Mysticism in the History of Mathematics

Ralph H. Abraham*

12 November 2015

Abstract

We examine the relationship between mysticism and mathematical creativity through case studies from the history of mathematics.

CONTENTS

*Mathematics Department, University of California, Santa Cruz, CA USA-95064, *rha@ucsc.edu*.

1. Introduction

Taking a broad definite of mysticism — including for example divination, Pythagoreanism, Platonism, and requiring only some connection with higher realms — we consider cases studies from the history of mathematics in which mystical influences seem to have been at work. In some cases the mathematician had overt mystical practices. In others, the mystical influence had diffused from the ambient culture.

2. Ancient, from 4000 BCE

Of the ancient wold we must consider the complex system of Egypt, Mesopotamia, and Greece.

2.1 Egypt

Meditating on the history of mathematics and mysticism in the ancient world, the first idea that came to my mind is the Great Pyramid of Khufu, the oldest and largest of the pyramids in the Giza Necropolis south of Cairo. Built around 2644 BCE, it was originally about 480 feet tall, the tallest man-made structure in the world until recently. Its design reflects an advanced knowledge of sacred geometry. But most of the mystical connections of its geometry have been denied by the best experts.

In *Mathematics in the Time of the Pharaohs* of 1972, historian Richard J. Gillings devoted a three-page appendix to debunking popular beliefs in Great Pyramid mysticism.

> Many writers have ... made extravagant prophesies about the Great Pyramid. ... It may therefore come as a surprise ... that most of the miraculous stories written by these writers have no foundation in scientific fact at all; that the remarkable mathematical properties attributed to the Great Pyramid measurements are nowhere attested by scholarly Egyptological studies.[1]

Although I still think there is mystical mathematics in this pyramid, I will not dispute the experts. So, on to the second idea that comes to mind, astrology. There is splendid evidence for astrology in Ancient Egypt, for example, the Dendera zodiac of around 50 BCE. But it is generally understood that astrology came late to Dynastic Egypt from Greece. And it came to Greece from Mesopotamia.

2.2 Mesopotamia

Prior to the arrival of the Sumerians, a pre-Sumerian culture had developed sophisticated painted pottery with aesthetic friezes, which disappeared before 3500 BCE.[2] Mesopotamian

[1](Gillings, 1972; p. 237)

[2]See (Wooley, 1965; p. 9 and Fig. 4).

cultures, all polytheistic with sky gods, followed this sequence.[3]

- Sumer, 3500 BCE
- Akkad, 2300
- Babylonia, 1800
- Assyria, 1300

The early astrology, or proto-astrology, began in the Babylonian period, around 1800 BCE, while accurate observations began around 700 BCE, toward the end of the Assyrian kingdom. On the early development of astronomy = astrology in antiquity, we have the following opinion of Jim Tester.

> While many and fantastic claims have been made ever since antiquity for the vast age of Babylonian astronomy, it seems safe to say that some sort of mathematical theoretical astronomy was only developed late in Mesopotamian history, from the fifth century B.C. on, and that the real development of the science was the achievement of the Greeks.
>
> Early Mesopotamian astronomy was purely descriptive, and the 'prehistoric' period lasted from about 1800 B.C. until the fifth century. ...
>
> So it seems that horoscopic astrology cannot be older than the fourth century B.C., ... The earliest truly astrological texts that we possess are from Hellenistic Egypt, in Greek, from the late third and second centuries; ...
>
> ... the earliest of the few known Babylonian horoscopes is dated 41- B.C. ...
>
> ... and two streams may be said to have mingled in the Greek schools, the Babylonian and the Egyptian. ...
>
> From the second millennium B.C. there was developed in Mesopotamia a vast bulk of omen-literature, which was collected and organized in a work known as the Enuma Anu Elish, about 1000 B. C. ... A typical such omen reads: 'When the Moon occults Jupiter (*Sagmigar*), that year a king will die (or) an eclipse of the Moon and Sun will take place. A great king will die. ...' These omens are taken from stars, sun, moon and planets, eclipses, clouds, thunder and earthquakes. They clearly presuppose that there is some relationship between what happens in the sky and what happens on earth, though they do not suggest that the relationship is one of cause and effect.[4]

So we see that the mystical roots of Greco-Babylonian astrology came all the way from Sumerian cosmology. At this point it may helpful to have at hand a brief Sumerian chronology, 3800-2000. Preceding the dawn of astrology, we have the dynasties:[5]

[3] See the Mesopotamian timeline website

[4] From (Tester, 1987; pp. 11-13). See also (Neugebauer, 1952; pp. 97-101). Regarding the *Enuma Anu Elish*, see (Berlinski, 2003; pp. 9-10).

[5] (Crawford, 1990; p. 13)

- Uruk, 3800-3200 BCE
- Jemdat nasr, 3200-2900
- Early dynastic (ED), 2900-2500?
- Agade (Akkad), 2317-2191
- Ur III, 2112-2000
- Hammurabi, the first Babylonian king, 1848 BCE.

So the ultimate roots of Western astrology are to be found in the proto-astrology (omen-literature) of Sumer, with written records dating from the Old Babylonian period, about 2000 BCE. These are based upon a cosmology of four components: heaven, earth, air, and sea. The Sumerian pantheon includes four creating gods corresponding to the cosmology, and many non-creating gods, including three leading astral deities.[6] These top seven deities are:

- An, heaven
- Enlil, air
- Enki, sea, wisdom
- Ninhursag, earth, mother goddess
- Nanna-Sin, moon
- Utu, sun
- Inanna, Venus, goddess of love, fertility, and warfare

The myths of Sumer indicate the extent to which the gods, including the astral deities, were able to create the universe with their divine words, and likewise influence events in the lives of humans. This was the basis for the perennial traditions of divination, prognostication, the omen-literature, and astrology of Mesopotamia, Egypt, and eventually, Greece. The vestiges of Sumeria mysticism — that is, the divinity of the planets — survives yet in Western astrology. Mathematical astrology emerged in the Greek (Hellenistic) literature, with Eudoxus, in the fourth century BCE.[7]

[6] Here we follow (Kramer, 1963; ch. 4). See also (Kramer, 1961; p. 95) and (Tester, 1987; p. 15).
[7] (Tester, 1987; p. 11)

2.3 Greece

Greek mathematics and astronomy/astrology evolved from Egyptian and Mesopotamian influences. The mystical connection is generally attributed to Thales and his student, Pythagoras. Following them there are many important individuals in the history of Greek mathematics, probably all influenced by mysticism, but we will consider here only three, Plato, Euclid, and Proclus.

Thales, 624–547 BCE

Thales of Miletus was among the first of the Greeks to write on mathematics, science, and philosophy. He visited Egypt, learned geometry from priests, and imported it to Greece.

> He discovered many propositions himself, and instructed his successors in the principles underlying many others, his method of attack being in some cases more general . . . , in others more empirical . . . [8]

Thales was able to measure the height of a pyramid from its shadow. He is also credited with some of the theorems of Euclid's *Elements*.

Thus the mystical connection of Greek geometry ultimately derives from the frequent rituals performed by the priests in the temples of ancient Egypt.

Pythagoras, 569–475 BCE

Not all historians agree that Pythagoras was an actual (as apposed to mythological) person. But I will proceed here as if real. His time coincides with the late period or 26th dynasty of ancient Egypt, and the early part of the Persian conquest.

Pythagoras of Samos was a student of Thales from age eighteen. Upon advice from Thales he went to Egypt to study with the priests. He visited many temples, learning from all the priests, prophets, and wise men, for 22 years. Then, in 525, he was captured by Persian soldiers and taken to Babylon, where he studied with priests and scholars for another twelve years. He returned home aged 56, in 515. and began teaching what he had learned of mathematics. Eventually, around 518, be moved to Croton in Italy where he founded a school and religious community based on the Egyptian theory of the reincarnation of the soul, and vegetarianism. Mystical arithmetic is an important part of his legacy.[9]

Plato, 427–347 BCE

Plato expanded the teaching of Socrates on the perfection of the soul into a complete system, in 25 dialogues. This system contains his hierarchical cosmology, including (from the top):

---

[8] (Heath, 1960; p. 128)

[9] See (Taylor, 1986; p. 4–13) and (Rutherford 1984 p. 35–44).

1. The Good, an integral principle with no spatial extent,
2. The Intellect, including the Ideas or Forms,
3. The World Soul (including individual human souls), and
4. The Terrestrial Sphere of matter and energy.

His model of the Terrestrial Sphere, our particular interest here, is presented in the *Timeaus*, one of the last dialogues. Here is a partial abstract, with page numbers in brackets.[10]

> Now we come to the nature of the elements. [48] In the creation process there are three natures: an intelligible pattem, a created copy, and the space in which creation proceeds. [49] Space can receive any form, that is, the impress of any idea. [50] The elements are affections of space, produced by the impression of ideas. [51] The four elements took shape in space, and God perfected them by form and number. They are solid bodies, and all solids are made up of plane surfaces, [53] and plane surfaces in turn are made of scalene and isosceles triangles. Three elements are made from equilateral triangles, the fourth from isosceles triangles. [54] The first and simplest solid is the tetrahedron, the second is the octahedron, the third is the icosahedron, and the fourth is the cube. God used a fifth solid to delineate the universe. [55]

The elements are shaped as follows: earth as the cube, water the icosahedron, air the octahedron, fire the tetrahedron. [56] At page [56], four of the five cosmic figures have been described, including all the details we have given above. Plato obviously knows all about these four solids, and knows also that there are five, but the dodecahedron is not described in detail in the Timaeus.

Here we have the beginning of mystical geometry. The first four of these five Platonic solids (tetrahedron, octahedron, icosahedron, and cube) were known to the Pythagoreans. The fifth (dodecahedron) was studied in Plato's Academy.

Euclid, c. 300 BCE

Euclid systematized all the arithmetic, geometry, music theory, and astronomy of his time. His *Elements* comprise, in 13 books, the mathematics of Platos Academy. Until very recently, it was used as a textbook in our schools, and every famous mathematician of note studied from it.

The first six of the 13 books are devoted to plane geometry, including 148 propositions. Of these, 48 are constructions (ending in QEF in the Latin editions of Euclid, or *that which was to have been done*), the rest are theorems (ending in QED in the Latin, or *that*

[10] Taken from the Jowett translation. See (Abraham, 2011; p. 42) and (Heninger, 1974/2013; pp. 48, 74).

which was to have been proven). That is, for Euclid, a construction is to be done, while a theorem is to be proven. ln fact, the main motivation of the theorems, in the beginning, was to prove that the constructions actually work. That is, if we follow the steps correctly to construct a square, then in the end, we have a figure that is actually square. In other words, Euclids goal is a set of constructions.

The next three books, VII, VIII, and IX, are devoted to number theory. Book X deals with irrational numbers. Book XI returns to plane geometry and begins solid geometry. Book XII treats volumes of solids with the method of exhaustion.[11] Regarding the last book:

> The object of Book XIII is to construct, and to 'comprehend in a sphere', each of the five regular solids, the pyramid (Prop. 13), the octahedron (Prop. 14), the cube (Prop. 15), the icosahedron (Prop. 16) and the dodecahedron (Prop. 17); 'comprehending in a sphere' means the construction of the circumscribing sphere, which involves the determination of the relation of a 'side' (i, e. an edge) of the solid to the radius of the sphere; in the case of the first three solids the relation is actually determined, while in the case of the icosahedron the side of the figure is shown to be the irrational straight line called 'minor', and in the case of the dodecahedron an 'apotome'.[12]

To me, it seems without doubt that the construction of the five *cosmic figures* (or Platonic solids) of Book XIII is the primary goal of Euclid's *Elements.* This was the view of Proclus, although some authorities disagree.[13]

Proclus, 412–485 CE

Following Plato there were a sequence of philosophical schools: Platonists (including Euclid), Middle Platonists, Late Platonists, and Neoplatonists, beginning with Plotinus (205–270 CE). Neoplatonism remained an important thread in medieval times, and into the Renaissance. Proclus was among the most important Neoplatonists of late Antiquity. He wrote influential works on religion, theology, philosophy, astronomy, astrology, and mathematics, including a commentary on Book I of Euclid's *Elements.*

> In this commentary, as the reader will see, mathematical reasoning and exposition is often interrupted by digressions pointing out the moral or metaphysical significance of a figure or a theorem under discussion, and its value in directing the mind upwards to the region beyond mathematical existence.[14]

[11]See (Heath, 1960; pp. 373–419) for a detailed analysis.
[12](Heath, 1960; p. 415)
[13](Heath, 1956; p. 2)
[14](Morrow, 1970; pp. xliii–xliv)

Here we rest our case for the connections between Ancient Greek mathematics and mysticism. Throughout the Middle Ages mathematics had significant developments all over the world.[15]

3. Middle Ages, from 1000

During the middle ages, mathematics was evolving primarily in India and the Moslem world. Islam, evolving from the revelation of Muhammad during the 7th century, attained a golden age in the 9th century. The Abbasid (the third) caliphate succeeded the Umayyad in 750 CE. Its capital was moved from Damascus to Baghdad in 762, beginning this golden age, which lasted until 1258.

The caliph Harun al-Rashid (786–809) created the *Bayt al-Hikma* (House of Wisdom), which was further developed by his son, caliph al-Mamun (813–833). In this major intellectual center, Greek, Syriac, Pahlavi, and Sanskrit sources were translated into Arabic.[16] It was here that al-Khwarizmi (780–850) created rhetorical agebra, after translating Euclid's *Elements*.[17]

The golden age culminated in the 10th and 11th centuries. This is explained by Seyyed Hussain Nasr (formerly of the Iranian Academy of Philosophy) in terms of the bifurcation of Muslim world into the Sunni and Shi'ite factions.[18] The Shi'ah — more open to pre-Islamic philosophies, including the spiritual dimension of Neoplatonic cosmology, alchemy, astrology and so on — became more powerful, especially in Persia, after the 9th century.

During this time mystical Sufism coalesced within Islam and influenced mysticism in Christendom, especially the Troubadours. A philosophical school, the Ikhwān al-Safā' (the Brotherhood of Purity) — explicitly neo-Pythagorean and Hermetic — were ascendent after about 1000.[19] Two mathematicians of note during this time were al-Bīrūnī and Omar Kayyām.

3.1 al-Bīrūnī, 973–1048

Abū Raiḥān al-Bīrūnī, was among the most learned scholars of his time. He wrote on philosophy, cosmology, medicine, and physics as well as mathematics.

al-Biruni and India

Besides his primary residence in Ghazna (in present-day Afghanistan) he spent many years in India in the interval 1017–1031, culminating in a book on Hindu culture. In this book

[15] A very helpful time-line may be found inside the front cover of (Katz, 1993).
[16] (Nasr, 1964; p. 12)
[17] (Katz, 1993; pp. 228–233)
[18] (Nasr, 1978; pp. 14, 17)
[19] (Nasr, 1978; p. 26)

there may be no clear evidence that Biruni encountered Kashmiri Shaivism, but his time (and likely travels) coincides with the composition of the *Tantraloka* of Abhinavagupta, a leading text of Kashmiri Shivaism.

al-Biruni's mathematics

Earlier works on trigonometry from Alexandria and India were combined by Arab mathematicians into the form basic to modern mathematics by the 9th century. al-Biruni made significant contributions to this new development.[20]

3.2 Kayyām, 1040–1123

Omar Kayyām was thrice great: poet, Sufi sage, and mathematician. Sufism is a mystical dimension of Islam, independent of the split between the Sunni and Shi'ite factions. The Sufis use meditation, ritual dance, and other devotional practices to access the spiritual dimensions. The love for the divine is a frequent topic in the works of the great Sufi poets, such as Rumi and Kayyām.

Kayyām, poet

He is best known in English for his poem, the *Rubaiyat*, thanks to the four marvelous translations from the Persian by Edward Fitzgerald (1809–1883). The title means quatrain in Persian, and his 75 quatrains each have four lines, three of which (excepting the third) rhyme. The Fitzgerald translations are so well-known in English literature, that even I memorized them in my youth, and recall them to this day.

Kayyām, Sufi sage

His philosophy is manifest in philosophical prose, as well as in his poetry.[21] Seyyed Hossein Nasr makes the case for Kayyām as Sufi sage.

> Man can be, know and act only now. Even the poems of the Persian sage Khayyām, long considered as a hedonist in the West, refer in reality to the metaphysical and initiative significance of the Eternal Now. When Khayyām sings,
>
> > Ah, fill the Cup: — what boots it to repeat
> > How time is slipping underneath our Feet:
> > Unborn, To-morrow and dead Yesterday,
> > Why fret about them if To-day be sweet![22]

[20] See (Katz, 1993; sec. 7.4) and (Joseph, 1991; cha. 9, 10).
[21] (Nasr, 1964; p. 20)
[22] Quatrain 37 of the Fitzgerald translation.

> he is not encouraging hedonism and Epicurean pleasure-seeking, which is the opposite of the attitude of the sage, but rather wishes to underline the significance of the present moment, of today, of the only moment when we can *be* and become what we are in reality in the Eternal Order. That is why the Sufi is called the son of the moment (*ibn al-waqt)*, for he lives in the Eternal Moment, already dead to the illusory life of forgetfullness.[23]

Kayyām, mathematician

In his lifetime, Kayyām was known primarily as a mathematician. His textbook on algebra, *Treatise on Demonstrations of Problems of al-Jabr and al-Muqabala* was published in 1070. This presented his original geometric method for the solution of the general cubic equation.[24]

3.3 India, 1300–1900

The history of mathematics in India, less well-known than that of Europe, is treated in abundant detail in *The Crest of the Peacock: Non-European Roots of Mathematics* by George Gheverghese Joseph.[25] The title is from a popular description of the Indian astronomer-mathematician, Bhaskaracharya (b. 1114), who anticipated the calculus of Newton and Leibniz by more than five centuries. Greek algebra and trigonometry were greatly advanced in India, and passed on to the Arabs.

The development of mathematics in India was particularly advanced by a school created in Kerala (on the south-west coast of India) by the mathematician-astronomer Madhava of Sangamagrama in the 14th century, the founder of mathematical analysis.[26]

The most important members of this school were:

- Madhava (1340–1425)
- Nilankantha (1445–1545)
- Jyesthadeva (150–1575)
- Putumanam Somayaji (1660–1740)
- Shankar Varman (177–1839)

As for the mystical connections of this school, it prospered in an atmosphere of orthodox Brahman religion with rituals, devotions, meditations, and so on.

[23] (Nasr, 1993; pp. 33)
[24] This is wonderful explained in (Katz, 1993; pp. 242–245).
[25] (Joseph, 1991; ch. 9) See also (Katz, 2007; ch. 4).
[26] (Joseph, 1991; p. 293)

A very curious coincidence is the similarity of the mathematics of this school, characterized by infinite power-series expressions for the trigonometric functions, to the astounding works of the Indian genius, Srinivasa Ramanujan Iyengar (1887–1920), who grew up close to the the birthplace of Madhava!

This concludes our case for the mystical infusion of mathematical creativity in the Middle Ages.

4. Renaissance

Ancient Greek wisdom reached the Renaissance through Byzantine and Islamic transmissions. The Platonic Corpus was brought from Byzantium to Florence in 1439. In 1452, following the revival of the ancient Greek tradition, a rapid development produced the beginnings of modern mathematics and science. Perhaps this period exhibits the greatest influence of the the mystical upon the history of mathematics. Astrology and astronomy diverged in this period.

4.1 Ficino, 1433–1499

Marsilio Ficino was chosen by Cosimo de' Medici to translate Plato from Greek into Latin. His commentaries on Plato were published in 1469. Ficino not only translated the Platonic Corpus, but also absorbed its spirit. His house evolved into a de facto Platonic academy, and he into the leading philosopher of the early Renaissance. Many architects, painters, sculptors, musicians, medical doctors, lawyers, and intellectuals of all kinds were attracted to him. His teaching was accomplished at soirees devoted to good wine and food, as with Pythagoras, Socrates, and Plato in ancient Greece.[27]

Ficino's originality derived from his syncretism of Pagan and Christian elements. His primary wells of inspiration were Plato, Plotinus, Proclus, the *Corpus Hermeticum*, the Areopagite, Augustine, and Aquinas.[28] Among the facets of this syncretism were:

- orphic music, music therapy (Ficino's personal practice)
- astrological psychology, a new science, and
- angel magic, a healing art.

He was heir to the long line of astrological magic — Synesius, Proclus, Macrobius, and Al Kindi — and was followed by Bruno and Agrippa. His cosmological model, the foundation for the whole of Renaissance philosophy, had five levels:

- the One,

[27](Abraham, 2000; Sec. 6)
[28](Allen, 1981; p. xi)

- the Intelligence or Cosmic Mind (*nous*),
- the World Soul (*psyche*),
- World Spirit or Quality (*pneuma*), and
- Matter, Body, or Nature.

The One is the undivided source of everything. The Intelligence contains Plato's ideas, the archetypes and blueprints for creation. The World Soul has three parts (rational, sensitive, and vegetative) and gives rise to individual minds (both human and angelic). Spirit intermediates between the World Soul and Nature. This hierarchy was adapted from Proclus.[29] It extends the four-level hierarchy of Plato and Plotinus by the insertion of the World Spirit between the World Soul and matter.

> Ficino's unusual insertion of a world-spirit analogous to our medical spirits between the World-soul and matter represents his personal addition to Plotinus.[30]

Ficino's astrological magic, psychology, and medical practice were based on his understanding of Spirit, and its relation to the stars and planets.[31] The subject has had a recent revival in the context of Jungian analysis, where the planet archetypes are manifest within the individual psyche, and afflicted planets may be appeased through therapeutic practices. The basis in Ficino's writings is his final three-volume work of 1489, *Libri de Vita Tres — Three Books About Life*, and especially its third book, *De Vita Coelitus Comparanda —How Life Should Be Arranged According to the Heavens.* This is called *The Planets* by Thomas Moore in his book, *The Planets Within*, of 1982, in which the whole of Ficino's astrological psychology is brilliantly explained.

While not usually regarded as a mathematician, we may rightly acknowledge Ficino as an applied mathematician/astrologer, with a strongly mystical connection.

4.2 Dee, 1527–1608

John Dee, although little known, was an important figure on the intersection of mathematics and magic in the late Renaissance.[32] This was the Hermetic time in English history, of Queen Elizabeth, Shakespeare, Giordano Bruno, and the Sidney Circle.

[29] (Allen 2001; p. xv)
[30] (Kaske and Clarke, 1998; p. 27)
[31] (Couliano, 1987; p. 28)
[32] Dee's role in the history of Renaissance mathematics is given fair treatment in (Katz, 1993; ch. 10).

Prologue

Chaos theory had a brief flicker of popularity around 1987. Journalists pestered me with questions on the origins of this new branch of math, resulting in my book *Chaos, Gaia, Eros* of 1994. As my colleagues at the University of California Santa Cruz learned that I was working on this book, they encouraged me to teach a course (Math 181) on the history of mathematics. In the first instance of this course, Spring 1989, I covered the whole story from Pythagoras to Chaos in ten weeks. In anonymous student feedback after the course ended, it appeared that this was too much material, so I decided to focus on a single historical figure in the next instance of the course.

Meanwhile, my friend Paul Lee, professor emeritus of philosophy, had got me interested in John Dee, and together we created the John Dee Society with its extensive website, www.johndee.org. So naturally, I chose Dee for the subject of the second instance of Math 181.

Background

John Dee was a a follower of Ficino's Neoplatonism in England in the time of Queen Elizabeth I. Astrologer, magus, and mathematician, he amassed one of the largest libraries in England in his home at Mortlake, and was the center of a circle of intellectuals that evolved into the Royal Society. An authority on Euclid, he encouraged the first translation of the *Elements* into English. This edition, translated by Henry Billingsley, the mayor of London, was published in 1570. Thanks again to Paul Lee, I was able to hold in my hands an original print of the work, without doubt the finest edition of Euclid. It included illustrations that popped up into three dimensions as one opened the pages of the book. It also included a lengthy preface by Dee.

Dee and astrology, 1558

Following the deaths of Marsilio Ficino and and his follower Giovanni Pico della Mirandela (1463–1494) there evolved an extensive literature on astrology, magic, alchemy, and the cabala. Dee, with his great library, became adept in these subjects, as well as in the mathematics of Euclid. As a working astrologer, he became the court astrologer to the English throne. As such, he was responsible for the astrological choice of the coronation day for Elizabeth I, 17 November, 1558. He remained an intimate of the queen until her death in 1603.

Dee's preface, 1570

In this long and florid essay, Dee exhibits a profound knowledge of the mathematical literature of his time, and lays out a program (his *Groundplat*) for the future of mathematics in which applications abound

In the Preface, which he claims was hurriedly written under constant pressure from the publisher, Dee manages to outline the entire state of science (see Plate 13) as it was known in the sixteenth century. The Preface opens with a discussion of philosophical mathematics and its mystical implications, which was of interest to magi; but when Dee begins to explain the practical applications of the mathematical sciences, he pointedly states:

> ... I will orderly recite, describe & declare a great Number of Artes, from our two Mathematicall fountaines [arithmetic and geometry], derived into the fieldes of *Nature*.

This he clearly does.

In the text accompanying the 'Groundplat', Dee explains the natures of the various sciences, the relationships among them, and the levels of advancement achieved in each. The explanations are usually trenchant rather than detailed. Dee also makes suggestions, which are sometimes prophetic, for future scientific developments.[33]

Dee and angel-magic, 1582

Dee believed that mathematics and the sciences, being the creations of God, should be revealed to humankind. He hoped that this hidden knowledge might be revealed by asking questions. And as angels intervened between the spheres of Ficino's cosmological model, he became interested in summoning angels by magical seances as informants in technical matters.

In 1582, the medium Edward Kelley arrived at Mortlake. Dee began a series of 'actions with spirits' with Kelley that continued for seven years. The notes of these sessions were published in 1659 as:

A True and Faithful RELATION OF What passed for many Yeers Between Dr. JOHN DEE (A Mathematician of Great Fame in Q. ELIZ. and KING JAMES their Reignes) and SOME SPIRITS : TENDING (had it Succeeded) To a General Alteration of most STATES and KINGDOMS in the World.

Among several spirit informants who responded during those years was one Madimi, a child angel who ran about among the books on the library shelves. She first came to them in Leiden on 28 May, 1583.

> Suddenly, there seemed to come out of my Oratory a *Spirituall creature*, like a pretty girl of 7 or 9 years of age, attired on her head with her hair rolled up before, and hanging down very long behind, with a gown of Sey, ... changeable red and green, and with a train she seemed to play up and down ... like, and seemed to go in and out behind my books, lying on heaps, the biggest ... and as

[33] (French, 1972/1987; pp. 166–167)

> she should ever go between them, the books seemed to give place sufficiently, dis ... one heap from the other, while she passed between them : And so I considered, and ... the diverse reports with E. K. made unto me of this pretty mine, and ... [34]

There follows about 450 pages with reports of incredible information ralated by Madimi and other spirits.

Dee represents the culmination of the late Renaissance evolution of mystical mathematics and the sciences, before the coming of the moderns, with whom alchemy begat chemistry, astrology begat astronomy, Euclid begat analytic geometry, and so on. To these first moderns we now turn.

4.3 Kepler, 1571–1630

Johannes Kepler was a contemporary of John Dee, William Gilbert, the scholar of magnetism (the first field of modern physics), Giordano Bruno, the champion of the cosmos as an infinite plenum, and Galileo, the first modern dynamicist.

In 1594, Kepler began his first job as professor of mathematics in Graz. Toward the end of his first year there, while lecturing to the young students, he had a vision of the solar system embedded in a nested sequence of the Platonic solids. This was published the following year as his first book, the *Mysterium cosmographicum*. The full title, in English, is: *Forerunner of the Cosmological Essays, Which Contains the Secret of the Universe; on the Marvelous Proportion of the Celestial Spheres, and on the True and Particular Causes of the Number, Magnitude, and Periodic Motions of the Heavens; Established by Means of the Five Regular Geometric Solids.*

In his later work on the elliptical orbits of the planets (especially Mars), Kepler proposed a theory of universal gravitation, the second field of modern physics. In his explanation of noncircular motion, he actually changed the word *spirit* (as in angelic influence) to *force* (that is, mechanism) in the manuscript for his most important work, *Astronomia Nova* of 1609. And here we may locate the death of the world soul, concomitant with the birth of modern physics.

From the seventeenth century to the present, the intelligence of the cosmos has been relegated to the physical force fields — electric, magnetic, gravitational, and more recently nuclear, which are all mathematical fictions — together with their mathematical models such as Newton's law of motion. This is the materialist world view.

While a principle agent of the bifurcation from the mystical to the materialist view of science, Kepler remained a Pythagorean to the end. His final work, the *Harmonice mundi* of 1619, was inspired by the Pythagorean idea of the music of the spheres.[35]

[34] (Dee. 1659; p. 1)
[35] See (Abraham, 2006a), Abraham, 2006b), and the website, www.visual-kepler.org.

In fact, this work of 1619 demonstrates another idea, namely, that travel before cars and airplanes provided opportunities for meditation. For Kepler's mother was accused of witchcraft in 1615. En route in a carriage to testify in her defense, Kepler read the *Dialogue on Ancient and Modern Music*, written by Vincenzo, Galelio's father, in 1581. Meditating at length on this book in his carriage, he developed his ideas for the harmony of the spheres, which found expression in *Harmonice Mundi.*

4.4 Galileo, 1564—1642

It seems impossible to pass over Galileo here without mention, even though he does not belong in this company. Here are a couple of reasons. First, while he is deservedly regarded the father of modern physics, he was not a mathematician. Secondly, he seems to have been opposed to everything mystical.

Galileo was condemned by the Inquisition in 1633. The official cause was his support for the Copernican model of the solar system, in his book *Dialogue Concerning the Two Chief World Systems* published in 1632.[36] However, an earlier offense has been proposed by the Italian historian of science Pietro Redondi in his book, *Galileo, Heretic* of 1987. This offense was Galilee's atomic theory of secondary qualities (such as smell and taste), which contradicted Aristotle, and also conflicted with the Catholic doctrine of the Eucharist (that is, the substance of the bread and wine). This appeared in Galileo's earlier book *The Assayer* of 1623.

In this book appears the much quoted paragraph:

> Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossibly to understand a single word of it; without these, one wanders about in a dark labyrinth.[37]

Here Galileo follows Pythagoras and Plato in comprehending the universe as a mathematical construction, implicitly, by the hand of God. So even if he denies the existence of the soul, he straddles the mystical thread of antiquity.[38]

---

[36]The main argument (on the fourth and final day of the dialogue) concerning the ebb and flow of the tides, had been published earlier, in 1616. See the Introduction by J. L. Heilbronn in (Galilei, 2001) at p. xiii.

[37]Quoted from (Katz, 1993; p. 354).

[38]See (Abraham, 2000).

4.5 Descartes, 1595–1650

Descartes is remembered for several contributions to mathematics and philosophy, especially his dream problem, analytic (coordinate) geometry, and the mind/body problem. Here we will consider only the dream problem.

The dream problem

The dream problem, like so much of Western philosophy, goes back to Plato. His late dialogue, *Theaetetus*, includes a dream theory, and the dream problem: How does one know if a conscious experience is in a dream or in reality? Descartes focused on this question in his *Meditations on First Philosophy* of 1641, where he casts doubt on the reality of sense perceptions in general.

Descartes' dream

He also had a youthful vision, in 1619, followed by three dreams. The third dream predicted the unification of the whole of knowledge by the method of reason.[39]

Dream states and mysticism

At one time it was conventional to order consciousness in three stages: waking, dreaming, and deep sleep. Then the meditation community suggested the addition of a fourth state for meditation, and the lucid dreamers added a fifth state. Thus, from the top down: waking, meditating, lucid dreaming, unconscious dreaming, and deep sleeping. It was Plato's opinion that dreaming consisted in communications from the soul, So we may propose that soul-mind communication, that is, mystical interaction, may occur in all five states of consciousness. This might include flashes of mathematical creativity, solutions to problems, and the like. For example, Descartes' idea for reducing math problems to geometry by means of coordinates.

4.6 Newton, 1642–1727

Newton, like many in Renaissance Europe, was a devout Christian. In his time — a few generations after Dee, Kepler, and Galileo — alchemy was giving way to chemistry, and astrology to astronomy. Nevertheless, besides his crucial participation in the miracle of modern science, Newton clung to some of the traditional concepts. One of these was his lifelong romance with alchemy.

[39](Davis, 1986; pp. 3-4)

Newton's mathematical epiphanies

Newton is known particularly for two sudden breakthroughs. His first epiphany occurred in his youth. During an outbreak of the plague, the university in Cambridge was closed from the Summer of 1665 through the Spring of 1667. Following his bachelor's degree in Spring 1665, he returned home. In his *marvelous year*, at age 23, he developed his calculus and law of gravity.[40]

The second miracle, deriving the elliptical orbit from Kepler's laws, occurred in August of 1684, and was published as Newton's major work, *The Mathematical Principles of Natural Philosophy*, in 1687, at age 44.[41].

Newton's alchemy

After his marvelous year, in 1668, Newton began his alchemical project.

> Alchemy never was, and never intended to be, solely a study of matter for its own sake. Nor was it, strictly speaking, a branch of natural philosophy, for there was a spiritual dimension to alchemy — a search for spiritual perfection for the alchemist himself or herself, or a search for an agent of perfection (the "philosopher's stone") that could transform base metals into silver or gold or perhaps could even redeem the world. It was in fact the spiritual dimension to alchemy that led Newton to study it, but his goal was not exactly one of the traditional ones. He perceived alchemy as an arena in which natural and divine principles met and fused, and he understood that through alchemy it might be possible for him to correct the theological and scientific problems of the seventeenth-century mechanical philosophies.[42]

Thus, Newton pursued a mystical connection, a conduit for divine ideas to descend into consciousness.

5. Enlightenment, from 1650 CE

The epochs of European art history are usually given something like this:

- Ancient, before 476 (the fall of Rome)
- Middle Ages, 476–1300
- Renaissance, 1300–1520
- Mannerism, 1520–1590

[40](Dobbs, 1995; p. 7)
[41]Read the full story, involving Edmund Halley, the Astronomer-Royal, in (Dobbs, 1995; p. 38)
[42](Dobbs, 1995; p. 21)

- Baroque, 1590–1750
- Classic, 1750–1820
- Modern, 1820–1970

What we have called Renaissance in Section 4 thus overlaps the Mannerist and Baroque periods of art history. Ficino is Renaissance. Dee is Mannerist. Kepler, Galileo, Descartes, and Newton are Baroque, along with Rubens (1577–1640) and J. S. Bach (1685–1750).

But as our quest involves mysticism, the European Age of Enlightenment (1715–1789) is especially important. This was a philosophical movement — overlapping the Baroque and Classical epochs of art history — in which intellectuals, besides championing individual liberty, distanced themselves from mystical ideas and the dogmas of the church. A sort of revival of Aristotle, opposed to the Neoplatoism of the Renaissance. This movement is crucial for our appreciation of the mystical connections of the mathematicians of this period. We now consider two cases, Leibniz and Euler.

5.1 Leibniz, 1646–1716

Leibniz was a polymath who contributed to philosophy, theology, history, ethics, and science as well as to mathematics.

In the history of mathematics he was primarily known for his development of the calculus, independently of Newton. Developed in Paris in the 1670's, Leibniz' calculus was published in full only in 1714, that is, shortly before his death, on the eve of the Enlightenment.[43] According to Michel Serres, both Newton and Leibniz were inspired by alchemy, which had been revived after Descartes.[44]

Leibniz' philosophy, derived from his calculus, was expressed in his *Theodicy* of 1710, and more concisely in his *Monadology* of 1714. It became the dominant philosophy of the Baroque. It comprises a highly original cosmology in which the geometric continuum of the universe is filled with atoms, or monads (variously called entelechies, simple substances, etc.), which are indivisible and soulful. These are ordered in a pyramidal hierarchy — created monads, souls with perception and memory, and spirits — with God as capstone. This amounts to a combination of Plato and Democritus, cross-dressed as a rational system, acceptable to the Age of Enlightment. The world full of monads was structured by a Pythagorean process of world harmony. This is a link in the great chain of harmony, from Pythagoras to Proclus, Ficino, Vicenzo, Kepler, Leibniz, and on to Bach.[45]

Chrsitian Wolff (1679–1754) was an eminent German philosopher. He and his followers developed an extensive monadology, which was quite different from that of Leibniz.

[43](Katz, 1993; p. 472)
[44](Serres, 1968)
[45](Deleuze, 1993)

The Wolffians were ultra-rational Enlightenment dogmatists, denying the harmony of the monads.[46]

5.2 Euler, 1707–1783

Euler, generally considered the greatest mathematician of all time. He worked in most areas of the subject. He remains the measuring stick for creativity in mathematics. More than 800 books and papers, filling 80 volumes in his collected works. Due to the restrictive milieu of the Enlightenment, his mystical connections are obscure.

However, he did write some 234 letters on philosophy in the 1760's, including eleven attacking Wolffian monadology. Euler adhered to the monadal harmony of body and soul.[47] Thus, both Leibniz and Euler were closet Pythagoreans.

6. Classic, from 1800 CE

Romanticism — roughly 1780–1850, overlapping Early Modernity — belonged to a group of Counter-Enlightenment movements. Both Poincaré and Hadamard were born into the penumbra of this era — including authors such as Alexandre Dumas, Schiiler, and Byron — and thus imbued with the reaction against the Enlightenment. An occult revival, for example, that of the magician Eliphas Levi (1810–1975), was characteristic of the time.

6.1 Poincaré, 1854–1912

Henri Poincaré was among the most prolific mathematicians of all time, producing more than 30 books and 500 articles in his short lifetime. Like Euler, his work covered nearly the whole of mathematics. He liked to publish hs work in a wide variety of journals, so as to reach a broad populace. Late in his life he published three books of science for the lay public which were widely read and influential. The third of these, *Science and Method* of 1908, sold more than 20,000 copies by 1913. Here he devoted a chapter of some twelve pages to the question of creativity in mathematics.

Poincaré's theory of mathematical discovery extrapolates his personal experiences. One amazing case formed the basis for his theory of mathematical discovery.

> ... I shall limit myself to telling how I wrote my first memoire on Fuchsian functions. ...
>
> For fifteen days I strove to prove that there could not be any functions like those I have since called Fuchsian functions. I was then very ignorant; every day I seated myself at my work table, stayed an hour or two, tried a great number of combinations and reached no results. One evening, contrary to my custom, I

[46] (Fellman, 2007; pp. 73–77)
[47] (Breidert, 2007; p. 103)

drank black coffee and could not sleep. Ideas rose in crowds; I felt them collide until pairs interlocked, so to speak, making a stable combination. By the next morning I had established the existence of a class of Fuchsian functions, those which come from the hypergeometric series; I had only to write out the results, which took but a few hours.

Then I wanted to represent these functions by the quotient of two series; this idea was perfectly conscious and deliberate, the analogy with elliptic functions guided me. I asked myself what properties these series must have if they existed, and I succeeded without difficulty in forming the series I have called theta-Fuchsian.

Just at this time I left Caen, where I was then living, to go on a geological excursion under the auspices of the school of mines. The changes of travel made me forget my mathematical work. Having reached Coutances, we entered an omnibus to go some place or other. At the moment when I put my foot on the step the idea came to me, without anything in my former thoughts seeming to have paves the way for it, that the transformations I had used to define the Fuchsian functions were identical with those of non-Euclidean geometry. I did not verify the idea; I should not have had time, as, upon taking my seat in the omnibus, I went on with a conversation already commenced, but I felt a perfect certainty. On my return to Caen, for conscience' sake I verified the result at my leisure.

Then . . . I went to spend a few days at the seaside, and thought of something else. One morning, while walking on the bluff, the idea came to me, with just same characteristics of brevity, suddenness and immediate certainty, that the arithmetic transformations of indeterminate ternary quadratic forms were identical with those of non-Euclidean geometry.

Returning to Caen, I meditated on this result and deduced the consequences. . . . I made a systematic attack upon them and carried all the outworks, one after another. . . . All this work was perfectly conscious.[48]

Poincaré's psychological theory is a direct abstraction of this experience. Reduced to an outline, it amounts to these four stages.

1. Fully conscious work, without success.

2. Incubation, unconscious work by the *subliminal ego*.

3. Illumination, spontaneous emergence of an idea into consciousness.

4. Conscious verification and elaboration.[49]

[48] See (Poincaré, 1908/1946; pp. 387–388) and also (Hadamard, 1949/1954; pp. 12–14).
[49] (Hadamard, 1954; pp. 31, 56)

During incubation, the subliminal ego performs experimental combinations of ideas, filtering the outcomes by criteria of mathematical beauty.

It remains to speculate on the nature of the multilayered unconscious system in the incubation stage. I am proposing here the connection of the individual unconscious system with higher layers, up to the collective unconscious (or CUC) system of the mystical traditions, especially, the Platonic.

6.2 Hadamard, 1865–1963

Jacques Hadamard was a few years younger than Poincaré but following in the same cultural milieu, still in the sway of the Counter-Enlightenment movement, in the Parisian mathematical world. He made important contributions in some of the areas where Poincaré has worked. He was involved in the Dreyfus affair, as his wife was related to Dreyfus. Poincaré had testified in the Dreyfus trials in 1899 and 1904, so certainly the two men were acquainted.

Inspired by a lecture of Poincaré in Paris, Hadamard analyzed and elaborated the theory of Poincaré in a series of lectures in New York City in 1943, which became his book *The Psychology of Invention* of 1945.

Poincaré wrote of mathematical creation, Hadamard of mathematical invention, while I prefer mathematical discovery. The Platonic perspective is merely my opinion; there can be no proof.

6.3 Ramanujan, 1887–1920

Srinivasa Ramanujan Iyengar, wonder autodidact of South India, prince of intuition — his genius and originality astonished the European mathematical community in 1913, when they first heard of his work. Here is a brief chronology of his short life.[50]

- 1987, born December 22 in Erode (now part of Tamil Nadu)
- 1900, first theorems of his own
- 1903, studied math textbook by G. S. Carr
- 1908, began writing his Notebooks
- 1909, married
- 1911, first publication
- 1914, arrived in England
- 1918, returned to India

[50] Abstracted from (Kanigel, 1991).

- 1920, died April 26 (age 32) of TB

His mathematical career, comprising nearly 4000 results, spanned two continents — six years in India, then four years in Cambridge, followed by two years of decline and death. His output in India was mainly in four Notebooks comprising more than 800 handwritten pages. Most of his publications were written in England, many with coauthor G. H. Hardy,

Considering that his birth place was close to that of Madhava, the founder of the Kerala mathematical school around 1400, it is amazing how similar are the writings of these two men.

The mystical dimension for Ramanujan consists in his Brahmanic upbringing and life-long association with Hindu temple worship, ritual, singing, and especially, the input from the goddess of Namagiri, his local temple in Tamil Nadu. She appeared in his dreams, wrote equations on a screen, and whispered them in his ear.

7. Conclusion

With our brief glances into the lives of mathematicians and the growth of mathematics over the ages, we have remarked on the relationship between mathematical creativity and mysticism. Both the mystical practices and attitudes of the individual mathematicians and those of their ambient cultural background have been taken into consideration. It remains to integrate these instances into a global chronological picture.

The ancient natural and mystical philosophies declined during the Middle Ages in Europe, and were revived in the Early Italian Renaissance through Ficino's translation of Plato's dialogues, among other discoveries. As alchemy became chemistry and astrology turned into astronomy, the spirit of Ficino, the crucial link in the great chain of being, was lost to the moderns. The Renaissance giants — Dee, Kepler, Galileo, Descartes, and Newton — participated in the demise of the Spirit, as well as attempting to preserve some aspects of it.

We may visualize these tendencies by comparing two graphs: one of mathematical creativity, the other of the strength of the mystical, as functions of historical time. For the first, we might just count the pages devoted to each century in a major text on the history of mathematics. Carrying this out with the text (Katz, 1993) yields the blue histogram in Figure 1. The large numbers along the the bottom of the figure indicate the times of the major bifurcations of European cultural history: the onset of the Ancient period, the Middle Ages, and so on.

My intuitive estimate of relative openness to the mystical is the basis of the black curve in Figure 1. My view of world cultural history, based on chaos theory, and presented in my book *Chaos, Gaia, Eros* of 1994, is the justification for this black curve. There I examined consciousness as a complex dynamical system, with bifurcations marking the major transformations between historical epochs. The role of gender was emphasized as a driving force in this complex system.[51]

The concordance of the blue and the black is strong, supporting my hypothesis of mystical illumination of mathematical discovery, up until recent times.

A similar idea of cultural transformation was promoted earlier by Riane Eisler, in her book, *The Chalice and the Blade* of 1987.[52] Here she introduced a theory of cultural transformation based on the changing balance of dominator society (androcracy, the blade) and partnership society (gylany, the chalice). Following the advent of the blade, in the patriarchal domination around 4,000 BCE, the chalice occasionally surges up from the collective unconscious, in waves of gylanic resurgence, or GR waves.

I believe that GR waves coincide with the ebb and flow of mystical practices, and thus also, mathematical creativity. These waves are shown in the black graph of Figure 1.

[51] See especially the Conclusion, (Abraham, 1994; pp. 219–220).

[52] (Eisler, 1987; ch. 10)

References

Articles

- Abraham, Ralph H. (2000). Galileo's leap into the future. Preprint: http://www.ralph-abraham.org/articles/Blurbs/blurb107.shtml
- Abraham, Ralph H. (2006a). The new sacred math. *World Futures*, 62(1); pp. 6-16 (2006).
- Abraham, Ralph H. (2006b). The broken chain. *Elixir*, 2 (2006); pp. 9-16.
- Breidert, Wolfgang (2007). Leonard Euler and Philosophy. In: Bradley, 2007; pp. 97–108.

Books

- Abraham, Ralph Herman (1994). *Chaos, Gaia, Eros.* Rhinebeck, NY: Epigraph.
- Abraham, Ralph Herman (2011). *Bolts from the Blue: Art, Mathematics, and Cultural Evolution.* Rhinebeck, NY: Epigraph.
- Allen, Michael J. B. (1981). *Marsilio Ficino and the Phaedran Charioteer.* Berkeley: University of California Press.
- Allen, Michael J. B., transl. (2001). *Marsilio Ficino, Platonic Theology, Vol. 1, Books I—IV.* Cambridge, MA: Harvard University Press.
- Berlinski, David (2003). *The Secrets of the Vaulted Sky: Astrology and the Art of Prediction.* New York: Harcourt.
- Bradley, Robert E., and C. Edward Sandifer (2007). *Leonard Euler: Life, Work and Legacy.* Amsterdam: Elsevier.
- Couliano, Ioan P. (1987). *Eros and Magic in the Renaissance.* Chicago: University of Chicago Pres.
- Crawford, Harriet (1990). *Sumer and the Sumerians.* Cambridge, UK: Cambridge University Press.
- Davis, Philip J., and Reuben Hirsch (1986). *Descartes' Dream: The World According to Mathematics.* Boston: Houghton Mifflin.
- Deleuze, Gilles (1988/1993). *The Fold: Leibniz and the Baroque.* Minneapolis, MN: University of Minnesota Press.

- Dobbs, Betty Jo Teeter, and Margaret C. Jacob (1995). *Newton and the Culture of Newtonianism.* Atlantic Highlands, NJ: Humanities Press.

- Eisler, Riane (1987). *The Chalice and the Blade: Our History, Our Future.* San Francisco: Harper & Row.

- Fellmann, Emil A. (2007). *Leonard Euler.* Basel: Birkhāuser.

- French, Peter (1972/1987). *John Dee: The World of an Elizabethan Magus.* London: Ark Paperbacks.

- Galilei, Galileo (1632/1953/2001). *Dialogue Concerning the Two Chief World Systems: Ptolemaic and Copernican.* New York: Modern Library.

- Gillings, Richard J. (1972). *Mathematics in the Time of the Pharaohs.* New York: Dover.

- Hadamard, Jacques (1945/1954). *An Essay on the Psychology of Invention in the Mathematical Field.* New York: Dover.

- Heath, Thomas l. (1921/1960). *A History of Greek Mathematics, Volume I: From Thales to Euclid.* New York: Dover.

- Heath, Thomas l, transl. (1925/1956). *Euclid, The Thirteen Books of the Elements, Vol 1 (Books I and II).* New York: Dover.

- Heninger, S. K., Jr. (1974/2013). *Touches of Sweet Harmony*, 2nd edn. Tacoma, WA: Angelico Press.

- Joseph, George Gheverghese (1991). *The Crest of the Peacock: Non-European Roots of Mathematics.* New York: Penguin.

- Kanigel, Robert (1991). *The Man Who Knew Infinity: The Life of the Genius Ramanujan.* New York: Washington Square Press.

- Kaske, Carol V., and John R. Clark, transl. (1998). *Marsilio Ficino, Three Books on Life.* Tempe, AZ: Renaissance Society of America.

- Katz, Victor (1993). *A History of Mathematics: An Introduction.* New York: Harper Collins.

- Katz, Victor, ed. (2007). *The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook.* Princeton, NJ: Princeton University Press.

- Kramer, Samuel Noah, ed. (1961). *Mythologies of the Ancient World.* New York: Doubleday.

- Kramer, Samuel Noah (1963). *The Sumerians: Their History, Culture, and Character.* Chicago: University of Chicago Press.
- Moore, Thomas (1982/1989). *The Planets Within: The Astrological Psychology of Marsilio Ficino*. Herndon, VA: Lindisfarne Books.
- Morrow, Glenn R., transl. (1970). *Proclus, A Commentary on the First Book of Euclid's Elements*. Princeton, NJ: Princeton University Press.
- Nasr, Seyyed Hossein (1964/1978). *An Introduction to Islamic Cosmological Doctrines: Conceptions of Nature and Methods Used for Its Study by the Ikhwān al-Ṣafā', al-Būrūnī, and Ibn Sīnā*, Revised Edition. Boulder, CO: Shambala.
- Nasr, Seyyed Hossein (1993). *The Need for a Sacred Science.* Albany, NY: State University of New York.
- Neugebauer, O. (1957). *The Exact Sciences in Antiquity.* New York: Dover.
- Poincaré, Henri (1908/1946). *The Foundations of Science: Science and Hypothesis, The Value of Science, Science and Method*, George Bruce Halsted, transl. Lancaster PA: The Science Press.
- Rutherford, Ward (1984). *Pythagoras, Lover of Wisdom.* Wellingborough, UK: Aquarian Press.
- Serres, Michel (1968). *Système de Leibniz et ses modèles mathématiques.* Paris: Presses Universitaires de France.
- Taylor, Thomas, transl. (1986). *Iamblichus' Life of Pythagoras.* Rochester, VT: Inner Traditions.
- Tester, Jim (1987). *A History of Western Astrology.* Rochester, NY: Boydell Press.

Websites

- Mesopotamian timeline, www.mesopotamia.co.uktimeexploremain_mes.html
- John Dee Society, www.johndee.org
- Visual Euclid, www.visual-euclid.org

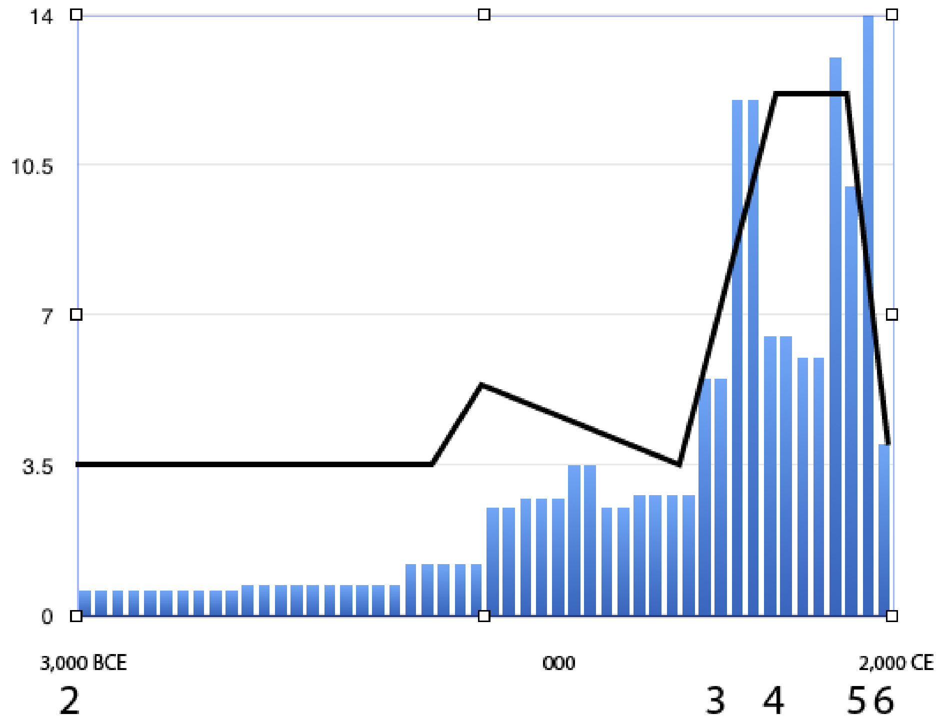

Figure 1: Mathematical creativity (blue) and mysticism (black) vs time. Blue from (Katz, 1993; endpapers). Red from intuition.

Bifurcations:
2 = Ancient, 3 = Middle Ages, 4 = Renaissance, 5 = Enlightenment, 6 = Classic.