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7. The Function of Mathematics in the Evolution of the Noosphere

Abstract

A model for the noosphere is proposed, in the spirit of geometry and dynamics, in which sociodynamics and information flow cause bifurcations, while mathematics is responsible for synthesis. The concordance of the noosphere is embedded in the model, all illustrated by recent events in mathematical physics. Some speculations on the future, including the impact of computers, are presented.

Bifurcation and Synthesis

The world of ideas -- evanescent bubble of knowledge, inflated by millenia of human thought, attached to our fragile culture, maintained on paper and in consciousness by words and drawings -- is too vast to grok. Our languages scarcely have words for it. So let us call it the noosphere, and distinguish it from the world of ideas of Plato -- if there may be such a universal store of form, beyond the emergence of knowledge into the consciousness of our planetary society. The noosphere evolves and bifurcates into independent domains, as required by the limited capacity of individual humans for the storage and manipulation of information. (A version of the recent history of the noosphere of our own culture is shown in Figs. 1 and 2.) The very dynamics of this process of growth and bifurcation itself evolves, as the culture develops information and communication technology such as printing, photography, electronics -- which extends the capacity of the individual servants of the noosphere. Thus it may occur, in the history of knowledge, that independent domains recombine and synthesis occurs.

We may think of bifurcation and synthesis as opposed forces in the evolution of the noosphere, as the masculine and feminine principles of evolutionary dynamics. And our

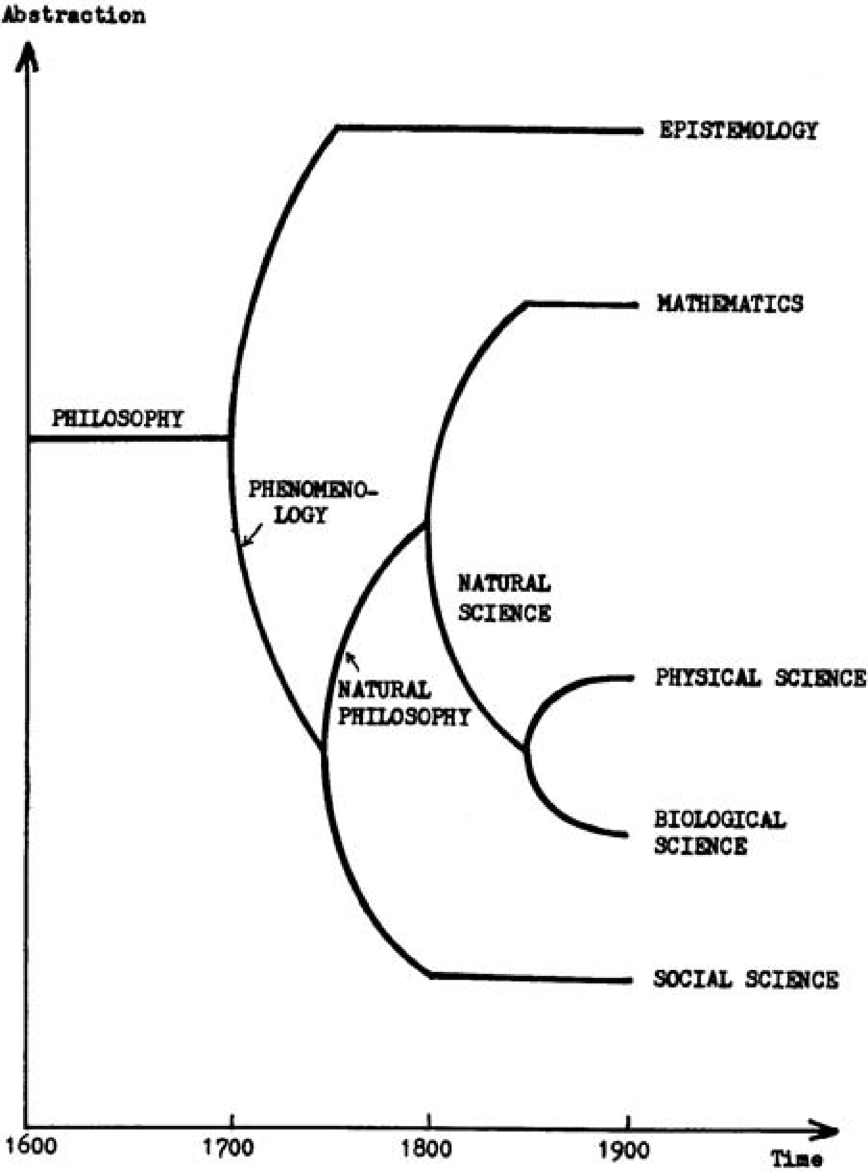

Figure 1. Bifurcation sequence of the noosphere: Philosophy in the 17th, 18th and 19th centuries. The bifurcations shown are "socio-informatic," that is to say, they indicate a separation of the scholarly community into subgroups which intercommunicate poorly (or, a reunion of such), as indicated in the text.

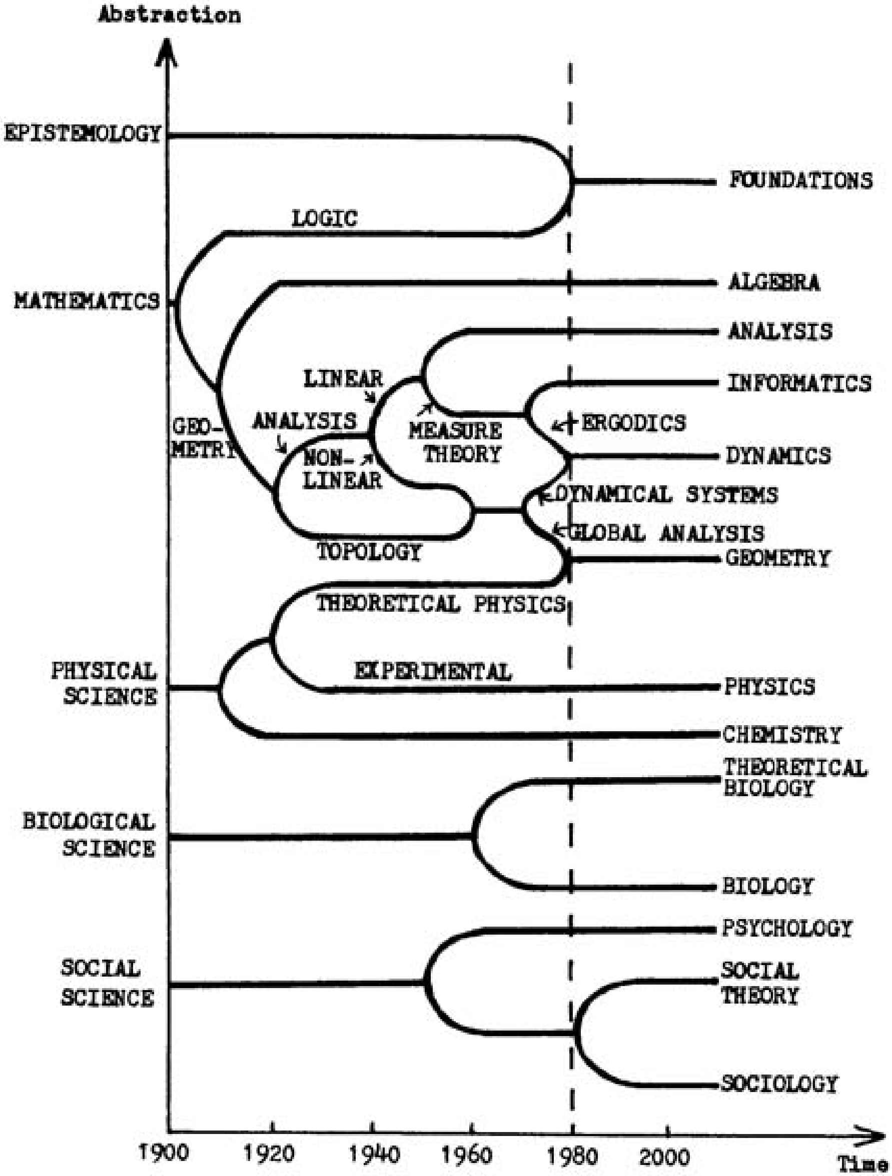

Figure 2. Bifurcation sequence of the noosphere: Philosophy in the 20th century. Fine structure is shown here in the mathematical zone only.

time is one of the domination of the noosphere by the masculine force, in this sense. But the computer revolution currently in progress may extend the information capacity of the servants of knowledge, if they succeed in adapting the emerging technology to this purpose, and thus enhance enormously the forces of synthesis. It may be that the future history of the noosphere in fact demands synthesis soon, to avoid dispersion of knowledge into superstition, or the knowledge death of the Egyptian, Chinese, Arabic, and Mayan cultures described by White (1979).

On the other hand, it may be that the rigidity of society will resist the emergence of the feminine principle (synthesis) or that counter-evolutionary forces will monopolize the new technology, and our cultural noosphere is doomed to follow these examples. My own experience as an extreme specialist in the world of ideas, attempting syntheses on a minute scale, has been discouraging. One feels pressures of all sorts, pushing backwards towards the security of specialization, conventional work, easy appreciation. It is thus with the greatest trepidation that I now put forward these trial ideas on evolution, the role of mathematics, the potential of information machines, and the future of our noosphere.

Geometric Models of the Noosphere

We may visualize the world of ideas divided conventionally into subjects, as in a library or university catalogue. Alternatively, we may view the noosphere sociometrically: A "subject" or domain is defined by a group of scholars, and we distinguish subjects as disjoint areas when the scholarly groups defining them intercommunicate poorly. Taking this latter point of view, a portion of the noosphere corresponding to "philosophy" is shown schematically in Fig. 1. As time progresses through the period 1600 - 1900 (roughly), the subject bifurcates successively into disjoint areas in the sociometric sense. The corresponding schematic diagram for the past century, Fig. 2, reveals numerous syntheses as well as bifurcations. These occur when two specialized groups get interested in each other, learn to intercommunicate, and combine as an informatic organism.

These schematic diagrams are not yet geometric models. We must imagine a representation of the noosphere with more dimensions, within which each "subject" is a surface (of two or more dimensions), and among which the "bifurcations" are such as those of catastrophe theory, or some other classification even more general. For example, the earliest bifurcation of Fig. 1 is represented in Fig. 3 as a cusp catas-

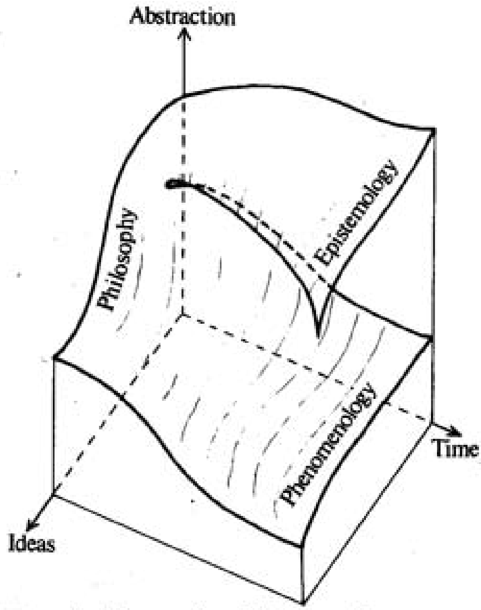

Figure 3. Representation of a bifurcation as a cusp catastrophe.

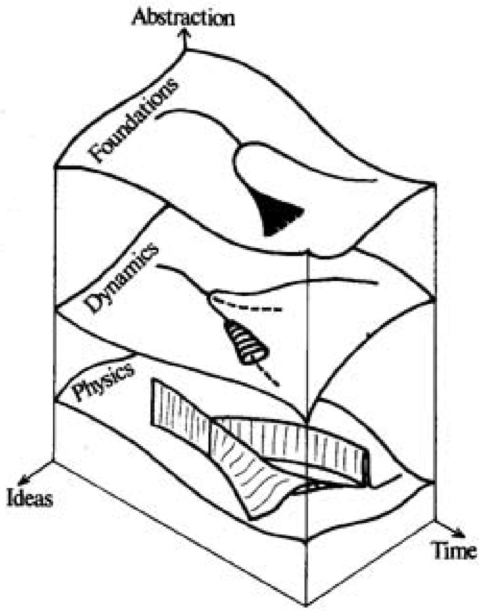

Figure 4. Morphogenesis of data, shown as concordant bifurcations on parallel sheets of the sociodynamic substrate.

trophe -- see Thom (1972), Zeeman (1977) or Poston and Stewart (1978) for the concepts of catastrophe theory. With the whole history of philosophic scholarship represented in such a pictorial scheme, we would have a geometric model of the noosphere.

We proceed now, without an actual geometric model, but as if we had one. Then a small piece of the model, a typical piece, would consist simply of a finite stack of surfaces of various dimensions. Such a "neighborhood" of our fictitious geometric model of the noosphere is shown in Fig. 4. Here, parallel small pieces of three "subjects" are depicted as surfaces of dimension two. A local chart of coordinates -- abstraction, morphology, time -- is shown. We imagine that, if our geometric model were fully described (as it is not), then these local coordinates would likewise be exactly defined (they are not.) But these vague coordinates are introduced in Fig. 4 just to give intuitive sense to this neighborhood in the world of ideas.

These surfaces represent the evolution in time of a subject defined by intercommunicating groups of scholars. This is the substrate of the information (data, texts) of their scholarly discourse, not the actual content of it. So we further imagine -- defined upon these "surfaces" or substrate -- some geometric objects which model (no matter how crudely) the informational content of the subject (field, area of knowledge.) And these geometric data structures also evolve in time, so that differentiation and morphogenesis occur on each surface. This morphogenesis of data, on the sociometric substrate of the model of the noosphere, is shown in Fig. 4.

Such a composite picture -- bifurcation sequence, geometric representation, and data structures including morphogenesis -- is what we mean by a geometric model of the noosphere.

Concordance

We suppose now that a geometric model is at hand, that it has been determined from the historical record -- the libraries of our culture -- according to a constructive algorithm and that the geometric dimensions are at least intuitively meaningful to future historians. If this seems far-fetched, remember that this is done daily by applied mathematicians, at least for very small neighborhoods. For examples, see Poston and Renfrew (1979), or Thompson (1979). What is proposed here is no more than a certain enlargement of scale. Let M denote the substrate of this model -- the geometric surfaces without the data.

For the sake of further discussion, we now make a restrictive geometric assumption about this model: the layered structure shown within a small neighborhood in Fig. 4 exists throughout the model. To be more precise, we assume there is a certain "master substrate", or space, S, and a map from the substrate of the model, M, into S x R (here, R denotes the real numbers)

$$\pi: \quad M \to S \times R$$

The master substrate, S, is an extension of the coordinate called "morphology" in Fig. 4, and the real line, R, represents historical "time". We further suppose that the layers of M are each divided into a finite number of pieces by reasonable boundaries, and that, restricted to any of these pieces P, the map π: $P \to S \times R$ is regular (that is, approximately a linear projection locally) onto an open subset of S x R. The map π: $M \to S \times R$, together with such an assumption of piecewise regularity, is a <u>concordance</u> of the model.

The idea behind our assumption of a concordance is this: We want to relate different descriptions of the phenomenal universe, perceived by the different disciplines, as if these were an objective reality beneath phenomena. Thus, the mathematical, physical, chemical, and biological descriptions of an "event", are imagined to correspond. The concordance of the model, π, represents this correspondence. Thus two different points of the model -- m_1 and m_2 in M -- which have the same labels assigned by π:

$$\pi(m_1) = \pi(m_2) = \pi(s,t)$$

are supposed to locate data structures describing the same event. We may suppose, in addition, that the data structures of the model are such as allow comparison at different points. Thus we could say: the data at m_1 are similar to those at m_2, or they are not. How this comparison may be accomplished is very difficult to describe in general. But we have excellent examples in the history of mathematical physics, which we will soon describe.

Mathematics and the Natural Sciences: An Exemplary Bifurcation

Our proposal for a geometric model of the noosphere is not solely a cognitive device. We have in mind a causative dynamics -- a sort of force field on the sociometric space -- to account for the bifurcation and synthesis of "subjects" in the course of time. As viewed here, this is a psychosocial process, due to the migration of interest of scholars, and their capability to communicate with each other. The coupling of intrinsic properties of subject areas with psychological and social factors are involved in the migration, and thus in the dynamics. We now consider, from this point of view, the bifurcation of natural philosophy into mathematics and the sciences.

We must characterize the differences between mathematics and physics, from the scholar's point of view. Formerly, one used to say that mathematics is more abstract -- a formal (axiomatic) system, its truth absolute and decidable-- while physics is based on phenomenal reality -- a conventional system, its truth relative and empirical. Lately, the emphasis on the empirical process in mathematics, by Lakatos (1976), inclines some to identify mathematics with physics. This is a sort of materialism, like the identification of mind and body. But from the psychological point of view, the feeling of truth of a mathematical theorem is based on faith in its proofs -- its conformity to the formal (logical) system at the foundation of mathematics -- which increases with time, as trusted workers check the logical proofs repeatedly, and testify to their completeness and accuracy. This is well described in Manin (1979).

In contrast, the feeling of truth in a physical theory depends upon faith in its tests -- its conformity to the phenomenal universe -- which increases with time, as trusted workers check the empirical tests repeatedly, obtaining consistent results. In spite of the similarity in these faith mechanisms, there is a difference of polarity. Mathematicians look upward, and physicists downward, for the impression of truth. Thus also individuals attracted to natural philosophy will be polarized towards mathematics or the sciences, according to their tendency to believe in inner (upward, logical, mental, personal) or outer (downward, empirical, physical, social) reality. This dynamics polarized the natural philosophers into two parties, Inwards and Outwards. Yet this polarized group of scholars is to be considered a single "subject" in our model for the noosphere, as long as the two kinds each are in mastery of the entire subject. When this ceased to be the case, in the history of our own culture, is difficult to pinpoint in time. In Fig. 1, we have set it rather arbitrarily around 1800. In any case, the post-bifurcation subjects are identified with two groups of scholars (mathematicians and scientists) which intercommunicate poorly, and suffer mutual suspicion, jealousy, and competition.

These two subjects eventually provide an excellent example of the sociodynamics of synthesis as well. In Fig. 2, a synthesis between global analysis and theoretical physics is indicated around 1980, that is to say, at the present time. We have seen a decade or two of struggle by individuals within these groups to learn the language of the other group, for pursuit of their own goals. The progress of global analysis, within its own domain, produced simplification to such an extent that its language became learnable by theoretical physicists. Meanwhile, successive generations of theoretical physicists learned bits of global analysis, and rewrote their subject in this language, making it accessible to the mathematicians. As these physicists seek a theory of a mathematical type, the psychological conditions for synthesis are favorable, and so it progresses at present.

Geometry and Physics: An Outstanding Concordance

Now, that the basic concepts of our essay -- sociometric domain, bifurcation, and synthesis in the universe of ideas-- have been illustrated with mathematics and the natural sciences, we return to the supplementary notion of concordance.

If two subject areas are actually a single organism in our sense -- as, for example, mathematics and natural science (that is, natural philosophy) before the 19th century (see Fig. 1) -- one would not be surprised by a correspondence in their morphology. In fact, an inconsistency in the data structures of two parties (such as the Inwards and the Outwards) would undoubtedly produce a bifurcation. However, after a bifurcation, the concordance of two disjoint subjects is a cause of awe, wonder, and the suspicion of miracles, divine works, and Platonic ideas.

Two outstanding examples from the literature of our century are provided by Einstein and Wigner. A century or so after the separation of mathematics and physics (according to the very rough scheme of Fig. 1), the success of general relativity theory as a geometric model for the solar system prompted Einstein (1921) to address the Prussian Academy of Science thus:

> At this point, an enigma presents itself, which in all ages has agitated inquiring minds. How can it be that mathematics, being after all a product of human thought which is independent of experience, is so admirably appropriate to the objects of reality? Is human reason then, without experience, merely by taking thought, able to fathom the properties of real things?

And more recently, inspired by the success of group representation theory in modelling elementary particle physics, Wigner (1959) wrote:

> Mathematical concepts turn up in entirely unexpected connections. They often permit an unexpectedly close and accurate description of the phenomena in these connections ... It is difficult to avoid the impression that a miracle confronts us here...

These two examples of the successful application of mathematics to physics are outstanding for the independence of the mathematical discoveries (tensor geometry, classification of representations of Lie groups) from the physical data (motion of Mercury, hadron multiplets) and for the precision of the concordance. But we should note that applied mathematics -- the art of exploring concordance -- had many other success stories in its history, and the high technology of our culture is based upon them. The concordance of geometry and physics continues to grow, for example, with the Ruelle-Takens (1971) model for turbulence.

Dynamics and the Sciences: Concordance on a Larger Scale

As a branch of mathematics, dynamics was born with Newton, launched into prominence a century ago by Poincaré, and became an autonomous subject, in our sense, within the past decade or two -- as shown in Fig. 2. The basic concepts of dynamics -- attractors (simple and chaotic), basins, separatrixes, robustness, and bifurcation -- are described, in historical perspective, in Chapter 8 of Abraham and Marsden (1978). An unusual feature of this area is the role played by computing machines, which emerged (along with information science) as a distinct subject in the same period of time (again, see Fig. 2) (Stein and Ulam, 1964).

This concomitance has polarized the dynamics community, which (unlike other branches of mathematics) has an essential experimental subgroup. Eventually, dynamics may bifurcate into theoretical and experimental camps. For example, one of the central ideas of dynamics is that of a chaotic attractor. This was discovered by experimentalists, but came to the specific notice of the theoreticians only a few years later. In fact, it received explicit attention neither in the influential survey of Smale (1967) nor in the futuristic book of Thom (1972). At present, the polarization of dynamics is not yet a bifurcation, as the two groups intercommunicate well. The information load may soon overwhelm the communication channel, or the storage capacity of individual dynamicists, and create a separation.

We may divide theoretical dynamics into three branches, considering the already extensive and rapidly growing application literature. These are:

- Dynamical systems (DS) theory, dealing with the classification of attractors, the characterization of robustness, and generic dynamical properties, as described in Smale (1967) or Abraham and Marsden (1978).

- Elementary catastrophe (EC) theory, the classification of general bifurcation of static attractors -- see Zeeman (1977) or Poston and Stewart (1978).

- Dynamic bifurcation (DB) theory, the classification of generic bifurcation attractors, outlined in Thom (1972).

These three theories may have the most impressive and extensive concordances in the history of applied mathematics. The lists of "admirably appropriate" applications and "unexpectedly close and accurate descriptions" is extended daily in the rapidly growing literature. By now there are exceptional concordances of:

- DS Theory with electronics, game theory, meteorology;

- EC Theory with sociology, naval architecture, mechanical engineering, linguistics, optics; and

- DB Theory with hydrodynamics and elastodynamics;

to list just a few. The pioneering text of Thom (1972) suggests very novel correspondence of the metaphors of DB Theory with numerous fields. And in fact, we have hinted here at a DB-theoretic model of the noosphere, in which DB theory would model itself, amid all the rest of our evolving knowledge, social structure, and psychohistory.

A Platonic Confession

We acknowledge the incredibility of so vast a concordance in the world of ideas. Some of the early publications of EC applications have been criticized as "wild claims", most notably in the epistomological megalith of Fussbudget and Znarler (1979). We admit a bias in favor of concordance, and moreover toward a Platonic idea reality -- an additional sheet in Fig. 4, high above philosophy, hidden from consciousness by clouds, yet pinned through to the fabric of the noosphere by an extensive concordance, as a pattern for life. Yet we come out of this closet -- like Plato, Einstein, Gödel and all other revealed believers before us -- in full confidence that extreme specialists and conservative informalists in great numbers will confirm the concordance of mathematics and all the sciences beyond question.

This Platonic faith obviates logical difficulties as well. It assumes that the exploration of mathematical reality and the discovery of its secrets by our society are a matter of revelation or creative intuition, as described by Hadamard (1964). The extent of this discovery process, being limited by the structural evolution of the inquiring minds of our times, yields a poor sketch of the terrain, expressed as a formal (logical) system. But the full flavor of Platonic reality is harmonious and consistent beyond the capability of our formal languages.

Thus, the Inward faith is essentially unassailable. Yet our view of the role of mathematics, in the evolution of our noosphere, inspired by Whitehead (1929), is independent of this bias. The Outward view -- that concordance grows from roots in phenomenal (ordinary) reality beneath the lowest sheets of the noosphere (as represented in our geometric model, for example, below Fig. 4) and that mathematics results from the abstraction process, applied to human perception of the real world, and carried to extremes -- equally admits the growth of a culturally determined, concordant noosphere. In fact, Inward and Outward scholars work side by side, harmoniously, unconscious of their faiths. Tirelessly serving the shared principle of concordance, they jointly erase conflicting data. So it is no wonder that our cultural noosphere is concordant.

When, however, in the future an Alexandrian library might be unearthed by archeologists, or a Mayan Codex overlooked by Bishop daLanda, or if a UFO was to land at a terminal of the galactic library in the sky, might we not be amazed by an Outwardly inexplicable, cross-cultural concordance? We might look first for the mathematical leaf of the alternative noosphere, seeking a correspondence with our own.

The Evolutionary Roles of Mathematics and Informatics

What is this special role of mathematics in the evolution of cultural noospheres? Mathematics is abstract enough to be central to an extensive concordance and yet precise enough for these to be meaningful, even amazing. Its own morphogenesis, -- whether Inward or Outward directed, or a random process -- leads the corresponding morphogenesis in scientific domains. In this way, Ricci calculus preceded Einstein relativity, Cartan classification preceded Gell-Mann quarks, and Lorenz attractor preceded Ruelle-Takens turbulence. But beyond this temporal leadership, in which metaphors emerge into the evolving consciousness of our culture on an abstract level and are mirrored on the more concrete planes a few years later, mathematics serves the feminine principle, synthesis, in a functional way.

As described above, specialization and bifurcation in the world represent an informational defect in the sociodynamics of the scholarly community. The limits of individual information handling capacity mandate the separation of a scholarly group into special subgroups. The subgroups drift apart, from the communication point of view, as the local language of a subgroup expands to fill its vocabulary capacity, thus pushing the vocabulary of the complementary subgroup out of local memory.

Mathematics, as a higher-order language which grows vertically, provides ever more compactification and efficiency in the technical languages it serves. Thus the "chaotic attractor" of mathematics may replace "turbulent, broad spectrum, stochastic, aperiodic, ergodic, noisy" and a host of other concordant concepts of the sciences. The compactification reduces memory requirements, permitting groups to learn some different words of each other's vocabularies. And the commonality of the mathematical metaphors allows a limited intercourse in a universal language, among all groups knowing some mathematics. Thus, mathematics decreases the informatic distances between scholarly groups at the same time that their intrinsic efforts tend only to increase them.

In fairness, we must admit that the growing role of computing machines, and the associated scholarly domain of information science, share exactly the same evolutionary roles in the future growth of our noosphere. First of all, mathematics itself relies increasingly on machines for proofs, management of literature, and experiments with algebraic, geometric, and dynamical systems. Further, the use of machines by the various disjoint scholarly groups increases their information capacity, and thus their power to intercommunicate. Similarly, the use of machines for communication networks will increase the information capacity of channels interconnecting these groups. And finally, and most importantly, the new concepts of information science, like those of mathematics, are highly compactive and efficient as linguistic elements for scientific use. In fact, we have used some in describing the role of mathematics above.

In both cases, mathematics as well as informatics, we have described qualitative utilitarian roles in addition to the obvious quantitative one: computation. So let us note here that computation has played a primary role in the growth of the natural sciences in the past and will remain important in the future, but the qualitative function of conceptual morphogenesis has in both cases, surpassed computation in its evolutionary importance. Thus we have emphasized here qualitative (especially geometric) mathematics, informatics, and machines.

The Future

We have looked at the noosphere from the coarse point of view of sociodynamics, and seen its morphogenesis -- including the special role of mathematics and informatics, in promoting syntheses, to balance the inevitable tendency toward fragmentation -- in the visual metaphors of dynamics and catastrophe theory. To this picture, a mathematical formulation of "concordance" has been adjoined and we have proposed that a vast concordance of unprecedented scale is presently emerging in our noosphere. This is an occasion of tremendous excitement in the scholarly community. Projecting into the future (if indeed the planetary political reality admits one) we see thus a catastrophic struggle between the masculine principle (fragmentation) and the feminine (synthesis). In our view, the future evolution of our noosphere will be possible only with a balance of these forces. At present, scientific patriarchy (specialization) dominates, and synthesis is oppressed. Thus, to nourish our future, extra fuel should be provided mathematics, informatics, the access of mathematicians to computing machines, the applications of mathematics to all fields, the intercommunication devices such as computer networks and interdisciplinary conferences, and the entry of feminists into the scholarly community. Yet the social climate for this nourishment of synthesis in the noosphere appears cool. The feminine principle in the scholarly world is starving. Perhaps the mathematical-informatical community should take more responsibility in the field of public education and aggressively seek support.

Our idea in this essay on sagacity theory, or psychohistory, is to begin the development of a model for the noosphere, aided by mathematics, with which in the future we may pick up the reins of evolution and choose our own future historv

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