MACROSCOPY OF RESONANCE

by

Ralph Abraham

University of California Santa Cruz

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1. Introduction.

This is a progress report on an experimental program, begun a year ago, in the exploration of resonant furcations (= catastrophes) by analog simulation and direct observation - the macroscope program. It was inspired by the ideas of THOM on psycholinguistics, ZEEMAN on Duffing's equation and the brain, and KENNEDY on EEG artifacts. At this meeting I learned of the prior work of FARADAY and BROOKE BENJAMIN from Lew HOWARD, and of the recent results on attractor furcations from Sheldon NEWHOUSE and Floris TAKENS. I am happy to acknowledge these influences. But I must especially express my gratitude to Hans JENNY, the great pioneer of experimental work in this area, who so generously shared his ideas and results with me in 1972 shortly before his death, and whose work I have continued in this program.

2. The coupling question.

A central theme in various applications of catastrophe theory (for example: communication, perception, memory recall) is the coupling question: if X_i is a vectorfield on a manifold M_i, $i = 1,2$, $X_1 \times X_2$ is the product vectorfield on $M_1 \times M_2$, and $A_1 \times A_2$ is the product of attractors A_i of X_i, what attractors can arise from the coupling of X_1 and X_2 (that is, the perturbation of $X_1 \times X_2$) - in other words, what are the generic furcations of $A_1 \times A_2$? We may translate this question into the context of catastrophe theory by introducing a simplifying idea: a flexible coupling is a generic, finite dimensional, perturbation of $X_1 \times X_2$ - that is, a stable map $\mathbb{R}^K \to \mathfrak{X}(M_1 \times M_2)$ through $X_1 \times X_2$. In applications, this occurs as a coupling device with controls.

For example, consider two oscillators with attractors $A_1 = A_2 = S^1$, coupled by a mechanical connection with stiffness $c \in \mathbb{R}$. In the product system, the attractor, $A_1 \times A_2 = T^2$, may pass through the vascillating furcations of Sotomayor as c is changed. If you suppose that X_1 and X_2 model distinct organisms, that X_2 percieves the state (occupied attractor) of the coupled system or at least its projection into his own state space M_2, and that X_1 can wilfully manipulate the control c, then X_1 can send X_2 messages consisting of words of an infinite alphabet. In another case, the product attractor may pass through the bifurcation found by Zeeman in the Duffing equation, in which case X_1 may send X_2 binary messages, like Morse code.

3. The case of forced oscillations. At this point I may make an aside for dynamical systems specialists: some geometric quantitative aspects of the phase portrait are important in applications. What are the proper definitions of the strength, amplitude, frequency, and speed of an attractor in a Riemannian manifold?

Now consider again the coupling question. The simplest case is $A_1 = A_2 =$ point. This includes the gradient case, and punctual furcations of resonant theory, such as the Hopf furcation and the blue sky catastrophe. Next consider A_2 = point, and A_1 arbitrary. Then the problem amounts to the usual furcation theory for A_1 for small perturbations, that is, before A_2 furcates. The case $A_1 = A_2 = S^1$ has been described above. Taking up the case $A_2 = S^1$ with A_1 arbitrary, we simplify the possibilities by supposing that A_2 is dominant, or very strong with respect to A_1, or in other words, only perturbations so small that A_2 does not furcate will be allowed. As X_2 is assumed to oscillate - that is, remain in attractor $A_2 = S^1$ although the frequency and amplitude of this oscillation may change - we may consider the product system in $M_1 \times M_2$ restricted to $M_1 \times A_2$. Therefore this case is equivalent to the classical model for forced oscillations: a periodic vectorfield, Ⓗ : $S^1 \to \mathcal{X}(M_1)$, of period, τ, equal to the period of A_2 in X_2.

4. Reduction to a cascade by stroboscopy. Having placed the problem of forced oscillations in a catastrophe scheme for coupled systems, and thereby causing a whole lot of probably unnecessary confusion, I will now connect it to something quite standard to clear the air. Recalling the procedure for transforming a time-dependent vectorfield into an autonomous system, we suspend the periodic vectorfield

$$Ⓗ : S^1 \to \mathcal{X}(M_1) : \theta \to X_\theta$$

to obtain a ring vectorfield, $Y \in \mathcal{X}(M_1 \times S^1)$ defined by

$$Y(m, \theta) = (X_\theta(m), \tfrac{1}{\tau})$$

where τ is the period of Ⓗ. But this ring vectorfield clearly has a global section and Poincaré map, Φ, a diffeomorphism of M_1.

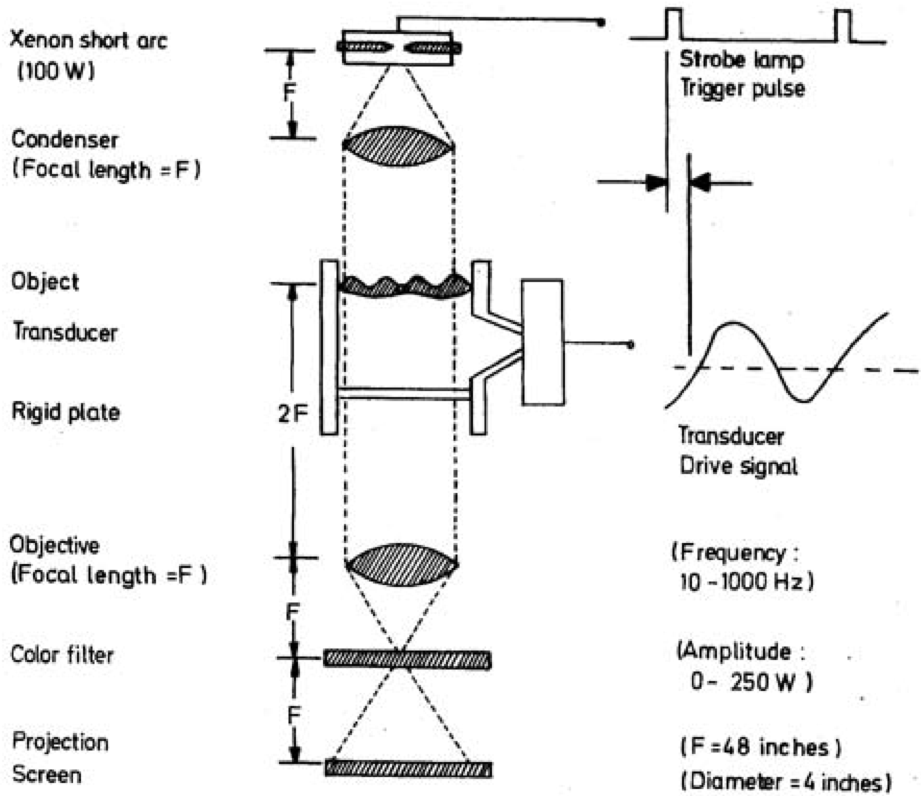
Thus Y is the suspension of Φ , and the periodic vectorfield, Ⓗ , is qualitatively equivalent to the cascade generated by the Poincaré diffeomorphism, Φ . Experimentally, Φ is revealed by stroboscopy: the ring system, Y , is observed only at times $t = n\tau$, $n \in Z$, when the orbitting point passes through the section. Obviously, some interesting quantitative information is lost in this reduction. For example - a fixed point, m , of Φ corresponds to a closed orbit, $\{\gamma_t = (m_t , t)\}$, of Y. Projecting this closed orbit into the section (phase space, M_1) we have a cycle, $\{m_t\}$, the diameter of which (assuming a metric) is the amplitude of the periodic motion of this point, M. Experimentally, this may be observed by changing the phase of the stroboscope, to observe successively all of the sections of the ring $M_1 \times S^1$.

So far, we have described the general question of coupling in catastrophe theory, singled out the special case of forced oscillations - coupling of an arbitrary dynamical system to a dominant closed orbit - and reduced this to a qualitatively equivalent cascade. The coupling is flexible, so the cascade depends on a parameter. The result of coupling is described by the furcations of the attractors of the section diffeomorphism. When the parameter is one or two dimensional, the furcations are partially known, through the results of HOPF, BRUNOVSKY, SOTOMAYER, NEWHOUSE-PALIS, RUELLE-TAKENS, TAKENS, and ZEEMAN. In my own view of applied mathematics, this question is of the greatest importance. Its exploration by simulation is the motivation for the macroscope project.

5. Simulation with Faraday's beer waves. The history of physics is punctuated with observations of fluids - especially powders - on vibrating plates and membranes. If the plate vibration is weak, standing waves are observed - these are the crispations observed by

FARADAY in beer. When the vibration is stronger, toral eddies are produced - the simmering observed by JENNY in lycopodium powder, and by VON BEKESY in the cochlea. Taking into account the model of RUELLE-TAKENS for hydrodynamical turbulence, we regard this system of forced oscillation as an analog computer simulating the Navier-Stokes equation with a periodic forcing term added. Ignoring the fact that the phase space is infinite dimensional, we expect attractors of finite dimensional vectorfields as observed states of the vibrating fluid, and to see their furcations as the frequency and amplitude (flexible coupling) of the dominant driving attractor are changed. The transition from crispating to simmering is an example, and furcations of HOPF-TAKENS (wobble) and DUFFING-ZEEMAN (jump) types are to be expected.

6. Observation by macroscopy. The macroscope consists of (1) an electronic function generator, producing a powerful sine wave (the dominant attractor, 0 - 50 kiloHertz by 0 - 250 Watts) and a synchronous trigger pulse (of adjustable phase) for the strobe light, (2) a control plane monitor, showing the frequency and amplitude of the driving signal as an illuminated point on a video tube, as well as digital meters, (3) an electromechanical transducer (loudspeaker, courtesy of Acoustic Research) coupled acoustically to (4) a transparent dish with a flexible bottom, containing a thin layer of glycerol thinned with water, and (5) a color schlieren optical system of SETTLES type, illuminated by a pulsed point source (100 Watt xenon arc lamp, courtesy of Chadwick-Helmuth). The arrangement, indicated in the Figure in linear equivalent, is folded with plane mirrors to fit in the laboratory, and the principal optical elements are a matched pair of f/10 telescope mirrors (courtesy of Lick Observatory). The macroscope design was inspired by the prototypes built by JENNY and VON BEKESY.



SCHEMATIC

THE FOUR INCH MACROSCOPE

The preliminary observations with this instrument verify the expectations - the effects photographed by JENNY can be replicated, as well as those reported previously by FARADAY, RAYLEIGH, VON BEKESY, BROOK BENJAMIN, and BAUER. Furthermore, the innovations of color schlieren optics and strobe phase control allow new and subtle furcations to be observed, and recorded in wax pencil on the face of the video screen modelling the control plane. A distinct fold, producing prolonged hysteresis, is observed on the furcation line between the elastic (crispation) and fluid (simmering) regions. A fine network of jump furcations, reminiscent of the TAYLOR-COUETTE situation, fills both regions. At large amplitudes, complicated wobble furcations abound. All of these are shown in the videotape, Introduction to the Macroscope.

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BIBLIOGRAPHY

Abraham, R. H., Introduction to Morphology, Dept. de Mathematiques, Univ. de Lyon (1972).

______________, Psychotronic vibrations, First Int. Congress Psychotronics, Prague (1973).

______________, Macrodynamics and morphogenesis, in JANTSCH and WADDINGTON.

______________, Introduction to the Macroscope (videotape), Univ. of California, Santa Cruz (1975).

Bauer, H. F., Chang, S. S., and Wang, J. T. S., Nonlinear liquid motion in a longitudally excited container with elastic bottom, J. Amer. Inst. Aeronautics and Astronautics, 9 (1971) 2333-2339.

Brook Benjamin, T. and Ursell, F., The stability of a plane free surface of a liquid in vertical periodic motion, Proc. Roy. Soc. (London) Ser. A. 225 (1954) 505-517.

Brunovsky, P., On one-parameter families of diffeomorphisms, Comment. Math. Univ. Carolinae 11 (1970) 559-582.

____________, On one-parameter families of diffeomorphisms II, Comment. Math. Univ. Carolinae (to be published).

Faraday, M., On the forms and states assumed by fluids in contact with vibrating elastic surfaces, Phil. Trans. 121 (1831) 319-346.

Hopf, E., Abzweigung einer periodischen Lösung von einer stationairen Lösung eines Differential systems, Ber. Math. - Phys. Kl. Sächs. Acad. Wiss. Leipzig 94 (1942) 1-22.

Jantsch, E. and Waddington, C., eds., Evolution in the Human World (to appear).

Jenny, H., Kymatik, Basilius, Basel (1967).

________, Kymatik, Band II, Basilius, Basel (1972).

Lyttleton, R. A., Stability of Rotating Liquid Masses, Cambridge (1953).

Magarvey, R. H. and MacLatchy, C. S., The formation and structure of vortex rings, the disintegration of vortex rings, Canadian J. Phys. 42 (1964) 678 - 689.

Newhouse, S., and Palis, J., Bifurcations of Morse-Simale Dynamical systems, in PEIXOTO, 303 - 366.

___________, Cycles and bifurcations (to appear).

Peixoto, M. M., ed., Dynamical Systems, Academic, New York (1973).

Rayleigh, Lord, On the crispations of fluid resting upon a vibrating support, Phil. Mag. 16 (1883) 50 - 58.

Ruelle, D., and Takens, F., On the nature of turbulence, Comm. Math. Phys. 20 (1971) 167 - 192 and 23 (1971) 343 - 344.

Settles, G., The amateur scientist, Sci. Amer. (May, 1971).

Sotomayor, J., Generic one parameter families of vector fields in two-dimensional manifolds, Publ. Math. I.H.E.S. 43.

____________, Structural stability and bifurcation theory, in PEIXOTO 549 - 560.

____________, Generic bifurcations of dynamical systems, in PEIXOTO 561 - 582.

____________, Saddle connections of dynamical systems (to appear).

Takens, F., Unfoldings of certain singularities of vectorfields: generalized Hopf bifurcations, J. Diff. Eq. 14 (1973) 476 - 493.

__________, Forced oscillations, Publ. Math. Inst. Utrecht (1974).

Thom, R., Language et catastrophes: eléments pour une sémantique topologique, in PEIXOTO, 619 - 654.

Turner, J. S., Bouyancy Phenomena in Fluids.

Von Békésy, G., Experiments in Hearing, McGraw - Hill, New York (1960).

Zeeman, C., Duffing's equation in brain modeling (this volume).