Mathematical Perception

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Abstract

We extend the cognitive theories of Husserl and Poincaé up the cosmological chain to the Platonic world of ideas.

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1. Introduction

Poincaré (1855–1912) was perhaps the most creative mathematician of all time. And he was among the first to write about his creative process. So naturally his article entitled *Mathematical Creativity* must be taken seriously. As the title indicates, he was a constructivist, believing that mathematics was a human creation. His article contains a theory of mathematical perception.

He was a contemporary of Husserl (1859–1938), who also wrote on perception. It is well-known that Husserl and Poincaré influenced each other on the philosophy of geometry, and here I contend that they also interacted on the theories of perception. Husserl's early ideas on perception appeared in his *Logical Investigations* of 1900 and 1902, while Poincaré's lecture on creativity in 1904 was the basis of his writing, published in 1908. While Poincaré died prematurely at age 57, Husserl enjoyed a long and prolific life, developing his early ideas into phenomenology, his mature philosophy of perception.

Meanwhile, Poincaré's ideas were carried further by the mathematician Jacques Hadamard (1965–1963) in a book *The Psychology of Invention in the Mathematical Field* of 1945. Briefly, the theory of Poincaré, reduced to an outline, amounts to these four stages.

1. Fully conscious work, without success.
2. Incubation, unconscious work by the *subliminal ego*.
3. Illumination, spontaneous emergence of an idea into consciousness.
4. Conscious verification and elaboration.[1]

During incubation, the subliminal ego performs experimental combinations of ideas, filtering the outcomes by criteria of mathematical beauty.

This may be regarded as a specialization of phenomenology to the case of mathematical ideas, or objects. My project in this paper is to question this theory from a Platonic perspective. The problem, as I see it, is posed by the universality of mathematical ideas, as supported by the many cases of independent discovery. I say discovery, rather than creation or invention, due to my personal preference for Platonism. The problem may be resolved by the cosmological concept of the great chain of being, common to both Eastern and Western philosophies.

The crux of the problem lies in the nature of the perception of objects in the mathematical world.

2. Theories of perception

We may begin with the ideas of Poincaré and Husserl, and attempt to update date them, following developments in the philosophy and psychology of perception since their time, a

[1] (Hadamard, 1954; pp. 31, 56)

century or so ago.[2]

2.1 Phenomenology

The trajectory of phenomenology has advanced from Husserl through the work of Merleau-Ponty, Francisco Varela, and their followers.[3] But the problem remains: How are mathematical objects perceived?

Recall Poincaré and Husserl. In Step 1, we have the manipulation of mathematical objects by the conscious mind. These presumably have been learned through reading the literature of the subject, and talking with colleagues. They are established as mental objects, and have no counterparts in the real world, no sense-data, no conventional perceptions. Thus, they skirt the central ideas of phenomenology completely.

To apply these central ideas, we must suppose the mathematical object received is a presentation of an existing external prototype object, but this is not in the physical world of matter and energy that may be sensed by the usual senses. Instead we must posit a Platonic world of ideas. And then the semiotic triangle may be applied to the perceptual process. This process of transcendental perception may be regarded as a kind of resonance process, as in the morphic resonance of Rupert Sheldrake,[4] or the spanda of Kashmiri Shivaism.[5]

In step 1 of Poincaré we have a sort of concentration exercise, or meditation, on mathematical objects selected from the collective storehouse of mathematical ideas. They are manipulated, or combined, according to established rules. Then Step 2 becomes a shadow of the conscious process of Step 1 in the unconscious system of the mathematician's mind. The process of trial combination of ideas and aesthetic evaluation proceeds in the background, and perhaps a discovery is made by the subliminal ego. Finally, in Step 3, a rent in the curtain between the conscious and unconscious systems allows the new object to be perceived in consciousness. This is similar to the recovery of a dream upon waking from sleep.

Mathematical perception more resembles a dream or an hallucination than a sense-perception. We may summarize the fundamental setup with the semiotic triangle shown in Figure 1. Here we have shown the semiotic connection as a bidirectional channel, in consideration of the enactive approach of Varela, Thompson, and Rosch. Hence, math objects may be immortal. as in Plato, or human creations, as in Poincaré, or a combination of these.

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[2] (Abraham, 2015)
[3] See (Varela, 1991), (Thompson, 2007), and (Thompson, 2015).
[4] (Sheldake, 2009)
[5] (Abraham and Roy, 2010)

2.2 Hallucinations

Psychedelics used to be called hallucinogens, but fans objected. It was thought that psychedelic visions are more real than hallucinations. In earlier writings, I compared mathematical perceptions and psychedelic visions. Here I am relating them to hallucinations. Although hallucinations have been with us forever, they comprise a branch of the philosophy of perception only since Descartes. And this recent explosion of articulate works on non-veridical (un-real) perception provides us a wealth of concepts that we may adapt to the problem of mathematical perception. Here I will briefly summarize some of these ideas, following Fiona Macpherson (2013).[6] There are many schools of thought, here are a few of them.

2.2.1 Common-kind theories

This category comprises most of the traditional writings on hallucinations before the advent of the divergent views called dysjunctivism. A visual perception of a physical object, for example, is regarded as a causal chain of reactions:

- light from the object, passing through the eye, impinges the retina,
- retinal cells fire through the optic nerve to the primary visual cortex,
- a complex of brain states is instantiated,
- a visual experience occurs via an unknown brain/mind process.

According to common-kind theories, if an intermediate step in the chain is caused somehow, the same visual experience will result as if it had resulted from the entire chain being involved. Thus, an observation of a red teacup will be indistinguishable from an hallucination of a red teacup. This is the common-kind view of hallucinations and dream experiences.[7] The sameness of the kind of the mental states in the two cases is the source of the name, common-kind.

According to this way of thinking, a mathematical visualization might be regarded as a sort of hallucination, in which the math object, a mental object, must impinge on the causal chain via a mind/brain process, proceed down the chain like a veridical perception, and then be returned to the mental world by an inverse brain/mind process. Such a morphic resonance chain seems possible, but perhaps unnecessarily complicated.

2.2.2 Dysjunctive theories

The dysjunctivists, on the other hand, view the two cases of perception (veridical and hallucinatory) as mental states of different kinds, even though the perceiver may not be able

[6] See her 36-page introduction in (Macherson and Platchias, 2013).

[7] (Macpherson, 2013; pp. 8–18)

to distinguish them. The common causal chain of the common-kind theories is dropped. One may imagine instead that there are two disjunct causal chains, one starting from the physical world, another that is entirely within the mind.[8]

This conception is better suited to an understanding of mathematical perception. A special chain may have its source on the world of mathematical objects, belonging to the collective consciousness perhaps, and its target in a mental zone that has evolved in the individual mind (conscious or unconscious) through the history and prehistory of mathematical thinking.

2.3 Mathematics

So here is a theory of mathematical perception. This is a theory, not a theorem, that is, there can be no proof, it is just an opinion, a cognitive strategy. We may use it to understand mathematical creativity in a way that is compatible with Poincaré's theory.

We begin by enlarging the world so as to include several levels, like Plato's cosmology, or the five kosas, or 36 tattvas, of the Sanskrit philosophers. Among them are the physical world of traditional theories of sense perception, and Plato's world of ideas, including mathematcial ideas.

Then, as in the dysjunctive theory of hallucination, we propose two chains of communication, or morphic resonance. Each begins in a level of the enlarged world, and terminates in the individual mind of the mathematician. As in the enactive theory of phenomenal perception, these channels may be bidirectional. Further, they may have cross connections, empowering synesthesias such as mathematical visualization.

Some support for this model is provided by the work of the mathematical biologists Bard Ermantrout and Jack Cowan. They showed in 1979 that a mathematical model for the reaction and diffusion of morphogens over the visual cortex may produce morphodynamic patterns of chemotaxis mimicking the visions seen during a migraine attack. This suggests a chain from an idea or mental state to a physicochemical brain state and thence to a visual perception. This also indicates cross-links among the levels of the cosmological model.

The five kosas of classical Vedanta philosophy possibly preceded and inspired Plato and the Neoplatonic traditions. These are:

- Annamaya Kosa (Physical Body)
- Pranamaya Kosa (Energy Body)
- Manomaya Kosa (Mind)
- Vijnanamaya Kosa (Intellect), and
- Anandamaya Kosa (Bliss)

[8] (Macpherson, 2013; pp. 18–25)

These five evolved — in the two hundred years from Shankar to Abhinavagupta — into the 36 tattvas of Kashmiri Shaivism, about 1000 years go.

It seems likely that the five kosas evolved from the three bodies — causal, subtle, and gross — that may in turn derive from the Shamanic traditions of the Epipaleolithic caves. The three bodies and the five kosas are usually correlated thus:

- Gross body = Annamaya Kosa (Physical)
- Subtle body = (Pranamaya, Manomaya, and Vijnanamaya) Kosas (Energy, Mind, Intellect)
- Causal body = Anandamaya Kosa (Bliss)

In any case, we may use the three bodies as the basis for an analysis of the work of Ermantrout and Cowan. The mathematical model and the computer model may be assigned the subtle body, the neurochemical morphogens to the gross body, and the hallucinatory visual perception back again in the subtle.

It will be helpful to use the five kosas as a framework for the dysjunctive view of mathematical perception, as shown in Figure 2.

Exemplary cases

We have considered the theories of perception as a foundation for the analysis of mathematical creation (from the human-centered point of view), or equivalently, discovery (from the Platonic view). We now consider two exemplary cases.

3.1 The case of the homoclinic tangle

This mathematical object came into consciousness along with Poincare's founding of chaos theory in 1890. A rough sketch of a tangle is shown in Figure 3. Poincaré intuited a fuzzier image in the context of his famous work on the stability of the solar system.[9]

The first drawings came a generation later, in a publication of The American mathematicians George D. Birkhoff and Paul Smith in 1928.[10] The firm connection with chaos theory came only after another generation, in the original discovery of chaotic behavior by Yoshisuke Ueda in 1961.[11]

In both cases, Poincaré in 1890 and Birkhoff and Smith in 1928, the image must have popped into consciousness after hard work by the subliminal ego. No such image had previously been known to mathematics, nor seen in Nature.

[9] For better drawings with interpretations see (Abraham and Shaw, 2005), and for more of the surrounding history, see (Abraham, 2013).

[10] Smith was my department chair when I was an assistant professor at Columbia University, 1962–64.

[11] See em Early...

3.2 Genericity of transversal intersection

Transversal intersection is a property of geometric objects seen only in the imagination of mathematicians specializing in differential topology, a branch of mathematics created by Poincaré. This particular property, first studied by the American mathematician Hassler Whitney around 1940, may be exhibited by subspaces in Cartesian three-dimensional space. For example two planes, when their intersection is as simple as possible, such as crossing each other in a common line.[12]

In 1960, the Fields Medalist Stephen Smale proved that dynamical systems were characterized by curved surfaces called stable manifolds, which might have multiple intersections. He conjectured that these Intersections, in general, were transversal. He presented this work at a conference in Urbino, Italy, in 1961, where I was in the audience. Shortly after, I was scheduled to present my own proof of this generic property of transversal intersection at another international conference in Bonn, Germany. The day before my scheduled talk, as I was reviewing my rather complicated proof, I realized there was a gap. I was petrified, because if I had to cancel my talk my reputation would be ruined.

I went into the library of the Institute where the conference was gathered to have some private time to collect my thoughts. I felt dizzy, and grasped the library shelves on both sides of the aisle, and there was a flash like a thunderbolt. Instantly there popped into my mind a totally new and very simple proof, complete with details of the presentation such as drawings for the blackboard and hand gestures to indicate the movements of the surfaces. Like Poincaré, I was so certain of the argument that I completely relaxed, and went out to dinner and dancing with some of the other participants. The next morning I gave the talk from memory with great success. The printed version of the proof filled an entire book that was published in 1967.

4. Conclusion

Attempts to model mathematical perception following in the footsteps of Poincaré and Hadamard after 1945 in the tradition of common-kind theories were foiled by the fact that most of the writings of this tradition seemed to identify a mental state with a brain state, whereas a mathematical object and its presentation in the mind (conscious or unconscious) are equally mental. This obstacle disappeared with the advent of the disjunctive theories in the 1960s. Now it is possible to speculate on mathematical perception using the tools of the disjunctive theories of hallucination, as we have done here.

However, a problem remains: what is the nature of the link between a mathematical object in the Intellect (Vijnanamaya Kosa, or Plato's world of Ideas). and its mental representation in the Mind (Manomaya Kosa)? This is the upper link in Figure 2. It is a subtle version of the mind/body problem, the lower link in Figure 2, and we have glossed

[12] I am sorry that this example contains technical jargon, which may be ignored.

it over it with the mysterious metaphor of morphic resonance. In Kashmiri Shivaism this problem is addressed explicitly with the notion of *spanda*, or vibration in the *akasha*, an immaterial field that connects all 36 tattvas. We have attempted an atomic model for this vibratory field elsewhere[13] but the problem remains.

[13](Abraham and Roy, 2010)

References

Articles

- Abraham, Ralph H. (2013). The Peregrinations of Poincaré. In: Christophe Letellier and Robert Gilmore, eds., *Topology amd Dynamics of Chaos: In Celebration of Robert Gilmore's 70th Birthday.* Singapore, World Scientific; pp. 23-38.
- Abraham, Ralph H. (2015). *Mathematicians and Mysticism.* Preprint, MS#146.
- Ermantrout, G. B., and J. D. Cowan (1979). A mathematical theory of visual hallucination patterns. *Biol. Cybernetics*, vol. 34; pp. 137-150.

Books

- Abraham, Ralph Herman (1994). *Chaos, Gaia, Eros.* Rhinebeck, NY: Epigraph.
- Abraham, Ralph Herman (2011). *Bolts from the Blue: Art, Mathematics, and Cultural Evolution.* Rhinebeck, NY: Epigraph.
- Abraham, Ralph Herman, and Sisir Roy (2010). *Demystifying the Akasha: Consciousness and the Quantum Vacuum.* Rhinebeck, NY: Epigraph.
- Abraham, Ralph H., and Christopher D. Shaw (2005). *Dynamics the Geometry of Behavior*, 4th edn. Santa Cruz, CA: Aaerial Press.
- Abraham, Ralph H., and Yoshisuke Ueda, eds. (2000). *The Chaos Avant-garde: Memories of the Early Days of Chaos Theory.* Singapore: World Scientific.
- Macpherson, Fiona, and Dimitris Platchias (2013). *Hallucination: Philosophy and Psychology.* Cambridge, MA: MIT Press.
- Sheldrake, Rupert (2009). *Morphic Resonance: The Nature of Formative Causation* Rochester, VT: Inner Traditions.
- Thompson, Evan (2007). *Mind in Life: Biology, Phenomenology, and the Sciences of Mind.* Cambridge, MA: Harvard University Press.
- Thompson, Evan (2015). *Waking, Dreaming, Being: Self and Consciousness in Neuroscience, Meditation, and Philosophy.* New York, NY: Columbia University Press.
- Varela, Francisco J., Evan Thompson, and Eleanor Rosch (1991/1993). *The Embodied Mind: Cognitive Science and Human Experience.* Cambridge, MA: MIT Press.

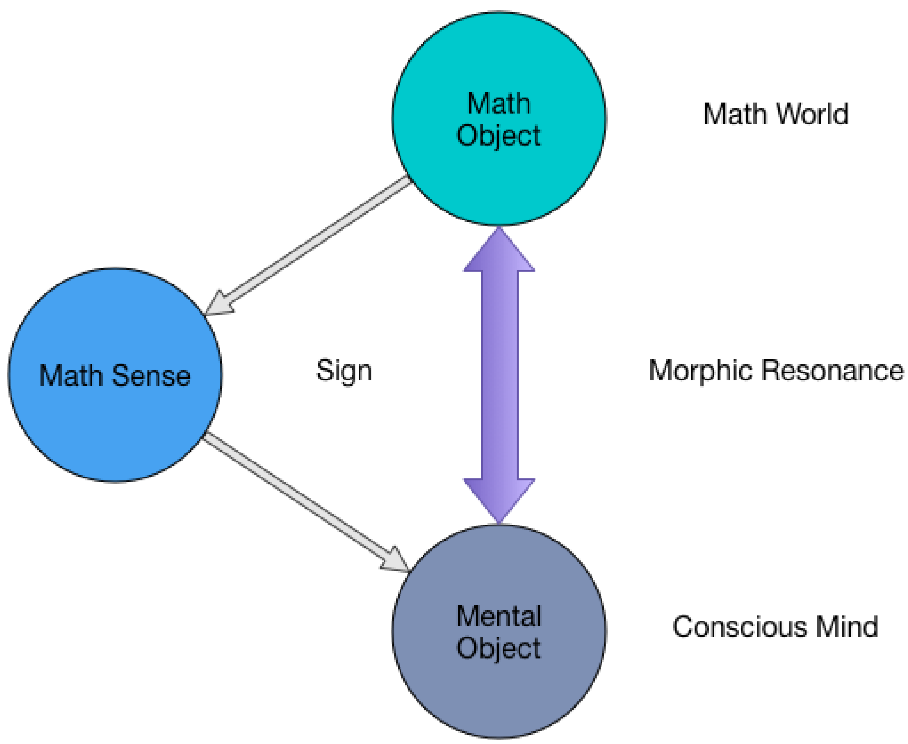

Figure 1: Mathematical perception, semiotic view.

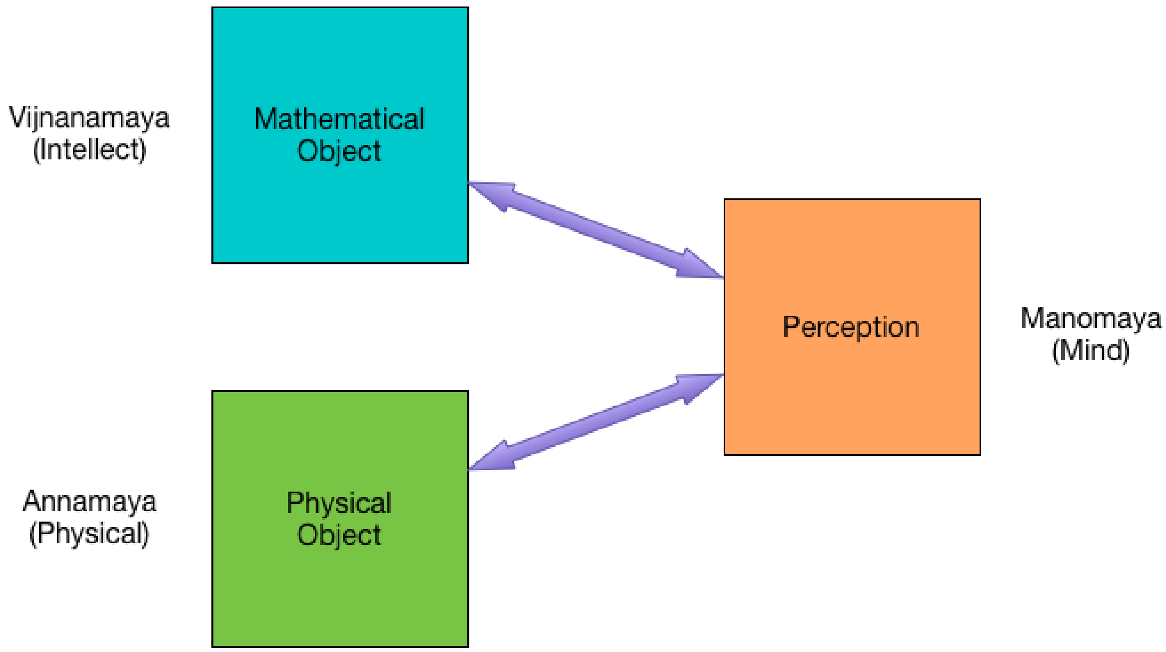

Figure 2: Mathematical perception, dysjunctive view.

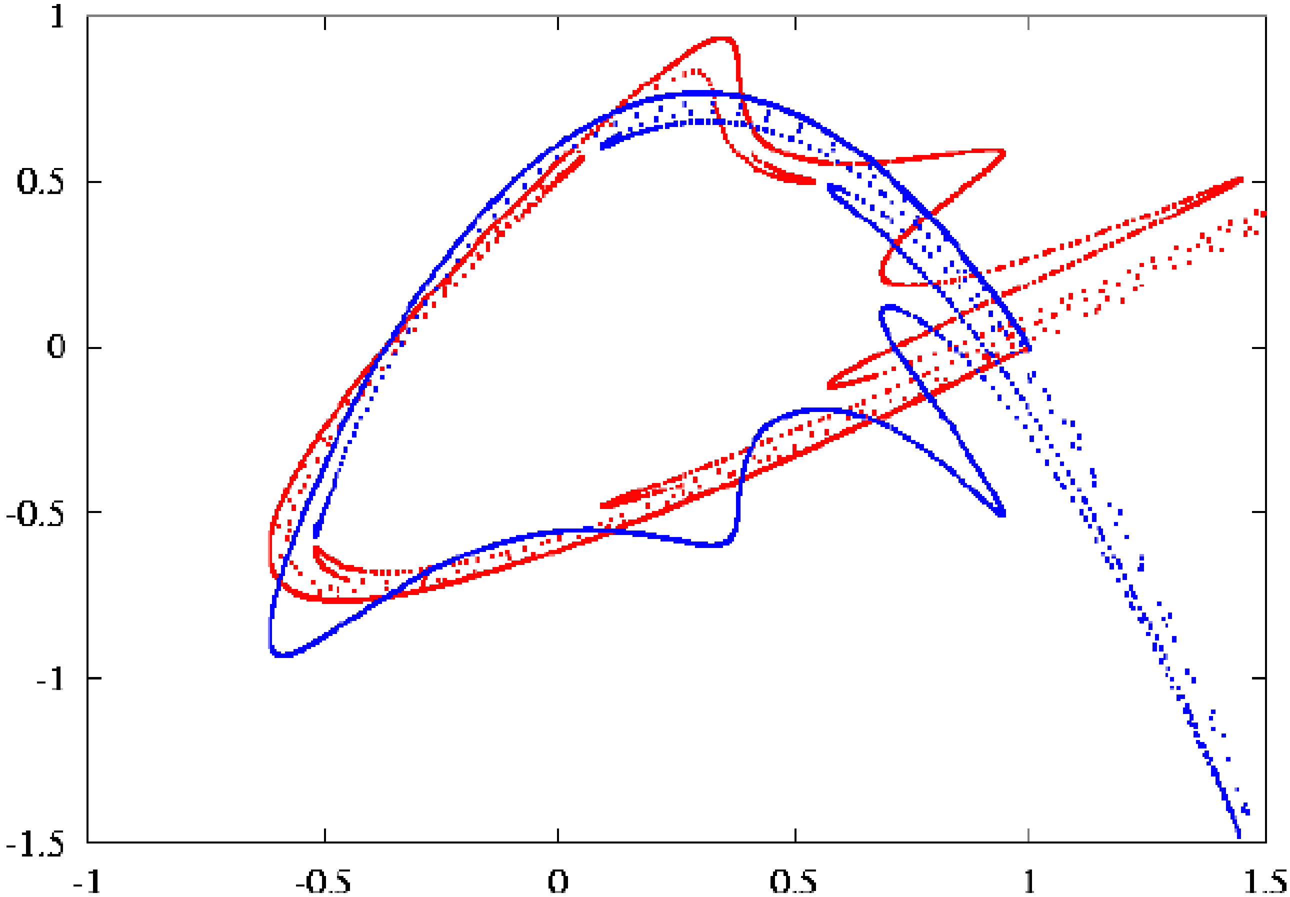

Figure 3: A typical homoclinic tangle in two dimensions.