CHAPTER 1

Complex Dynamical Systems Theory: Historical Origins, Contemporary Applications

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Editor's Introduction: In this opening chapter Ralph Abraham, a pioneer of the mathematical modeling and simulation of complex systems, traces the origins of the latest mathematical theories applicable to the systems that emerge and evolve in various domains, in nature as well as in society.

A concise and definitive account, the chapter serves as a general introduction to the mathematical underpinnings of general evolution theory, comprehensible to the layman as well as to the specialist. A number of the basic concepts that appear throughout this volume are here described and defined, including the concept of dynamical system itself, and the modeling and simulation of its various stable and unstable states with the help of static, periodic and chaotic attractors and the processes known as bifurcation.

Aware of the practical potential of the mathematical simulation of real-world systems as well as of the mushrooming problems that face contemporary people and societies, Abraham stresses the urgency and importance of researching and developing applications of the mathematical models of complex systems in the human and social sciences.

INTRODUCTION

Since the last glaciation, we have extensive records of some ten thousand years of the struggles of the human species for survival within the ecosystems of Terra. We have coextensive records of the evolution of consciousness, wisdom, intelligence, arts, sciences, and technology. The mutual interactions between these two levels of history have been critical to the survival of our colony up to the present moment, and will

Dedicated to Erich Jantsch (1929–1980)

continue to be critical, as we face the challenges to come. In this essay, we examine the cognitive strategies entwined in the historical records of the sciences, and propose an extrapolation for the near future which may be essential for our survival: *the mathematical acceleration of social theory.*

We will begin with a brief history of the role of mathematics in the development of the sciences since Newton, from the viewpoint of modeling and simulation. Then, we will outline three case studies: dynamics, physiology, and sociology. Finally, we propose an inexpensive project for the accelerated development of a large-scale model of our emerging planetary society, suitable for high-speed simulation by existing supercomputers.

The motivation of this essay, and the proposed project, is the challenge of meeting the oncoming evolutionary crisis, and surmounting it, through a timely increase of our understanding of complex systems and their transformations. For we feel that this increase in understanding will come soon, or never.

HISTORICAL INTERACTIONS BETWEEN MATHEMATICS AND THE SCIENCES

Mathematics is not a science, nor is science mathematical. The applications of mathematics to the sciences involves, in fact, a relatively small part of our mathematical activity, and an even smaller smaller part of our scientific efforts. Yet historically, this interaction has been particularly important in the development of each. This is particularly true since Huyghens, Newton and Leibniz, who were primarily responsible for the cognitive style which dominates scientific theory today.

Applied mathematics, as we may call the interaction between mathematics and the sciences, has two aspects: *modeling and simulation.* Modeling denotes the creative activity of building a mathematical model for a given phenomenon, or experimental domain. It may involve any branch of mathematics in the architecture, construction, testing, and evaluation of a model. Simulation, on the other hand, denotes the operation of an existing mathematical model for purposes of prediction, or study, of the target system. The computer revolution has changed the dominant method of simulation from classical analysis to numerical computation and graphical presentation. We wish now to focus on the modeling aspect of applied mathematics, which was called *mechanics* in ancient Greece.

According to this *mechanical paradigm*, our cognitive strategy in technical matters is mechanical. That is, we understand complex phenomena by constructing models, rather than by verbal, symbolic, or other representations. Models may be physical machines (such as orreries or planetaria), pictorial representations (such as photographs) or mathematical models (symbolically represented, as in $F=ma$). The relationship between the model system and the real target system is a conventional (fictitious) one, and need not be an ideal analogy in order to be cognitively useful. Many different models of the same target system (a *spectrum* of models) may be used at once, to advance understanding. In fact, this may actually *be* understanding. We call this the *mechanistic* approach to science.

This approach differs from that of dogmatic science, in which the model comes, over time, to be identified with the target system. For example, a traditional physicist may assume that the electrostatic potential of Maxwell's model has an actual existence in the phenomenal universe.

Accepting the mechanistic approach, let us review the role of the modeling aspect of applied mathematics in the history of the sciences since Newton.

Throughout the period 1680–1930, there was a growing list of spectacularly good models for physical phenomena. These have become, with surprisingly little evolution since their original creation, the cornerstones of mathematical physics: dynamics of particles and continua, electrodynamics, gravitational theory, thermodynamics, statistical mechanics, quantum theory, and so on. In each case, history follows the same pattern: experimental evidence mounts, cognitive strategies form and dissolve, data are increasingly numerical, models become increasingly mathematical, and so on. Eventually, someone has a revelation or intuitive leap, and theory emerges in a new simplicity of understanding, clothed in a splendid model (Maxwell's equations, Einstein's tensor, etc.) which stands as an ideal model for a long time. In this pattern of punctuated evolution in the sciences, the mathematical models play a key role in the formative stages and cognitive strategies, through interaction with the experimental and theoretical developments. This common pattern is a central point in this essay, and can be learned in detail from a single case study. An ideal case is d'Alembert's wave equation for the vibrating string, which established the dominant modeling style of mathematical physics in 1752.

We will now go on to consider three other cases, one each from the physical, biological, and social sciences.

THEORETICAL DYNAMICS

The word *mechanics* meant model-making to the ancient Greeks, as we have noted above, while the word *dynamics* referred to the medicinal power of plants. In the context of the physical sciences, these two words have become synonymous, and denote the science of force, mass and motion begun by Galileo. From the point of view of mechanics (model-making), the history of mechanics (dynamics of particles and continua) provides outstanding examples of the role of models in the creation of theory. It is very instructive to study them in detail, but here we will be satisfied with a brief listing.

In 1560 or so, Galileo made use of real (physical) models (marbles, inclined boards, leaning tower of Pisa, and so on) to elucidate the basic principles of motion. After creating the calculus in 1665, Newton used it to make mathematical models for the same phenomena in 1685. From the study of these models grew *classical analysis*, one of the main branches of mathematics. The goal of analysis was to obtain predictions (that is, simulated data) from the models (differential equations) by symbolic integration (that is, from explicit functions).

In 1865, James Thompson invented the first mechanical analog computer for the simulation of these same models, providing a second simulation strategy. In the 1920's, Van der Pol began using electronic analog computers for modeling and simulation, and these became fast enough to compete with classical analysis as a practical method. Later, during World War II, they became fast enough to be used as bombsites, simulating trajectories according to Newton's model. Shortly thereafter, digital computers replaced them as the simulation strategy of choice for most dynamic models.

The models created by Newton (coupled systems of nonlinear differential equations) are basic to all the simulations which followed, whether by classical analysis or analog or digital computation.

SIMPLE DYNAMICAL SCHEMES

An outstanding problem of theoretical dynamics is the stability of the solar system. In 1885, Poincaré showed that Newton's methods of classical analysis were inadequate to resolve this fundamental problem. He went on to establish totally new mathematical methods for the study of dynamical systems. These were geometric, rather than analytic, and gave rise to new branches of mathematics such as differential topology. The new methods, applied to systems of ordinary differential equations, are now known as *dynamical systems theory* (or qualitative nonlinear dynamics). They have provided a synthesis of all the outstanding models of the physical sciences into a single modeling strategy.[1]

A *dynamical system* is based upon a *state space*, or geometrical model for the virtual states of the target system. Each point of the state space represents a single, instantaneous state, perhaps through some number of observable parameters. The *dynamic* is a infinitesimal rule of evolution: each state is characterized by a unique evolutionary tendency, described by a velocity vector.

The behavior of these mathematical systems is well-known, through three centuries of experimental and theoretical findings. A given initial state evolves along a unique trajectory. After a temporary phase, the *transient response*, this trajectory approaches asymptotically to a limit set called an *attractor*, and a dynamical equilibrium is attained. These occur in three flavors: static, periodic, and chaotic.

Static attractors, also called *rest points*, have been extensively applied since the time of Newton. A system under the influence of a static attractor approaches the final destination and slows to a halt.

Periodic attractors, also called *oscillations*, have dominated dynamics for the past century. A system approaching an oscillation will behave more and more like a perfect oscillation as time goes on.

Chaotic attractors, also called *strange attractors*, are newly discovered, and provide for an understanding of many kinds of aperiodic behavior. Much is now known to be signal, which was previously considered to be noise.

In a given dynamical system, there are usually several attractors. As each initial state will evolve to one of them, the state space may be decomposed into sets sharing the same final fate, which are called *basins*. The basins are divided by *separatrices*. The state space, with the attractors, basins, and separatrices drawn upon it, is called the *portrait* of the dynamical system. This portrait comprises the full understanding of the dynamical behavior of the model, at least as far as long-run prediction is concerned.

Most useful models contain adjustable constants, or *control parameters*, which may be used to adjust the dynamic on the fixed state space. Such a model is called a *dynamical scheme*. As the controls change, so does the portrait. The *response diagram* of the scheme is a graph showing the dependence of the portrait upon the control parameters. The response diagram is the master map which gives this kind of model great power in applications. Points in this diagram where the portrait changes in a particularly significant way are called *bifurcations*.

Catastrophe theory has provided excellent pedagogic examples of response diagrams for various schemes, establishing their importance as graphic representations in many scientific disciplines. Further, it demonstrates the usefulness of mathematical theory in these applications, as the theory *excludes* many bifurcations which might otherwise be expected.[2, 3]

For our present purposes it suffices to observe that dynamical schemes unify all the best-known models of the physical sciences within a single modeling strategy.

THEORETICAL PHYSIOLOGY

After the introduction of dynamical systems theory by Poincaré in 1882, and the maturation of mathematical physics from mechanics to quantum theory, a disastrous gap opened between pure mathematics and the sciences. Although mathematical physics was two centuries old, biological science had hardly begun. Thus, unlike physics, biology was forced to evolve with little support from mathematical models. Of course a few biologists had extensive mathematical training, perhaps from backgrounds in physics or engineering. But the mathematics which had evolved in that arena was not ideally suited to biological modeling. So overall, one might say that theoretical biology and mathematics were both retarded by 50 years or so by the lack of interaction. Only in the past twenty years has there been a significant interaction between theory and modeling, and by now the journals of mathematical biology are filled with very sophisticated models.

The bulk of these models are, in fact, simple dynamical schemes. And their style is much influenced by the historical models of the physical sciences. And yet, when we try to transcend the reductionist models for isolated parts of whole systems, the strategies of the physical sciences fail us. Physical systems are too simple to guide us. Thus, new mathematical strategies have recently evolved for modeling the complex systems encountered in biology, such as general systems theory, systems dynamics, nonlinear control theory, urban dynamics, cybernetics, and so on.

One such strategy, a straightforward extension of the dynamical systems theory of Poincaré to complex systems such as networks and membranes of simpler systems, is called *complex dynamical systems theory*. It evolved in efforts to create high-fidelity models of physiological systems, such the endocrine control systems for the regulation of sleep, eating, stress, and immune responses of mammals.[4]

COMPLEX DYNAMICAL SCHEMES

Given two dynamical schemes, each with its own control and state spaces, a simple kind of *coupling* may be defined by a function from the state space of the first to the control space of the second. The first is the *driver* in this coupled system, as its states operate the controls of the other, *follower* system. By making such couplings among several simple schemes, coupled networks may be built. These provide an excellent modeling strategy in scientific areas where reductionist experiments have led to good dynamical models for the individual parts of a whole system. Each part-model is characterized by a response diagram, which may be well-mapped through extensive computer simulations.

Then the challenge to the theory of complex dynamical systems is *to predict the behavior of the complex system, from a knowledge of the behavior of the parts, and their couplings.*

At present, this theory is in its infancy. Even if the component schemes are stable linear ones, as is frequently the case in systems dynamics for example, the behavior of the complex system may be chaotic. Yet the emerging theory of bifurcations of dynamical schemes is very promising here, as it provides the beginnings of an encyclopedia of atomic bifurcations, of which all response diagrams are made. Viewed as exclusion rules, this encyclopedia may be very helpful in interpreting the results of computer graphic simulations of large-scale complex models. As it grows, a useful theory will become available, and although the behavior of a complex may not be predictable from the behavior of its parts, *it may be obtainable from an affordable amount of computer simulation*.

Other contributions to complex dynamical systems theory may be expected from differential topology and geometry, and practical experience will accelerate when personal supercomputers become available in the near future.

THEORETICAL SOCIOLOGY

This subject, beginning its ascent half a century behind that of theoretical biology, may be expected to grow at a faster rate. For the gap between mathematics and the sciences has been bridged here and there. Thus, the further advance of social theory could be meteoric, if it makes uninhibited and interactive use of innovative mathematical models in the spirit of Newton, Euler, the Bernoulli's, d'Alembert, and so on. For 70 years sufficed for the creation of mathematical physics as we know it today, while a like period in the history of mathematical biology advanced us relatively little.

Due to the explosive growth of social problems, *the fastest possible advance of social theory,* including an adequately predictive model, is mandatory. Thus, we need to nurture the maximum interaction between the ingredient subjects (complex dynamical modeling and simulation, pure mathematics, all of the social sciences, computer science) with adequate resources.

In the growth of social theory, what sort of mathematical models might be useful? Just as the simple dynamical schemes of physics had to be extended to the complex schemes of physiology, further extension may be necessary to build successful models for a planetary society. One modification has already been introduced by Stephen Smale, in his microeconomic model for a trading society.[5] In this model, the *dynamic* (that is, the rule of evolution) is specified not by a unique velocity vector at each point in the state space, but by a cone of favored directions instead. Another extension which may be necessary for the modeling of very large and complex systems is a hierarchy, or *spectrum,* of models.

SPECTRAL DYNAMICAL SYSTEMS

By this invented phrase we mean a whole family of complex dynamical models for the same target system. For example, we may have a hierarchy of models, ranked by differences of physical scale in the state space: microscopic, fine-grain, coarse-grain, macroscopic, thermodynamic, and so on. Or, we may have parallel models of the whole system, but seen from the perspective of disjoint local regions. There may be similar models, distinguished by separate hypotheses, decision strategies, policy-making styles, and so on.

This situation is already familiar, not in modeling practices, but in the verbal analyses of social systems. These are parsed by aspects belonging to separate subjects, such as political analysis, economic description, resources, needs, climate, foreign interactions, and so on.

Thus, we may foresee a further extrapolation of dynamical model structure, in which the cognitive styles already firmly fixed in the various social sciences may separately be embedded, each within one of a spectrum of interlocking complex dynamical models. The models of the spectrum must be made in a universal strategy and style so that they may be successfully combined, or coordinated, for purposes of computer programming, for simulation and prediction, for policy-making, and so on.

The master map or *hypermodel* which coordinates this spectrum of models will probably be a known structure from differential topology or geometry. But all this can be elaborated only in the context of the actual future of mathematical social theory.

PROSPECTS FOR AN ACCELERATED DEVELOPMENT OF A SUCCESSFUL PSYCHOHISTORICAL MODEL

We believe that a successful model of planetary society is an attainable goal for our species, complete with accurate predictions for millenia, simple models for chaotic states and transformations, and short lists of alternative futures at the bifurcation points of psychohistory. Indeed, the achievement of a satisfactory social theory, following in the footsteps of physics and biology, *must* provide us with such a model, and perhaps the extension of natural intelligence by the computer revolution is a necessary prerequisite.

However, we may not wish to wait for a century or two for the spontaneous development of this model, from science fiction to the boardroom computer. Indeed, we may not be able to. So we must ask: what are the prospects for the intentional acceleration of this natural development by a large factor, such as ten?

Certainly the exigencies of World War II created maximum acceleration efforts for various physical technologies, such as radar, rocket propulsion, and nuclear reactions. The respective strategies of England, Germany, and the United States for these accelerations were very similar: draft the best people, combine them in an isolated think tank with extensive resources and all the funding that can be spared, provide inspiring leadership and desperate motivation, and hope for the best. In all three cases, luck prevailed.

The history of analog and digital computing machines provides a second precedent. Several centers in England and the United States gambled on different strategies and took their chances, as in roulette. Here too, luck prevailed.

Our current situation may be very similar to wartime, but with all of us on the same side. As the battle for survival intensifies, the defense budgets of the world may be redirected to a desperate program to accelerate the development of the psychohistorical model. The earlier wartime efforts may serve as the organizing plan for a new crash program.

Yet those were based upon applications of sciences with theories already well developed. It may not be possible to accelerate the early stages of emergence of theory. At least, we cannot guess how long this might take.

We have much at stake. *Should we trust to luck?*

CONCLUSION

The history of the three sciences from the point of view of mechanics (model-making) has been considered, to suggest the possible importance of complex dynamical models and supercomputer simulations in the development of social theory, and an adequate model for supporting the emergence of a peaceful planetary society.

It appears that mathematics, computer science and the social sciences are poised for a rapid growth. But the normal rate of this growth may be much too slow to assist us in coming crises. We have no precedent for the intentional acceleration of the formative stages of a science.

As Einstein said: *One can organize to apply a discovery already made, but not to make one.*

It is time to begin.

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