SOCIAL AND INTERNATIONAL SYNERGY

a mathematical model

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Dedicated to Ruth Fulton Benedict (Ann Singleton), 1887-1948

Abstract. Lecturing at Barnard College in 1941, Ruth Benedict introduced a unique and important idea of synergy in a human social context. In this paper, striving towards a mathematical anthropology, we develop a complex dynamical model for her concept of *social synergy,* and discuss its application to *international synergy,* in the emerging planetary society of nations.

Ruth Benedict's Idea of Social Synergy

In 1941, Ruth Benedict, the distinguished American anthropologist, was asked to give the Shaw Memorial Lectures at Bryn Mawr College. Then age fifty-eight, she presented some mature and highly original ideas to the audience. Distracted by the war, Dr. Benedict's plan to bring these ideas together in a book did not materialize before her death seven years later. Further, her original lecture notes were lost. These lectures, and their central concept of social synergy, to which this paper is devoted, were described briefly in print by Margaret Mead in 1959[1] and by Abraham Maslow in 1964.[2] [3] Eventually, Maslow and Honigman published extracts of the only copy of Benedict's notes of the seven lectures, with a very short introduction by Margaret Mead giving some background.[4] [5] [6] The abstract preceding their paper is worth quoting in full.[7]

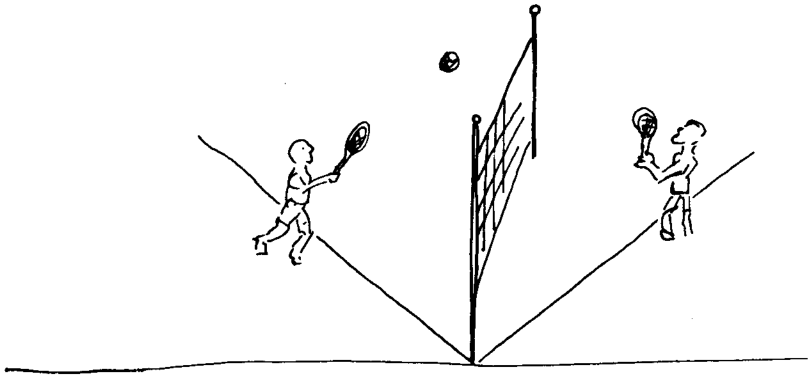
> Excerpts from 1941 lectures by Ruth Benedict call attention to the correlation between social structure and character structure, especially aggressiveness. Social orders characterized by high or low synergy, by a syphon or a funnel system of economic distribution, are compared for their different capacities to support or humiliate the individual, render him secure or anxious, or to minimize or maximize aggression. Religion, an institution in which people apotheosize the cooperation or aggression their cultural life arouses, differs between societies with high and low synergy.

The phrase *social synergy* is used here in a precise sense, defined by Dr. Benedict in the third lecture of her Bryn Mawr series. It is not just a special case of the usual usage of the word synergy, *mutual catalysis* in an interactive social group, or corporate merger. Nor is social synergy identical to *mutual aid,* Kropotkin's version of social altruism.[8] What Ruth Benedict meant by social synergy is the following (Reference 7).

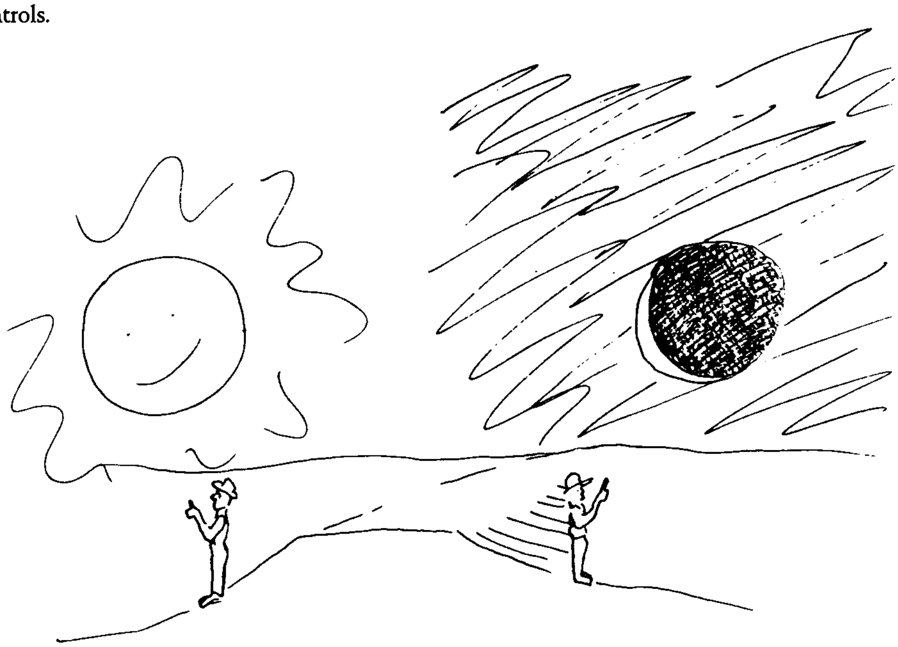
> "I shall need a term for this gamut, a gamut that runs from one pole, where any act or skill that advantages the individual at the same time advantages the group, to the other pole, where every act that advantages the individual is at the expense of others. I shall call this gamut synergy, the old term used in medicine and theology to mean combined action."

We now want to model this concept in the context of *complex dynamical systems (CDS).* This context is the basis of a strategy for building mathematical models for complex systems in nature, and combines the dynamical systems theory of modern mathematics (including chaos and bifurcation theories[9]) with the systems dynamics of general systems theory.[10] It is particularly appropriate for the modeling and simulation of social systems.[11] Our CDS model for social synergy is based on the econometric models of Steve Smale[12] [13] [14] [15] [16] and may be regarded as a cellular version of Lewin's field theory.[17] A complex dynamical system is a network (or directed graph) of dynamical schemes (dynamical systems depending upon control parameters) in which the output (some of the state variables) of one node determines the input (some of the control parameters) of another.

In this application, each node will represent one individual of the social group. Besides the control parameters determined by other individuals, some control parameters are imagined to remain free. We will assume the *self-regulation hypothesis*: each individual may make small changes in his own free controls at will. These small changes are further supposed to be made according to a system of preferences, due to values, utilities, tax relief, cognitive maps, etc. The key step in the construction of our model for social synergy will be the representation of preferences in the context of a single dynamical scheme, preferences which determine the choices an individual makes in changing his own free controls. This is described in the next two sections. We then return to our model for social synergy in a complex dynamical system.



The key step in the construction of our model is the representation of preferences which determine the choices an individual makes in his own free controls.



Preferences in a Dynamical Scheme

In the context of a community of economic trading partners, a mathematical model for the choices made by individual traders has been constructed by Steve Smale. This model is appropriate to a situation in which the control parameters are changed by an individual in small steps, or trades. It consists of a field of cones on the control space **C**. That is, at each point of **C**, a cone of preferred directions is specified. It is assumed that the individual will move only in small steps, in directions contained in the cone of preference. This cone is usually defined by a set of *utility functions*, all of which are increased by small steps of the control parameters in the directions of the preference cone. This preference cone model has been adapted to voting preferences by Chichilnisky.[18] Both Smale and Chichilnisky consider a situation in which the only dynamic is that of the trades or votes.

We now wish to generalize this to a dynamical scheme. Let **S** denote the state space, and **C** the control space of a dynamical scheme. This means that for each choice of a point **c** of **C** (representing chosen values of all of the control parameters) a unique dynamical system **D(c)** is specified on the state space **S**. To this intrinsic dynamic we now wish to add a preference cone dynamic, corresponding to trades or votes, on the control space **C**. We may call such a system a *dynamical scheme with preferential self-regulation*. This is similar to the concept of self-regulation introduced by Zeeman.[19]

The behavior of a dynamical scheme may be visualized by its *response diagram*, in which the *attractors, basins,* and *bifurcations* of the scheme are clearly revealed (Reference 8). The response diagram is drawn in the total space, **CxS**, of the scheme, and the *bifurcation set,* **B** belongs to the control space **C**.

Now imagine a utility function **(u)** defined on the total space of a scheme, **u:CxS->R** (where **R** denotes the real number line). Over a chosen point **c** in the control space **C** there may be several attractors of the dynamical system **D(c)**. The result of the choice of a control

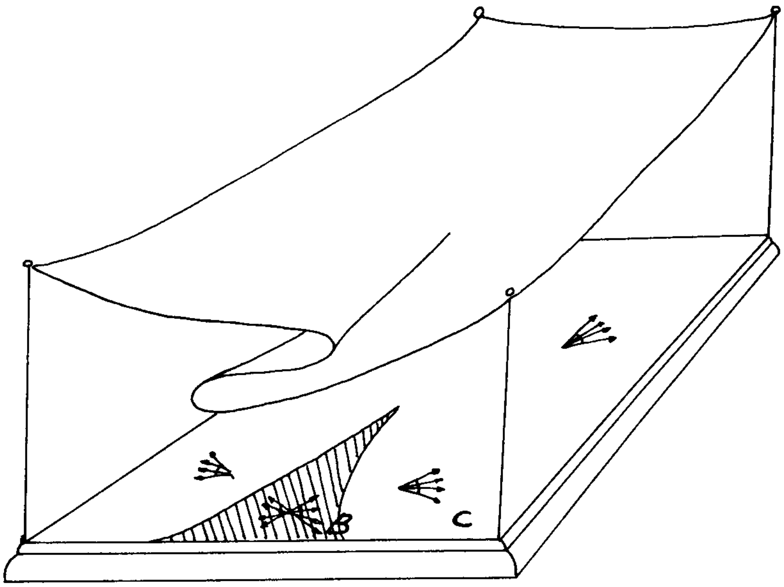


Fig. 1: Example of a preference cone field for the cusp. B is the bifurcation set, and the preference (utility direction) is to "move uphill."

parameter c, and of an initial state, will be one of the attractors of D(c). The motion of a trajectory over this attractor will result in a static, periodic, or (usually) chaotic time series of values of the utility u(t). We will take the time average of this data as the effective utility for this attractor, U(c). Note that U is not a function on the total space CxS but only on a part of C containing the chosen point, c, and avoiding the bifurcations set B in C. For each utility function u and attractor of D(c), such a function U is obtained by this averaging process.

Assuming that the chosen control is not a bifurcation point (c is not within the subset B of C), we may assume that a small variation in c will produce a small variation in the attractor, and thus in its average utility U(c). In case of a set of utility functions instead of a single one, there results a cone of preferred directions at the point c in the control space. Note that for each attractor of D(c), there will be a different preference cone. This completes the construction. An example, for the scheme known as *the cusp catastrophe*, is illustrated in Figure 1. Note that for points within the cusp curve (the bifurcation set B within C) there are two preference cones, one for each attractor.

Self-Regulation Habits

Under the self-regulation hypothesis described above, the control parameters will be wandering about during simulation of the scheme. In case there is no bifurcation in the response diagram, we may assume that reasonable hypotheses would guarantee the existence of attracting sets for this self-regulation dynamic, similar to the Pareto optima established by Smale. However, the preference cones may, in the general case which we are considering, conduct the control parameters through a bifurcation. In this event the averaged utility function U(c) will probably suffer a jump discontinuity. This complicates the theory of optima, and we may anticipate attractive cycles, or chaotic sets, in general. For example, a self-regulation trajectory may exhibit increasing average utility U(c) as c moves along. Suddenly, a catastrophe lowers the average utility. Then, it resumes a smooth increase, as in Zeeman's heart model.

We will assume in what follows that global attractors exist for the self-regulation dynamic, and refer to this assumption as the *habit hypothesis*. Thus, we may speak of these self-regulation attractors as habitual patterns, as opposed to optima. They are habits, as it were, of self-regulation.

Social Synergy in a Complex Dynamical System

We now consider a network of individual dynamical schemes, each with its own utility functions, and thus, fields of preference cones. We want to construct two preference cone fields for each individual, its original *individual preference field* defined by its *individual utility functions*, and another *collective preference field* coming from another set of utility functions defined only on the fully coupled complex system, the *collective utility functions.*

The Individual Preference Field. Consider a scheme with control space of two factors C = ExF where control parameters in the factor E are to be entrained by coupling from other individuals, and controls in the factor F are to remain free, subject to the choice of the individual. The cone fields of C may be projected down to F, providing preference cones for the free parameters. Note that cones from many points of C project to a single point of F, so the cones of free choice on F may be quite fat, and may be disconnected as well.

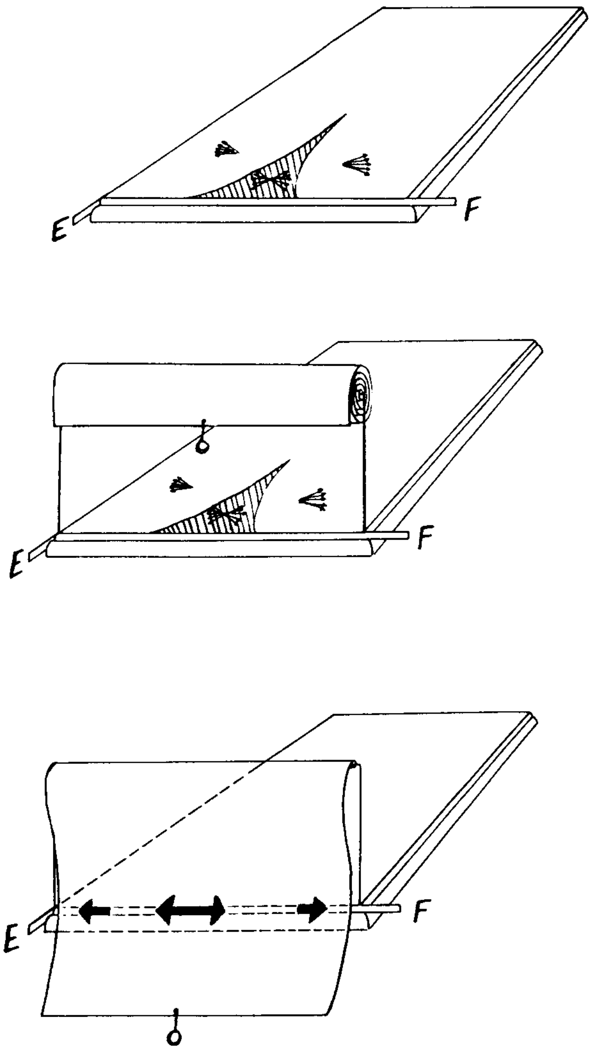
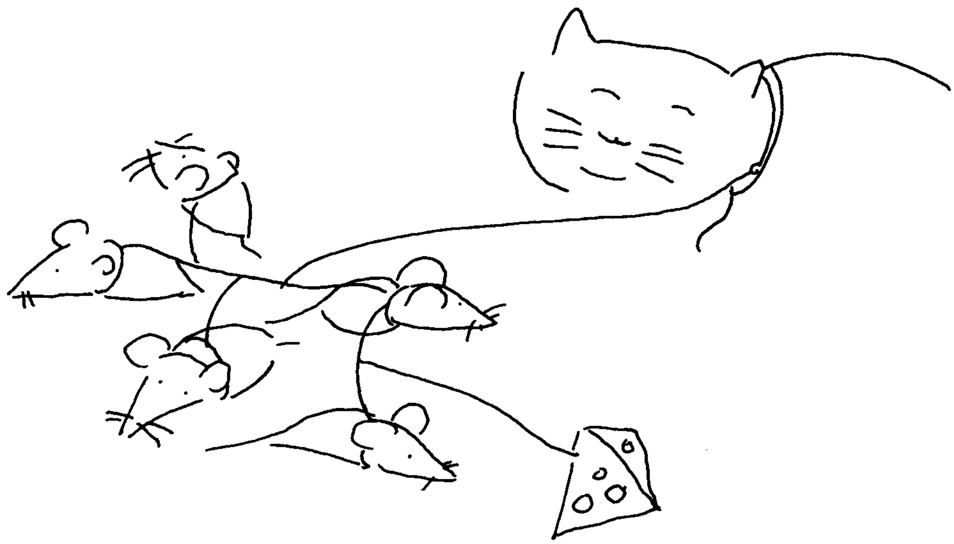


Fig. 2: The individual preference cone field of Figure 1, but with the control space expressed as a product C = ExF. Projecting onto F, we have three intervals. On the left, all the cones point to the left. The preference is to choose movement to the left. In the center, the preference cones point both ways, and on the right, the preference cones point to the right.

The Collective Preference Field. And now, let us connect the component schemes into the fully coupled complex dynamical system. The result is one giant scheme. Let $E_1xF_1xS_1, \ldots E_kxF_kxS_k$ denote the total spaces of the component schemes. After coupling, the giant scheme has as control space $G = F_1x \ldots xF_k$ and as state space, $S_1x \ldots xS_k$. Consider a set of collective utility functions defined on this giant space. These define a collective preference cone field on the collective control space $F_1x \ldots xF_k$ by the process of averaging over the attractors of the coupled scheme. (In the case of a strongly coupled system, such as a neural net, these collective attractors may be far from the product of the individual attractors. This is the interpretation of Prigogine's phrase *far from equilibrium* in the context of complex dynamical systems.) For the ith individual (i is a number between 1 and k), the collective cone field projects to a (fat, disconnected) cone field on the individual's free choice control space.

Finally, we may define social synergy in this context. It is simply the intersection of these two cone fields, defined upon the individual's space of free self-control parameters. An example, based upon a complex of two cusp schemes coupled as in an arms race model,[20] [21] is shown in Figures 1, 2 and 3. In Figure 2, we see the individual preference cone field of Figure 1, but with the control space expressed as a product of two factors C = ExF. Projecting onto F, we have three intervals. On the left, all cones point to the left. The preference is to chose movement to the left. In the center, the preference cones point both ways. (Here we assume that the individual is ignorant of the dynamics within the state space, and has accumulated experiences of success with variation in each direction, without knowing that two different attractors are involved.) And on the right, the preference cones point to the right.



In the case of a strongly coupled system, a collective attractor may be far from the product of the individual attractors.

In Figure 3, two identical cusp schemes have been coupled into a small complex system. The combined state space $S_1 x S_2$ is two dimensional, and the collective space of free controls $F_1 x F_2$ is two dimensional as well, with one linear factor in the hands of each individual. In this illustration, the planar state space is portrayed on edge, that is, as a line. A cone field defined by two collective utility functions is shown, along with its projection on each of the individual control spaces F_1 and F_2. Note that the projected collective preference cones all point to the right for the first individual, and all to the left for the second individual. We see that the first individual has *low synergy*, in the left of three intervals in free control space, *medium synergy* in the central interval, and *high synergy* on the right. The order is reversed for the second individual. This example shows the variation of social synergy in different ranges of individual free choice, and from individual to individual, which is characteristic of this model, even in the simplest possible society. This variation was not a feature of Ruth Benedict's original definition.

Habits and Synergy

Simulation of the model will result in the discovery of regions of high, medium, and low synergy, within the free control space of each individual system. Further, simulation with random choices of small changes in control of each individual, always made in preferred directions, may reveal control attractors of the self-regulation dynamic, or habitual patterns, for each individual, as described in Section 3. It is the relationship between the habitual self-regulation patterns and the synergy regions which determine the long-run synergy of the complex dynamical system. The location of habits within regions of high synergy may achieve stability of the global system in a region of high collective utility. We might further assume that, after the establishment of a habitual pattern, control choices might then come under the influence of the collective preference cone field, or any other lower priority preference system.

International Synergy

Although Ruth Benedict abstracted the social synergy concept from exemplary societies of primitive humans, it may be adapted directly to our emerging planetary society of primitive nations (Reference 2). This may be a useful step in the successful self-organization of the international community, as well as furthering the science of psychohistory.[22] [23]

The main reason for interest in social synergy in the international context is the correlation found by Ruth Benedict between high synergy and low aggression. (For the extension of this correlation to our complex society see Gorney,[24] [25] [26] and Marmor[27]). A further connection with high synergy and global peace and stability has been suggested by David Loye, who identifies the peaceful, cooperative and agrarian societies of the past[28] with high social synergy.[29]

Thus, a society emerging in an ambiance of high synergy may have an enhanced chance of stability, peace, and continued evolution. This is not to say that the evolution of a peaceful, stable, planetary society could not occur by accident; but rather, such an evolution may be hastened through the self-conscious application of the developing theories of psychohistory and mathematical anthropology, complete with complex dynamical models and computer simulations, to intentionally achieve a situation of high social synergy (Reference 22).

At this point we must admit that such a theory of psychohistory is inchoate at best. Although the technology and data have been available for fifty years or more, the federal budgets for science and technology have been directed elsewhere. So we are not able now to continue this essay with a specific and fully detailed exemplary model, as we would like. Instead, we will describe a dummy model, for

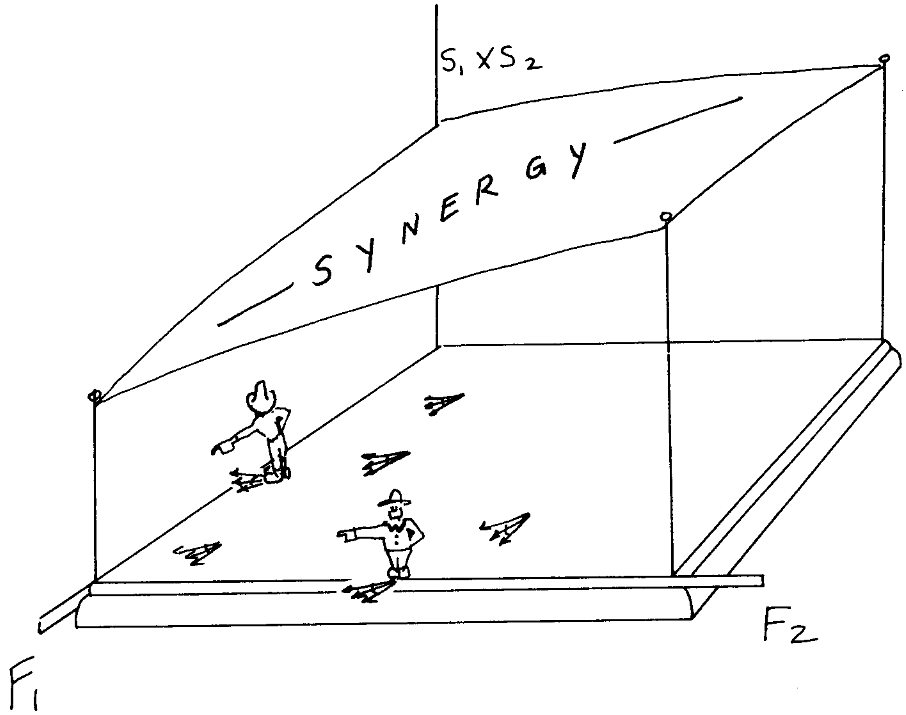
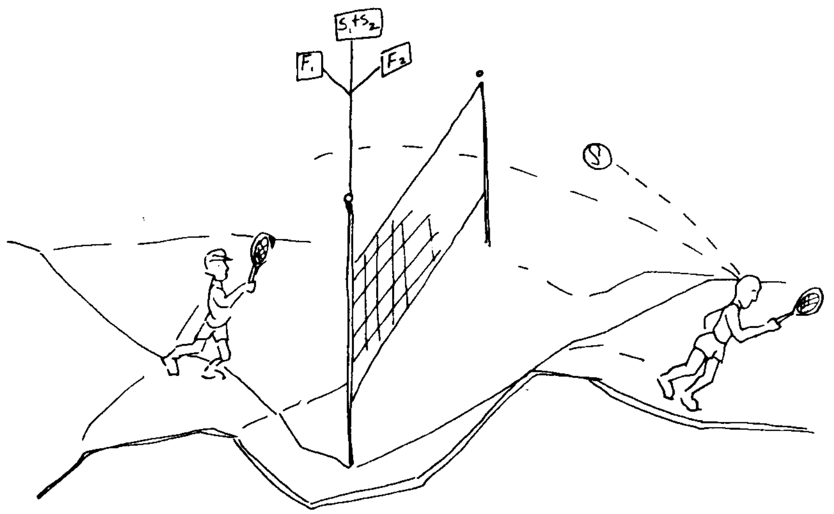


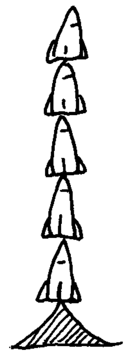
Fig. 3: Two identical cusp schemes coupled together into a small complex system. The projected collective preference cones all point to the right for the first individual and to the left for the second individual.



A tough court at a low synergy moment

pedagogic purposes. There are three steps in the construction of a model for a community of nations in the style of complex dynamical systems with synergy.

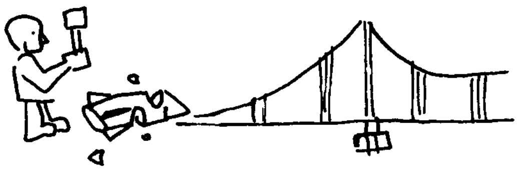
Step One. We begin with a prototype model for a single nation. This is a dynamical scheme, with utility functions. Then, this is specialized to model each individual nation in the community. Lacking a real model from the literature of political science, we may take, for pedagogic purposes, Zeeman's cusp model for a nation of hawks and doves, with preference cones as shown in Figure 1.[30] In this case, the total space is $E \times F \times S = R \times R \times R$.



The state space $S = R$ corresponds to aggressiveness or deterrent capability of a nation: say measured by amount of armaments . . . .

The control space to be coupled, $E = R$, corresponds to the perceived cost of an aggressive action: say estimated societal losses if attacked; E is coupled to S (E_1 to S_2, E_2 to S_1) . . . .

The control space to remain free, $F = R$, corresponds to sensitivity to threat: say with two attractors . . . .
1) saber rattling, and 2) bridge building.



Step Two. Next we must link up the model nations into a network. This will probably be done, when such a theory really evolves, by connecting each nation's state to a universal set of control parameters of each other nation, with an adjustable coupling strength, as is now common in the connectionist approach to neural nets. For our pedagogical example, we will simply connect two identical nations, each modeled by Zeeman's cusp as in *Step One*, obtaining the complex of Figure 3. (Synergy in a system of two individuals has been discussed by Maslow. [Reference 2.]) The state (aggressiveness) of one nation is coupled to the cost control of the other here, leaving the two threat parameters free[31], (References 20 and 21). (Again, other choices could be made for this coupling).

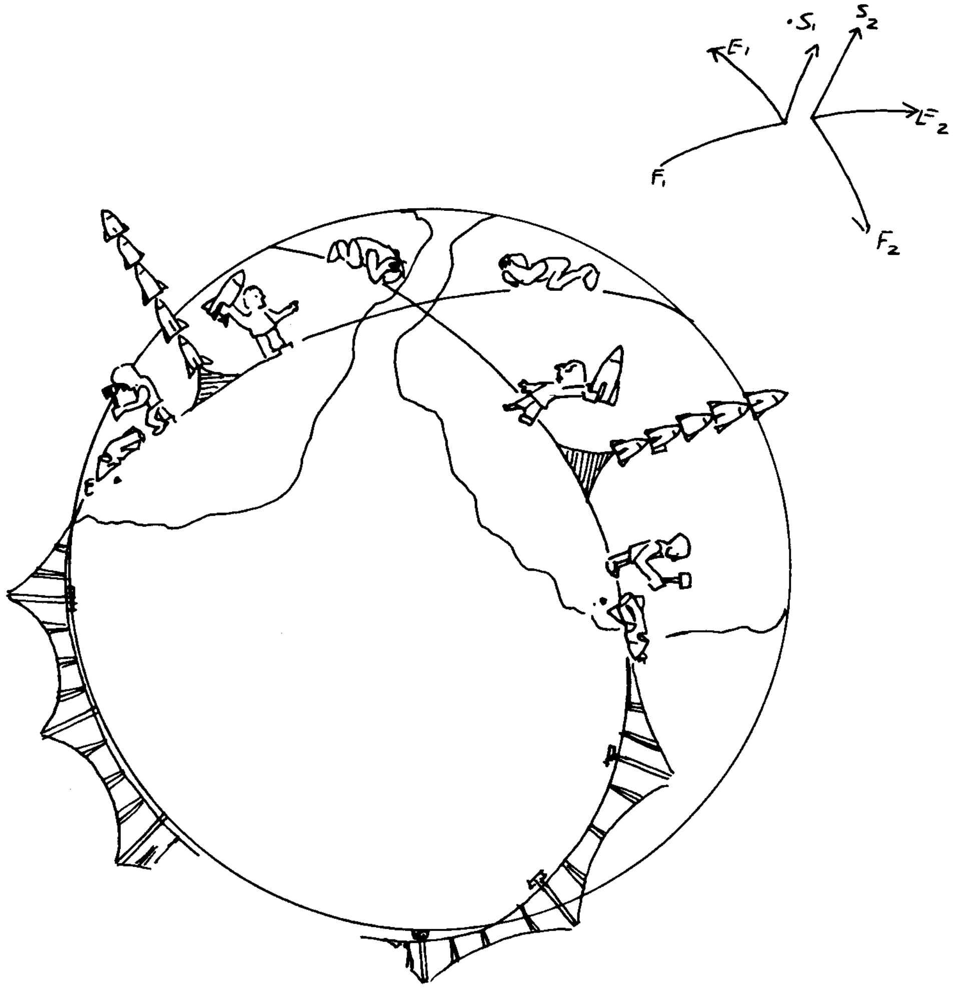
Step Three. Finally, we specify the collective utility functions on the total space of the fully coupled complex system, in this case, $S_1 \times S_2 \times F_1 \times F_2 = R \times R \times R \times R = R_4$. The simplest choice, for our pedagogical model, might be a single function based on the sum of the two armament states, $U: R_4 \to R; (x, y, f_1, f_2) \mapsto M - (x + y)$, where M is a positive constant.

Now the model is made, we should proceed to simulation and analysis. As this model is just a pedagogical device, we have not carried out the simulation, but will proceed to an analysis related to the habit hypothesis posed above, in Section 3. (See Kadyrov [Reference 21] for some two-nation simulation results, or Mayer-Kress [Reference 31] for three nations.) Thus, let us suppose that, from given initial conditions, the model runs to stationary habits of low synergy, low collectivity, and high aggression, and instability. And yet, from other initial conditions, it would have evolved to a more satisfactory equilibrium. We may then consider a global intervention, perhaps undertaken by unanimous consent of all of the partners, to make large scale changes in the individual choices. In this way, stability and consensual utility may be enhanced, by a mutually agreed *intervention*, that is, a large scale jump to a more desirable basin of attraction of the self-regulation dynamic, and thus, a more desirable habit pattern.

Conclusion: International Research Support.

Hello, anybody there? It gets lonely sometimes on the bridge. An essay like this relies too much upon mathematics for a lay person and too little for a mathematician. If any reader remains, let's talk about the support of science. The support of science by the individual governments of the world seems misdirected. Hardly a cent has been spent on the development of the social sciences of peace, global stability, world economy, and so on, while huge fortunes are devoted to war, defense, and international intrigue. No matter which country we consider, the pattern is the same: war and greed seem to be institutionalized as the national priorities.

It is not reasonable to expect private individuals, nor large corporations, to support peace research. Nor is it the real business of governments. For from the viewpoint of international synergy (that is, social synergy in the community of nations), the goals of individual nations may be in opposition to the collective goals. *Research on international synergy may be regarded as the business of governments, and may be expected to be funded only in a situation of high synergy in the international arena.* But the arms race currently underway in our world may be read as a sign of low synergy in the international arena. The situation now is like a train rushing down the track toward a broken bridge. Is this the death wish of a sick society? Or is it just an historical accident, in which our global political system got stuck in a habitual control pattern of low international synergy?



A coupled Hawk/Dove world of choices

Is it time for an international intervention? At least in the laboratory of dynamical modeling and computer simulation, the means are here. Forgiving the individual nations for avoidance of these means, we should look to a consortium of nations to fund research in this area. This eventuality has been foreseen by William Irwin Thompson, who predicts the transformation of the United Nations into a Research Institute for Mathematics and Society (RIMS) devoted to international peace and stability.[32]

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1 Margaret Mead, *An Anthropologist at Work: Writings of Ruth Benedict*, Houghten-Mifflin, Boston (1959, 1966).

2 Abraham H. Maslow, "Synergy in the society and in the individual," *J. Indiv.Psychol.* 20 pp. 153-164 (1964).

3 Abraham H. Maslow, *The Farther Reaches of Human Nature*, Penguin Books, New York (1971, 1976, 1977).

4 Margaret Mead, *Cooperation and Competition Among Primitive Peoples*, McGraw-Hill, New York (1937, 1961).

5 Gregory Bateson, *Naven*, Cambridge University Press, Cambridge (1936, 1958, 1966).

6 T. G. Harris, "About Ruth Benedict and her lost manuscript," *Psychology Today* 4 pp. 51-52 (1970).

7 Abraham H. Maslow and John J. Honigmann, "Synergy: Some Notes of Ruth Benedict," *Am. Anthropol.* 72 pp. 320-333 (1970).

8 Peter Kropotkin, *Mutual Aid: A Factor of Evolution*, Alfred A. Knopf (1925).

9 Ralph H. Abraham and Christopher D. Shaw, *Dynamics, The Geometry of Behavior*, Aerial Press, Santa Cruz, CA (1982-1988).

10 E. O. Attinger, *Global Systems Dynamics*, Wiley-Interscience, New York (1970).

11 Ralph H. Abraham, "Complex dynamics and the social sciences," *Journal of World Futures* 23 pp.1-10 (1987).

12 Steven Smale, "Global analysis and economics I, Pareto optimum and a generalization of Morse theory, " in *Salvador Symposium on Dynamical Systems*, Academic Press, New York (1973).

13 Steven Smale, "Global analysis and economics IIa, extension of a theorem of Debreu," *J. Mathematical Economics* 1 pp. 1-14 (1974).

14 Steven Smale, "Global analysis and economics III, Pareto optima and price equilibria," *J. Mathematical Economics* 1 pp. 107-117 (1974).

15 Steven Smale, "Global analysis and economics IV, finiteness and stability of equilibria with general consumption sets and production," *J. Mathematical Economics* 1 pp. 119-127 (1974).

16 Steven Smale, "Global analysis and economics V, Pareto theory with constraints," *J. Mathematical Economics* 1 pp. 213-221 (1974).

17 Kurt Lewin, *Field Theory in Social Science: Selected Theoretical Papers*, (1971).

18 Graciella Chichilnisky, voting.

19 E. C. Zeeman, "Differential equations for the heartbeat and nerve impulse," pp.81-140 in *Catastrophe Theory*, ed. E. C. Zeeman, Addison-Wesley, New York (1977).

20 Ralph H. Abraham, "Mathematics and evolution: a proposal," *International Synergy*, 2 (2) pp. 27-45 (1987)...

21 Kadyrov, "A mathematical model of the relations between two states," *Global Development Processes* 3 Institute for Systems Studies, (1984).

22 Issac Asimov, *Prelude to Foundation*, Bantam Spectrum, New York (1987).

23 Lloyd Demause, *Foundations of Psychohistory*, Creative Roots, New York (1982).

24 Roderic Gorney, *The Human Agenda*, Simon and Schuster, New York (1972).

25 Roderic Gorney, "Interpersonal intensity, competition, and synergy," *Am. J. Psychiatry* 128 pp.68-77 (1971).

26 Roderic Gorney and John M. Long, "Cultural Determinants of Achievement, Aggression, and Psychological Distress," *Arch. Gen. Psychiatry* 37 pp.452-459 (April, 1980).

27 Judd Marmor, "Psychiatry 1976, the continuing revolution," *Am. J. Psychiatry* 133:7 (July, 1976).

28 Rianne Eisler, *The Chalice and the Blade: Our History, Our Future*, Harper and Row, San Francisco, CA (1987).

29 David Loye, *Moral sensitivity and the evolution of higher mind*, preprint (1989).

30 C. A. Isnard and E. C. Zeeman, "Some models from catastrophe theory in the social sciences, " pp. 303-359 in *Catastrophe Theory*, ed. E. C. Zeeman, Addison-Wesley, New York (1977).

31 GottfriedMayer-Kress, *Personal communication.*

32 William Irwin Thompson, "Gaia and the politics of life: a program for the nineties," pp. 199 in Gaia, *A Way of Knowing: Political Implications of the New Biology*, ed. William Irwin Thompson, Lindesfarne Press, Great Barrington, MA (1987).

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