon such as the behavior of birds, entrails, winds, etc. These related words may be regarded as approximate synonyms of omen: sign, portent, forewarning, prediction, forecast, prophesy, harbinger, prognostic, prognostication, augury, or revelation.

An *oracle* is more severe than an omen, it is a fate.[42] Usually, an oracle is the response to an inquiry. Also, the specialist, shrine, or institution making the inquiry is called an oracle, or prophet, or seer. For example, the oracle at Delphi in Ancient Greece. The oracle in story form is thought to have originated in ancient Egypt.[43] Non-verbal oracles going back to Paleolithic shamanism are attested in the painted caves. We would like to distinguish *visions* and *prophesy*, for the visual verses verbal forms of oracles.[44]

6.2. Logic, math, and divination

The prehistory of divination, whether in Egypt or Babylonia, may forever be hidden from us. However, from 3500 BCE or so, oracular literature emerged in Mesopotamia.[45] This writing-based divination has been contrasted with the divination literature of

[41](Stoneman, 2011; p. 225 fn 3)
[42](Stoneman, 2011; p. 7)
[43](Stoneman, 2011; p. 9)
[44]Re ancient Israel, see (Noegel, 2010; p. 143, fn 2) and (Noegel, 2007).
[45](Manetti, 1993; p. 169, fn 2 for Sec. 1)

the later oral culture of Ancient Greece.[46] Mesopotamian gods wrote oracles, while Greek gods spoke them.

From a large number of Mesopotamian oracles recorded in cuneiform on surviving clay tablets, historians have analyzed a few recurring styles, all of the form *if P, then Q*. This is seen as the beginning of semiotics, as well as being a precursor of propositional logic.[47] The invention of propositional logic is credited to Aristotle around 330 BCE. The dependence of mathematics on theorems and proofs based on formal logic came in the generation after Aristotle in the geometry of Euclid. The line of transmission from Mesopotamia to Greece is uncertain, but probably passed through Egypt.

It must also be noted that Mesopotamian astronomy and astrology contributed to the early development of mathematics.

6.3. Math and semiotics

Semiotics is a large and complex field. The history of semiotics may divided into two threads, based on two distinct models for a sign. These are the dyadic and the triadic models. For our purposes it will be convenient to trace the evolution of just one branch, the triadic.

In a dyadic model, a sign comprises a sign vehicle and a meaning, idea, or thing. Champions of the dyadic model include Augustine, Hobbes, Locke, and Saussure.[48] In a triadic model, a sign comprises a sign vehicle, a thing, and also a sense or idea or concept. This third component is interpolated between those of the dyad, forming a triangle, the *semiotic triangle*.[49] See Figure 7, in which the semiotic triangle may be seen superimposed upon a cosmological map. Champions of the semiotic triangle include Plato, Aristotle, the Stoics, Leibniz, and Peirce.[50]

Ancient divination, in which an omen, a sign of the gods, is interpreted in the context of the enquiry by a priest as meaning, is clearly a semiotic exercise, and has been described as the origin of semiotics.[51] Astrology must be counted as another mantic

[46](Manetti, 1993; p. 3)
[47]See (Manetti, 1993) and (Annus, 2010).
[48](Noth, 1990; p. 88)
[49](Noth, 1990; p. 89)
[50](Noth, 1990; p. 90)
[51]See for example (Maneti, 1993) and (Annus, 2010).

art participating in the early development of semiotics. The astral constellations are still regarded by many people as signs of the gods.

We may now interpret mathematics, in the Platonic view, on the basis of the semiotic triangle. A mathematical object, existing in the intellectual sphere, is perceived by the mathematician as a mental concept, and then represented by a sign vehicle (word phrase, symbol, or image) for purposes of remembering, further cognitive processing, or communication with other mathematicians.

Thus, we may regard research work on the frontiers of mathematics as a semiotic practice, like divination, consistent with meditation or psychedelic voyaging.

7. How the brain-mind complex might work

It was Walter Freeman, in his 1970s research on the sense of smell, who first proposed that a smell was a spatiotemporal pattern of electrical activity within the olfactory bulb. In fact, he recorded such activity with a grid of electrodes in live animal studies. Shortly thereafter, Candace Pert proposed such patterns of neurotransmitters as the basis of emotional perception.

It is in this context that the brain-level mechanism of Ermentrout and Cowan may be interpreted. Thus, dynamic patterns of excitatory and inhibitory connections on the retina and the visual cortex, resulting from the mathematically determined dynamic pattern activity of electrical and biochemical concentrations, including those that accompany meditation, DMT, or oracle interpretation, are perceived by the mind as internally produced visions.

Reciprocally, an idea (for example, a mathematical object) in the mind might modulate the physical state of the brain to produce a visual, audible, or emotional representation of that idea. All this suggests a mathematical role in the phenomenology of perception. At base, it is the mathematical object or idea that underlies a perception. A computer simulation of this sort of modulation has been reported in (Abraham, 1996) and (Abraham and Broadwell, 1997).

An image, shown in Figure 5, shows the modulation in the morphic field of a black triangle produced by a white square. Each agent in this interaction may observe the perturbation of its own field due to a signal sent by the other. A simulation by Michael Nivala of a weakly coupled lattice of chaotic Rössler attractors produced

an image, shown in Figure 6, which resembles some mental imagery experienced in altered states.

And thus, following Freeman, Ermentrout, and Cowan, we may understand a materialist scenario in which a material smell or cortical state is translated into a physical, bodily action.

Now let us suppose that I am playing chess, and a mathematical analysis has led me to a mental dichotomy, and I must decide whether to move my bishop or my knight. After consulting the I Ching, I decide on the knight move, and my hand reaches out.

For the physicalist, there is no mystery here. A thought is a physical state of the biological brain, and so is the signal to the muscles, and one day soon neuroscience will elucidate all the mechanisms from the idea to the action.

But I am not a physicalist, so for me there is a mystery. There is the great chain of being, linking the idea in the immaterial mind to the vibrating electrochemical state in the material brain. The mystery of the brain-mind connection may be accepted as a phenomenon, even if not understood as a physical mechanism.

For the nondualist, it is all consciousness, all one, all mystery.

8. Conclusion

Regarding the brain-mind complex in the largest frame, such as that of Plato or Kashmiri Shaivism, we have a conjectural topology in which a journey – meditative, psychedelic, or oracular – may be regarded as an exploration. According to the entheogenic hypothesis, entheogenic substances in the diet of early humans precipitated the major bifurcations, or biological and cultural transformations, of the human story. The births of art, mathematics, the sciences, agriculture, the urban revolution, and religion – including shamanism, meditation, and divination – in ancient days ... all might derive from the ingestion of natural entheogenic materials.

Following the advent of agriculture and patriarchal domination, there has been a downswing of the spiritual aspect of life, especially in the western world. The potential for a radical upsurge, called by Terence McKenna the *archaic revival*, and by Timothy Leary et al the *birth of a psychedelic culture*, has been gaining steam around the world since the 1960s.

Based on personal experience, I have made a case for the contributions of meditation, entheogens, and divination to contemporary mathematics, particularly in the evolution of new branches, such as chaos theory and fractal geometry, that are based on computer graphics.

A mechanism for this contribution has been put forward in which the parallel worlds of alternate realities and mathematical apperception are interconnected by an immaterial medium (similar to an elastic fluid) through which information is transmitted by vibration-like activity.[52]

Along the way I have speculated on the very important differences among entheogens – LSD, DMT, and psilocybin – in the direction of their penetration into the higher worlds. These differences suggest a research topic on psilocybin similar to the DMT project of Rick Strassman, and another on the neurophysiology of math research.

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[52]Like the *spanda* of Kashmiri Shaivism.

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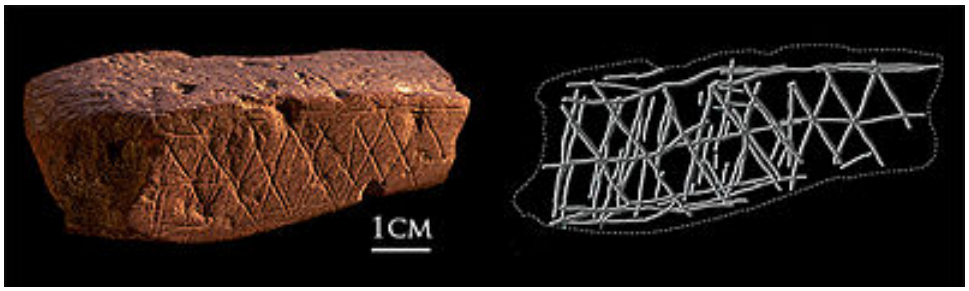

Figure 1: Engraved ochre, 100,000 BP, from Blombos Cave.

Figure 2: Engraved wooden idol, 9500 BP, Siberia.

Figure 3: Animal images from the Chauvet Cave, 37,000 BP, France.

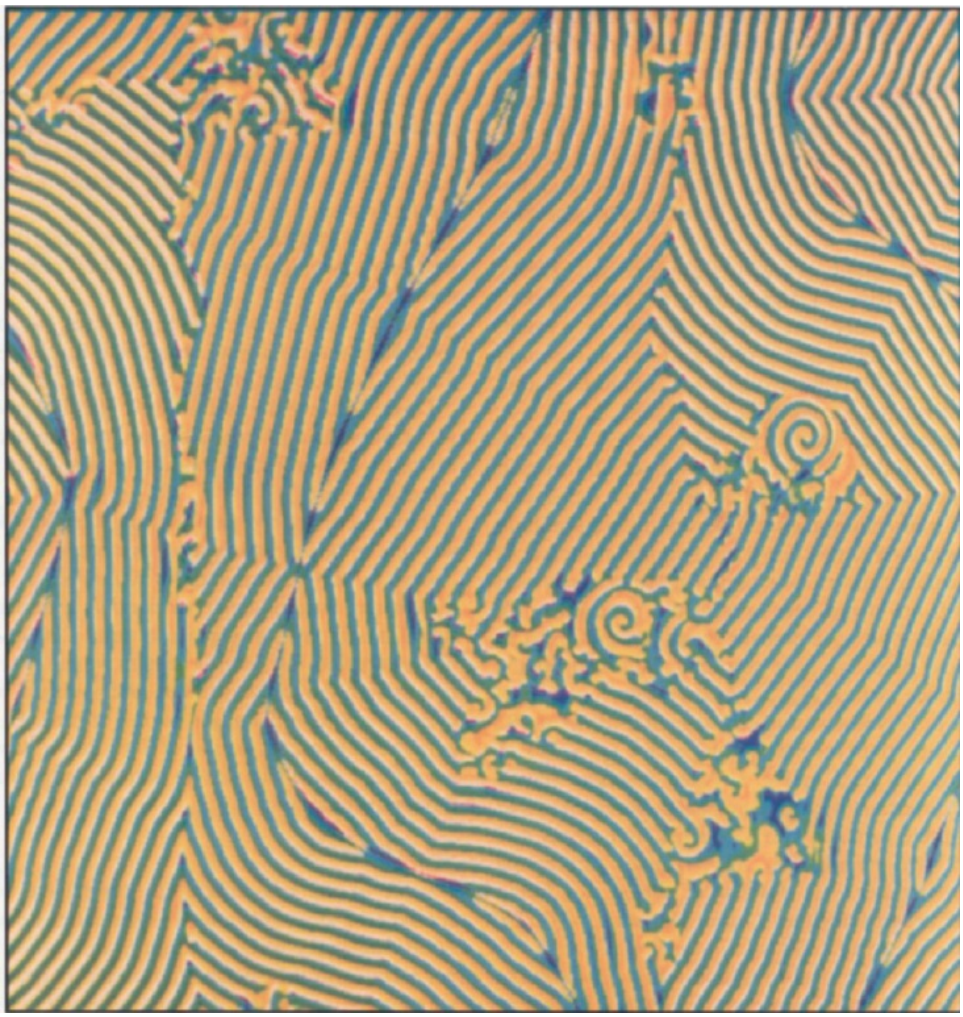

Figure 4: Super-computer simulation of a two-dimensional lattice of chaotic dynamical systems. Figure 4 from (Abraham, Corliss, and Dorband, 1991).

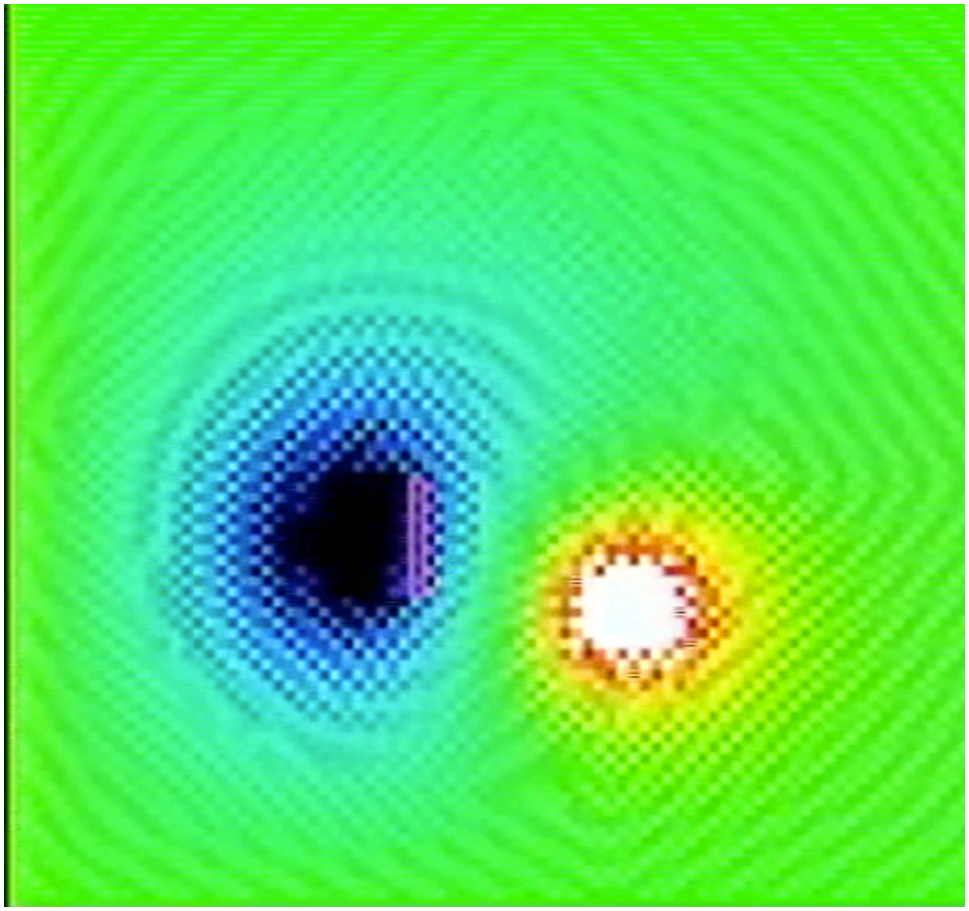

Figure 5: Simulation by Peter Broadwell of a vibration in a two-dimensional lattice of oscillators perturbed by two clamped shapes, a triangle and a square. See (Abraham, 1996) for further explanation.

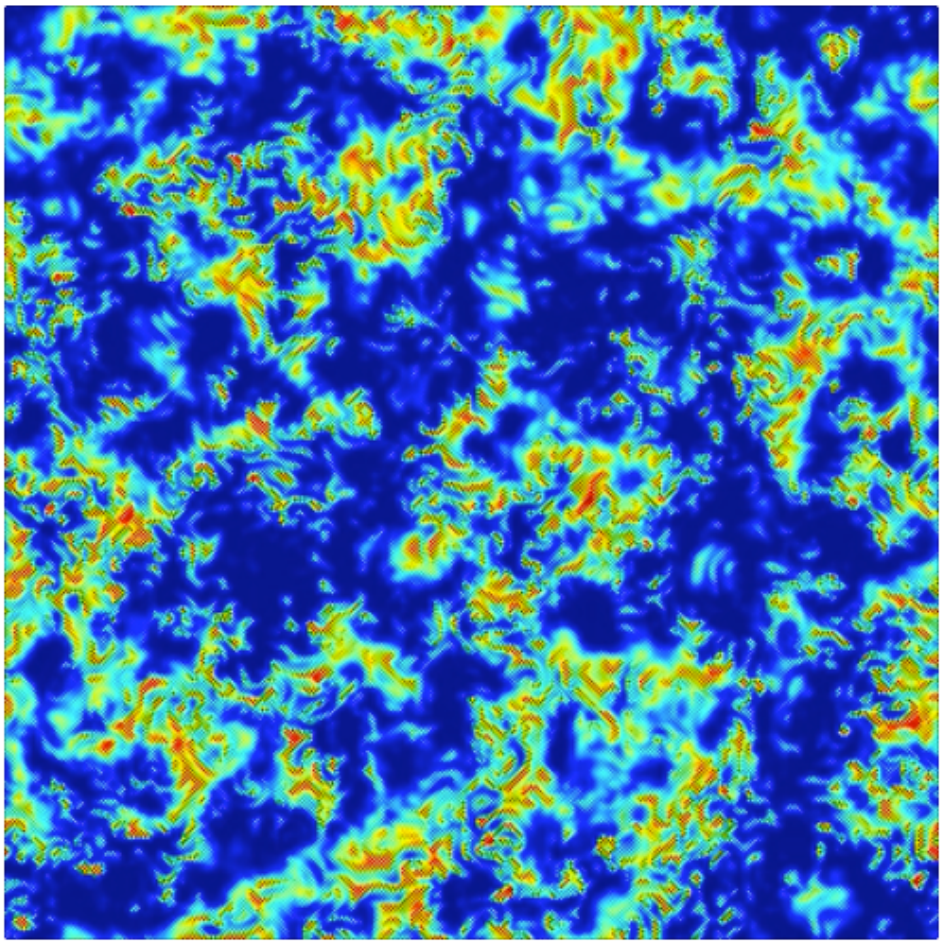

Figure 6: Super-computer simulation by Michael Nivala of chaotic synchronization in a two-dimensional lattice of Rössler attractors. See (Abraham and Nivala, 2014) for further explanation.

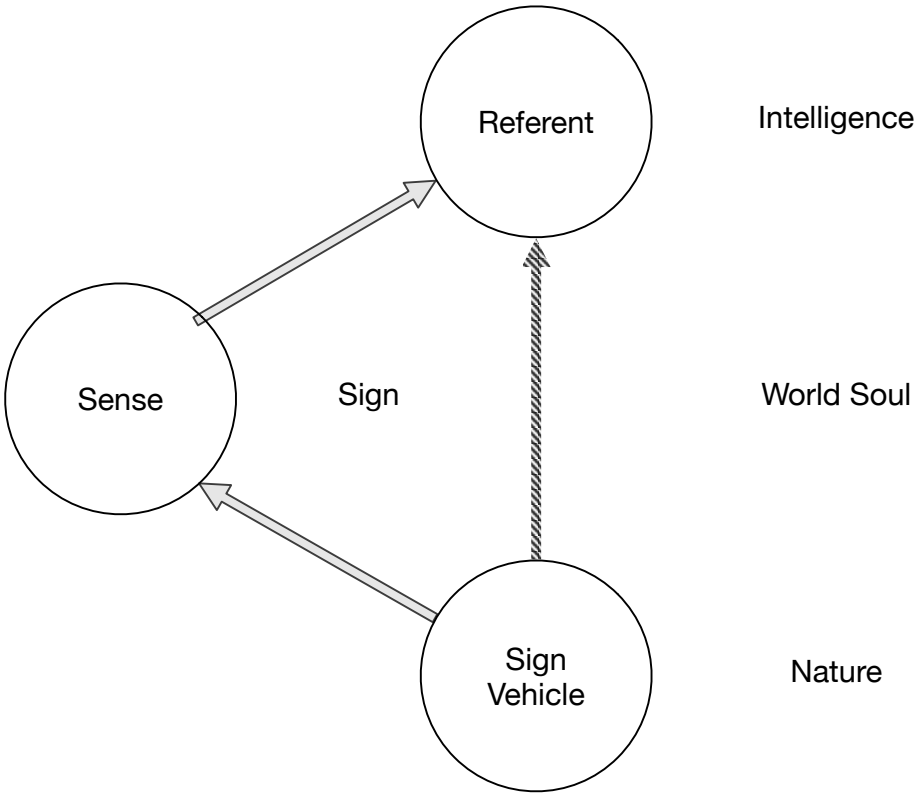

Figure 7: The semiotic triangle drawn over a cosmological map. See (Nöth, 1990; p. 89).