SIMULATION OF CASCADES BY VIDEO FEEDBACK

by

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<u>1. Video from the Topological Viewpoint</u>. The video process is based upon the piecewise linear approximation of a rectangle via scanning. We shall ignore the fact that geometrically, this rectangle is a torus with two equators removed. Thus a standard black and white video event - achieved with a video camera and monitor - is approximately a bundle map

$$\begin{array}{ccc} I^2 \times I & \xrightarrow{v} & I^2 \times I \\ \downarrow & & \downarrow \\ I^2 & \xrightarrow{b} & I^2 \end{array}$$

where I^2 is the rectangular screen, and the fiber, I, is the brightness on the screen. Here the base map, b, is supposed to be the identity. For color video, the fiber must be replaced by I^3.

<u>2. Synthesis of an Endomorphism</u>. To simulate a map $I^2 \to I^2$, we

first eliminate the map on the fiber. The video hardware (camera plus monitor) must be modified to binary mode - that is, brightness either on or off. To approximate binary mode, insert stair-step (grey scale) to the monitor. Adjust contrast control at maximum, and brightness control near minimum, so that half the monitor screen is black and half maximum white. This unit simulates a map - approximately the identity - as follows: Aim the camera at a white rectangle - the source, S. Adjust distance or zoom so that the image of S fills the screen, T, of the monitor. Darken the room. Illuminate a point $s \in S$ with a penlight, and observe $b(s) \in T$.

At this point, we have constructed a simulator for an accidental endomorphism of the rectangle, close to the identity map. To simulate an arbitrary endomorphism it is only necessary to reprogram the periodic scan function, $S^1 \to \mathbb{R}^2$, of the monitor. The most practical way of achieving this is by using a minicomputer to generate the desired scan function, but magnetic tape (providing external synch for camera and monitor as well as horizontal and vertical deflection signals for the monitor) can be used as well.

3. Interation by Feedback. Having completed the simulation of a map of interest, say the horseshoe diffeomorphism of Smale, for example, we would certainly be curious to study its asymptotic behavior. With two identical units, and aiming the second camera at the first monitor, we can simulate the square of the map, but soon we will run out of hardware. If we operate the first unit - freeze the result on the screen - and then reuse the first unit for the second iterate, we achieve simulation of the square of the map without iterating the hardware. Furthermore, the iteration process can be repeated indefinitely. This suggests the following experiment: take the binary video unit described above, and aim the

camera at the monitor screen, $T = S$.

In principle, the endomorphism will be iterated at the rate of 60 frames per second. The attractors may be observed by turning the system on, darkening the room, removing the lens cap, and lighting a penlight at a point $s \in S = T$. The orbit of s appears on the screen, moving swiftly to the ω limit set, which remains lit on the screen after the action dies away.

4. The Moiré Problem. Unfortunately, it's not that easy. Back at the start, we observed that the video event is approximately a bundle map over I^2 . Actually, linear scanning is not a good approximation to a rectangle. The video camera looks at the monitor and sees a row of bright line segments moving slowly downward. It scans this row with its own (not parallel) moving line. A beautiful moiré results - which video artists utilize in their productions - but to dynamical systems theory this is noise.

Attempts to eliminate moiré noise were made by inserting a time delay unit between the camera and the synthesizer, with partial success. The ideal solution is to replace the video monitor with a storage scope. Then the monitor image is frozen while the camera scan process is carried out, and no moiré is produced. The camera scan data must be stored on tape - as in the time delay unit - just until the scan is completed. Then the storage scope is erased, and the recorded camera scan read off the tape into the storage scope again. Only a factor of 2 in interation rate is lost. Not having a storage scope yet, we have not implemented this scheme so far. The other schemes described are illustrated on the videotape, Video Feedback.

The goal of this simulation device is the observation of attractor furcations. With a minicomputer programming the monitor scan, the endomorphism could be changed with controls, and furcations observed on the monitor screen, and simultaneously recorded on videotape.

5. Postscript on Migraines. After viewing the moiré patterns in the videotape, René THOM remarked their similarity to the Airy patterns of migraines. This suggests the following model for the migraine phenomena: a spiral or circular pattern - caused by a periodic biochemical reaction, or an electrochemical diffusion wave, or a liquid crystal flexoelectric vibration - sweeps over the visual cortex. Moiré interference arises between this extrinsic pattern and an intrinsic cortical pattern, such as the hexagonal cells described by RICHARDS, or feature detection structures (especially, edge detectors). A similar situation occurs in the well studied spreading depression of LEÃO. This analogy suggests that over-stimulation of the visual cortex might initiate migraine, and this is in fact reported by some migraine subjects.

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