ECONOMICS AND THE ENVIRONMENT: GLOBAL ERODYNAMIC MODELS

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1. Introduction

Simple dynamical systems theory evolved from celestial mechanics in the work of Poincaré a century ago. Complex dynamical systems theory (also known as systems dynamics) began during World War II, with the work of Von Bertalanffy on general systems theory and Wiener on cybernetics. Cellular dynamical systems theory developed in the early days of biological morphogenesis in the work of Rashevsky and Turing. Since the advent of massively parallel computation, these modeling strategies have been increasingly used to simulate highly complex natural systems. The challenge to understand our global problems — combining physical systems of the atmosphere and ocean (Chaos) with biological systems of the biosphere (Gaia) and the social systems of human and other species (Eros) — will test and extend our mathematical and scientific capabilities. The name *erodynamics* has been coined to describe this application of dynamical systems theory to the complex global system of our human civilization and environment. In this minicourse we develop the basic concepts of erodynamics in the frame of economics and the environment.

2. Types of dynamical systems

In mathematics, there are three categories of dynamical systems: flows, cascades, and semi-cascades. A *flow* is a continuous dynamical system, generated by a *vectorfield,* in which the trajectories are curves, parameterized by real numbers. A *cascade* is a discrete dynamical system, generated by a *diffeomorphism* (smooth invertible map with a smooth inverse), in which the trajectories are discrete point sets, parameterized by integers (positive and negative). A *semi-cascade* is a discrete dynamical system, generated by an *endomorphism* (smooth map, not neccesarily invertible) in which the trajectories are discrete point sets, parameterized by natural numbers (non-negative). In any case, the dynamics occurs on a smooth manifold called the *state space,* which may have dimension one or more. In Fig. 1, the three categories and the state space dimensions are used to spread out the family of all dynamical systems in a tableau. In this tableau, a three-dimensional flow may sometimes be sectioned, in a procedure invented by Poincaré, to a two-dimensional cascade. This, in turn, might be projected into a one-dimensional semi-cascade, in a procedure introduced by Lorenz. Each of these constructions is reversible. Thus, these three boxes, shaded in Fig. 1, are closely related, and have similar behavior. For example, each is the lowest-dimensional box in which chaotic behavior is observed. Thus, we call it *the stairway to chaos.* All of the parallel stairways are interrelated by similar constructions.

Let F, C, and S denote the rows, and 1, 2, 3, $\cdots$, denote the columns of the tableau. Thus, S1, C2, and F3 denote the stairway to chaos, and S2, C3, and F4 the adjacent stairway, and so on. The most familiar dynamical systems are distributed as follows:

Pendulum, Van der Pol: F2
Forced pendulum, forced Van der Pol, Rössler, Lorentz: F3
Hénon: C2
Logistic: S1

The exemplary dynamic models of mathematical economics are described in (Goodwin, 1990), the

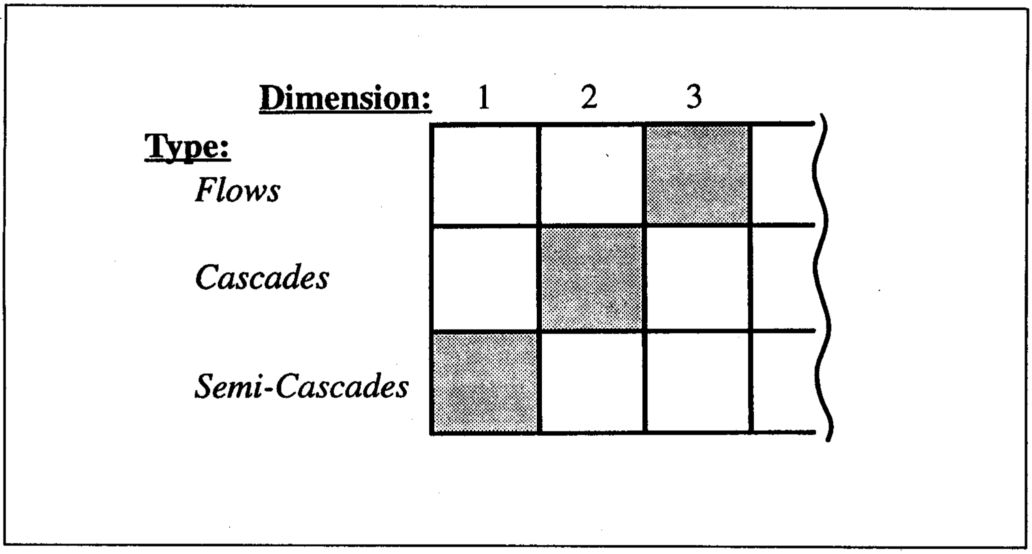

Figure 1. The dynamical system tableau.

chapters of which are distributed over the tableau as follows:

Róssler: F3 — Ch.1, 4, 5, 6, 7, 8, 9
Forced Van der Pol: F3 — Ch. 10
Forced Róssler: F4 — Ch.10
Logistic: S1 — Ch. 1, 2
Von Neumann: S2 — Ch. 3

The latter (Ch. 3) belongs to the frontier of current research on discrete dynamical systems.

In this outline of dynamical systems theory and modeling practice, we will concentrate on the top row of the tableau, flows, keeping in mind that the discussions apply without significant modification to the other two rows, cascades and semi-cascades. In any case, a dynamical system may be visualized by its *portrait,* in which the state space is decomposed into *basins,* with one *attractor* in each.

3. Simple dynamical schemes

We begin with the basic building block of complex dynamics, the simple scheme. This is a dynamical system depending on control parameters. Just as the portrait of attractors and basins is the visual representative of a dynamical system, the response diagram is the visual representative of a scheme.

Definitions. Recall that a *manifold* is a smooth geometrical space. (Abraham, 1988) Let C be a manifold modeling the control parameters of a system, and S another manifold, representing its instantaneous states. Then a *simple dynamical scheme* is a smooth function assigning a smooth vectorfield on S to every point of C. Alternatively, we may think of this function as a smooth vectorfield on the product manifold, CxS, which is tangent to the state fibers, $\{c\}xS$. For each control point, $c \varepsilon C$, let $X(c)$ be the vectorfield assigned by the scheme. We think of this as a dynamical system on S, or system of first order ordinary differential equations.

Attractors and basins. In each vectorfield of a scheme, $X(c)$, the main features are the *attractors*. These are asymptotic limit sets, under the flow, for a significant set of initial conditions in S. These initial states, tending to a given attractor asymptotically as time goes to plus infinity, comprise the *basins* of $X(c)$. Every point of S which is not in a basin belongs to the *separator* of $X(c)$. The decomposition of S into basins, each containing a single attractor, is the *phase portrait* of $X(c)$. Attractors occur in three types: *static* (an attractive limit point), *periodic* (an attractive limit cycle, or oscillation), and *chaotic* (meaning any other attractive limit set). The phase portrait is the primary representation of the qualitative behavior of the simple dynamical system, and provides a qualitative model for a natural system in a fixed (or laboratory) setting. Its chief features are the basins and attractors. The attractors provide qualitative models for the observed states of dynamical equilibrium of the target system, while the basins model the initial states, which move rapidly to the observed states as startup transients die away.

Response Diagrams. For each point c of the control manifold, the portrait of $X(c)$ may be visualized in the corresponding state fiber, $\{c\}xS$, of the product manifold, CxS. The union of the attractors of $X(c)$, for all control points $c \varepsilon C$, is the *attractrix* or *locus of attraction* of the scheme. These sets, visualized in the product manifold, comprise the *response diagram* of the scheme. The response diagram is the primary representation of the qualitative behavior of the dynamical scheme, and provides a qualitative model for a natural system in a setting with control variables. Its chief features are the loci of the attractors as they move under the influence of the control variables, and the *bifurcations* at which the locus af attraction undergoes substantial change. The response diagram provides the qualitative model for the dynamical equilibria of the target system, and their transformations, as control parameters are changed. A typical response diagram, for a model with a single control parameter (the stirred fluid system of Couette and Taylor). For a discussion of this diagram, and many others, see the pictorial *Bifurcation Behavior. (Abraham, 1992)*

Catastrophes and Subtle Bifurcations. For most control points, $c \varepsilon C$, the portrait of $X(c)$ is structurally stable. That is, perturbation of the control parameters from c to another nearby point cause a change in the phase portrait of $X(c)$ which is small, and qualitatively insignificant. In exceptional cases, called *bifurcation control points,* the phase portrait of $X(c)$ significantly changes as control parameters are passed through the exceptional point. Many cases, generic in a precise mathematical sense, are known, and the list is growing. These *bifurcation events* all fall into three categories. A bifurcation is *subtle* if only one attractor is involved, and its significant qualitative change is small in magnitude. For example, in a *Hopf bifurcation,* a static attractor becomes a very small periodic attractor, which then slowly grows in amplitude. Other bifurcations are *catastrophic.* In some of these, called blue-sky catastrophes, an attractor appears from, or disappears into, the blue (that is, from a separator). In those of the third category, called *explosions,* a small attractor suddenly explodes into a much larger one. All of these events are very common in the simplest dynamical schemes, such as forced oscillators. The bifurcations are clearly visualized in the response diagram of a scheme, which is sometimes called the bifurcation diagram. The theory up to this point is adequately described in the literature (see the picture books, and references therein). (Abraham, 1992)

4. Complex dynamical systems

A complex dynamical system is a network, or directed graph, of nodes and directed edges. The nodes are simple dynamical schemes or dynamical systems depending on control parameters. The directed edges are static schemes, or output/input functions depending on control parameters. These provide the serial coupling from the instantaneous states at one node into the control parameters of another.

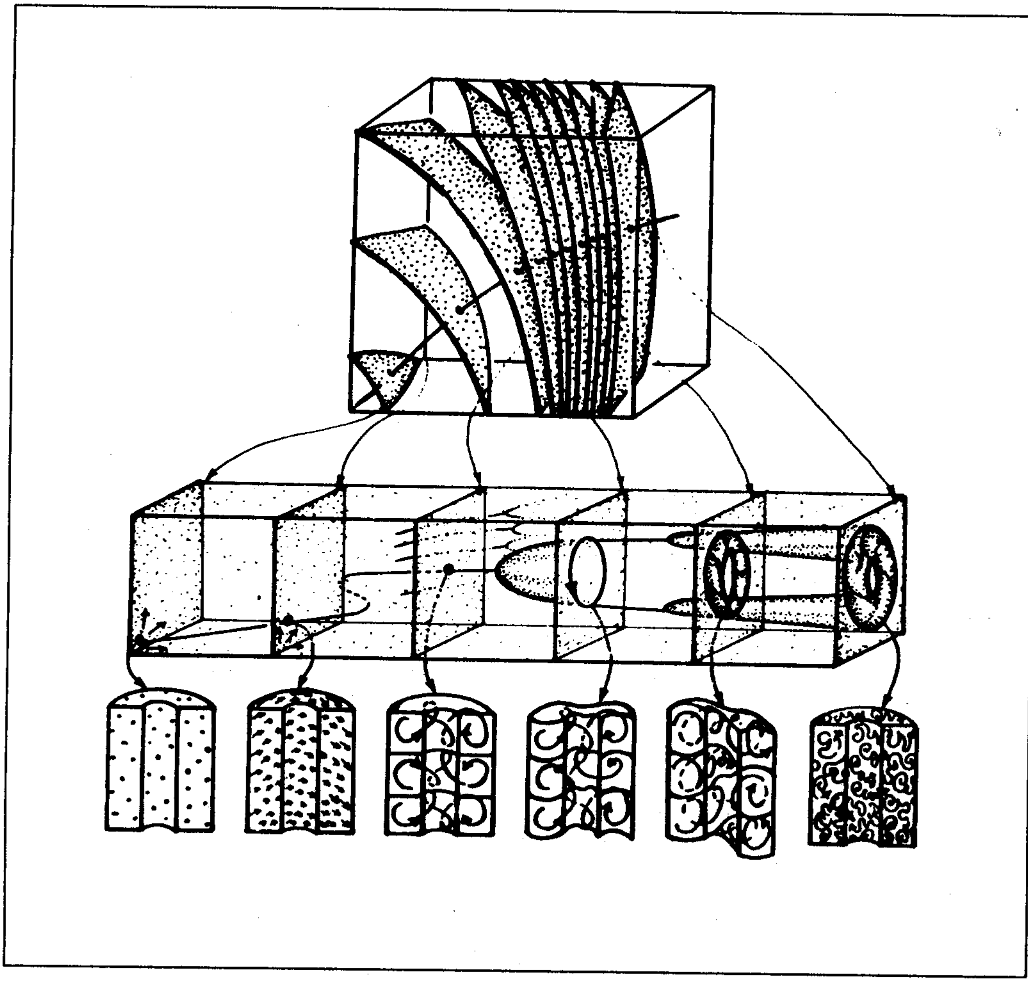

Figure 2. A typical response diagram

Static Coupling Schemes. Consider two simple dynamical schemes, X on CxS and Y on DxT. The two schemes may be serially coupled by a function which, depending on the instantaneous state of the first (a point in S), sets the controls of the second (a point in D). A static coupling scheme is just such a function, but may also depend on control parameters of its own. Thus, let E be another control manifold, and $g : E \times S \rightarrow D$. Then the serial coupling of X and Y by the static coupling scheme g is a dynamical scheme with control manifold CxE, and state space SxT, defined by

$$Z((c,e)(s,t))=(X(c,s),Y(g(e,s),t))$$

This is the simplest example of a complex dynamical scheme, symbolized in the literature by schematic diagrams such as those shown in Fig. 3 or equivalently in Fig. 4. In Fig. 3, the bullet icons represent dynamical schemes, with the state spaces, S or T, vertical, and the control spaces, C or D, horizontal. The

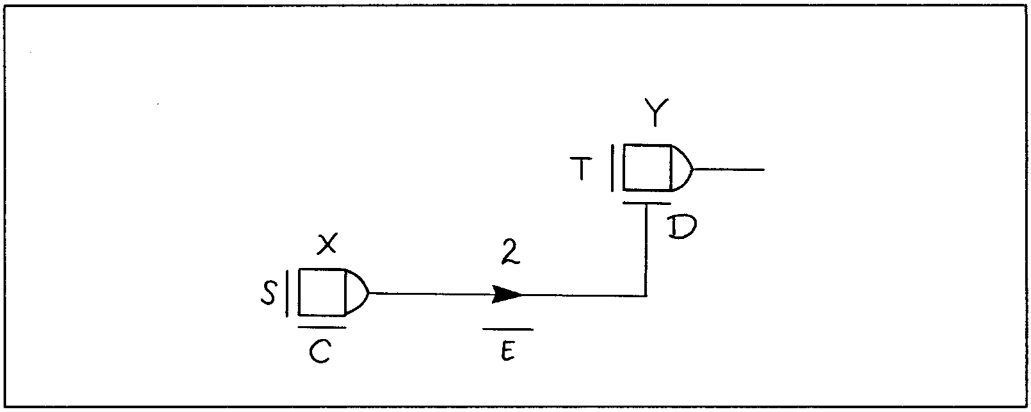

Figure 3. Static coupling of two simple schemes, pictorial

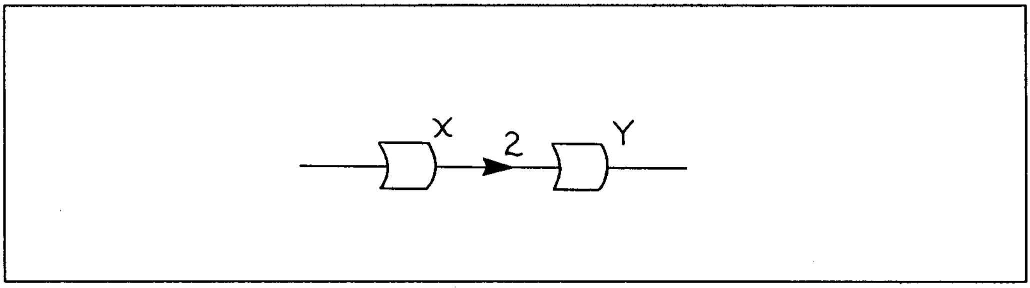

Figure 4. Static coupling of two simple schemes, schematic

total spaces of the dynamical schemes are cartesian product manifolds. For example, $C \times S$. The solid triangle represents a static coupling scheme, g, with its control space, E, horizontal. That is, g is a mapping, $g: S \times E \to D$. In Fig. 4, the symbols are further abstracted.

* *Other coupling schemes*

Coupling by static functions is the most common device found in applications, but there are others. The most frequent variation is a *delay function.* Thus, the coupling device must remember inputs for a given time (usually fixed), the *lag,* and at a present time, must deliver a function calculated from past inputs. This uses up memory in the simulation machine, but otherwise is straightforward and familiar. In many applications, delays are unavoidable. One unfortunate aspect of this neccessity is the fact that the completed CDS model is then a dynamical-delay scheme (system of differential-delay equations depending on control parameters), rather than a standard dynamical scheme.

Another coupling variation sometimes encountered in complex systems is an *integral function.* In this case, the coupling device must accumulate inputs over a fixed time, but need not remember the incremental values. This kind of static node can be replaced by a dynamical node, in which the integration of inputs becomes the solution of an equivalent differential equation. Thus, a static node may be regarded as a trivial type of dynamic node, in theory at least.

* *Serial Networks*. A large number of simple dynamical schemes may be coupled, pairwise, with appropriate static coupling schemes. The result, a serial network, may be symbolized by a directed graph, at least in the simpler cases. The purpose of the CDS, as a mathematical construction, is to create qualitative simulation models for complex dynamical systems in nature. (Abraham, 1988) The full scale complex dynamical system may be symbolized by a graph with two distinct types of directed node, static and dynamic. (A directed node has separate input and output panels.) As controls at a given node or coupling scheme may be segmented (parsed into a product of different control manifolds) and some of these connected to other directed edges, we may have multiple inputs arriving at nodes and at couplings. Multiple outputs from nodes or couplings may also occur. Some examples are shown in Fig. 5. The edge directions of these graphs of two types of directed nodes may be inferred from the connections because of the rule: each edge must connect an output panel to an input panel. Further, an edge from a dynamic output may only be connected to a static input.

5. Exemplary complex systems

Several pedagogic examples have been presented in the literature listed in the Bibliography. We review some of them here.

* *Master-Slave Systems* The simplest complex scheme consists of the serial coupling (as illustrated above) of two simple dynamical schemes. The behavior of these simple examples is notoriously complicated. Suppose that the control parameters of the first (or master) system are fixed. After startup, from an arbitrary initial state, the startup transient dies away, and the master system settles asymptotically into one of its attractors. We consider the three cases separately.

Static master. If the attractor of the master system is a static (point) attractor, and the control parameters of the coupling scheme are left fixed, then the control parameter of the second (slave) system are likewise

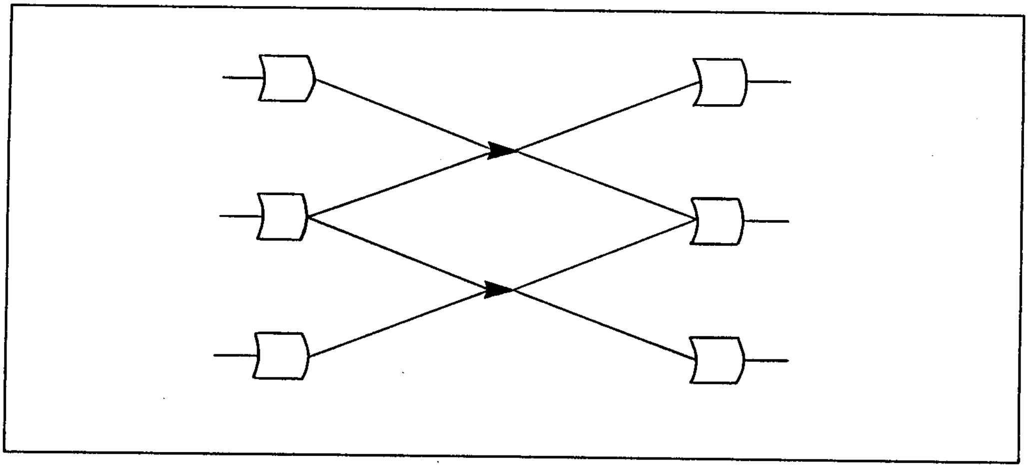

Figure 5. Multiple couplings.

fixed. Typically, this static control point of the slave system will be a typical (nonbifurcation) point, and the slave system will be observed in one of its attractors (static, periodic, or chaotic).

Periodic master. With fixed controls of the master and the coupling function, a periodic master attractor will drive the slave controls in a periodic cycle. This is the situation in the classical theory of forced oscillation. Experimental study of these systems began a century or so ago, and continues today. Here are the two classic examples.

1. Duffing systems. If the slave system is a soft spring or pendulum, the coupled system is the classic one introduced by Rayleigh in 1882, in which Duffing found hysteresis and catastrophes in 1918. (Abraham, 1992) The bifurcation diagram is very rich, full of harmonic periodic attractors and chaos. (Ueda, 1980)

2. Van der Pol systems. If the slave system is a self-sustained oscillator, the coupled system is another classic one introduced by Rayleigh, in which Van der Pol found subtle bifurcations of harmonics and Cartwright and Littlewood apparently found chaos. Both of these classical systems have been central to experimental dynamics, and research continues today.

Chaotic master. This situation, chaotic forcing, has received little attention so far. Many experiments suggest themselves, in analogy with forced oscillations. One situation which has been extensively studied is the perturbation of a conventional dynamical system by noise.

* *Chains of Dynamical Schemes.* If three schemes are connected in a serial chain by two static coupling schemes (Fig. 6) a complex system with a very complicated bifurcation diagram may result. If the first pair comprises a periodic master forcing a simple pendulum, as described above, the terminal slave may be either a periodically or chaotically forced system. Of course, if all three systems are pendulum-like (one basin, static attractor) the serial chain is also pendulum-like. But a periodic attractor in either the first or second dynamical scheme is adequate to produce rich dynamics in the coupled chain.

* *Cycles of Dynamical Schemes.* If the directed graph of a complex scheme contains a cycle (closed loop) then complicated dynamics may occur, no matter how simple the component schemes. The minimal example is the serially bicoupled pair (Fig. 7). Even if the two dynamical schemes are pendulum-like, the complex system may have a periodic attractor. For example, Smale finds a periodic attractor (and a Hopf bifurcation) in exactly this situation, in a discrete reaction-diffusion model for two biological cells. (Smale, 1976) A cycle of three pendulum-like nodes is discussed next, as we turn now to more complex examples.

* *Intermittency in an Endocrine System Model.* Models for physiological and biochemical systems have a natural complex structure. A recent model for the reproductive system of mammals (hypothalamus, pituitary, gonads) is a very simple network (Fig. 8). (Abraham, 1985) Although the simple dynamical scheme at each node is a point attractor in a one-dimensional state space, the complex system may have two periodic basins, each containing a periodic attractor. This phenomenon, sometimes called

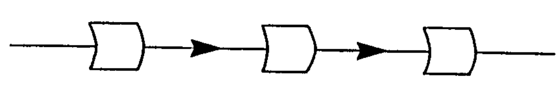

Figure 6. A chain

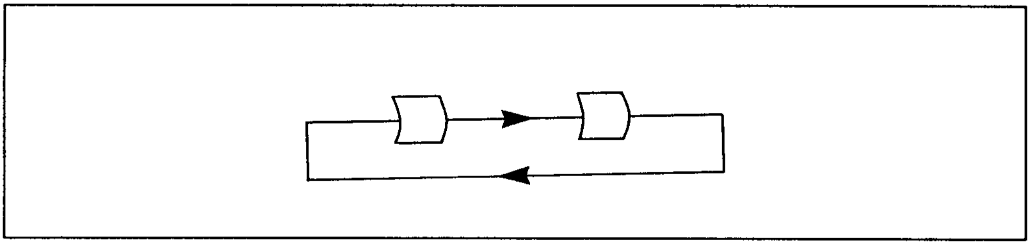

Figure 7. Bicoupled pair.

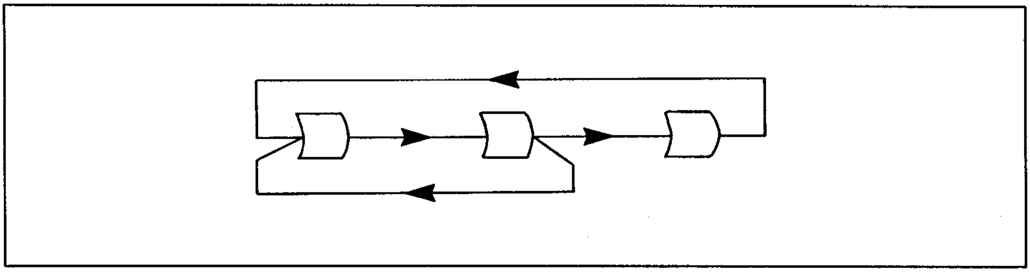

Figure 8. An endocrine system model

birhythmicity, has also been found in a biochemical model. (Decroly, 1982) Small changes in the control parameters of the coupling functions cause intermittent jumps between the two distinct oscillatory states.

6. Cellular dynamical systems

Here we introduce *cellular dynamical systems theory,* a mathematical strategy for creating dynamical models for the computer simulation of biological organs and membranes, and other systems exhibiting natural intelligence. Reaction/diffusion equations were introduced by the pioneers of biological morphogenesis: Fisher (in 1930), (Fisher, 1930/1958) Kolmogorov-Petrovsky-Piscounov (in 1937), (Kolmogorov, 1937) Rashevsky (in 1938), (Rashevsky, 1938) Southwell (around 1940), (Southwell, 1940) and Turing (in 1952). (Turing, 1952) Rashevsky introduced spatial discretization corresponding to biological cells. These discretized reaction/diffusion systems are examples of cellular dynamical systems, probably the first in the literature. Further developments were made by Thom (1966-1972) (Thom, 1975) and Zeeman (1972-1977). (Zeeman, 1977) The latter includes a heart model, and a simple brain model exhibiting short and long-term memory. The ideas outlined here are all inspired by these pioneers. The strategy is based upon CDS concepts. (Abraham, 1986)

* *Definitions*. By *cellular dynamical system* we mean a complex dynamical system in which the nodes are all identical copies of a single dynamical scheme, the *standard cell,* and are associated with specific locations in a supplementary space, the *physical substrate,* or *location space*. Exemplary systems have been developed for reaction/diffusion systems by discretization of the spatial variables. In these examples, pattern formation occurs by Turing bifurcation. One of the most-studied examples of this class is the *Brussellator* of Lefever and Prigogine. Other important examples of this construction are the heart and

brain models of Zeeman. These models have something in common with the *cellular automata* of Von Neumann, yet possess more structure. We might call them *cellular dynomata.*

The behavior of a cellular dynamical system may be visualized by Zeeman's *projection method:* an image of the location space (physical substrate) is projected into the response diagram of the standard cell, where it moves about, clinging to the attractrix, or locus of attraction. Alternatively, the behavior may be vizualized by the *graph method:* attaching a separate copy of the standard response diagram to each cell of the location space. Within this product space, the instantaneous state of the model may be represented by a graph, showing the attractor occupied by each cell, within its own response diagram. In either case, the behavior of the complete cellular system may be tracked, as the controls of each cell are separately manipulated, through an understanding of the standard response diagram provided by *dynamical systems theory:* attractors, basins, separators, and their bifurcations.

7. Exemplary cellular systems

Cellular dynamical systems began in the context of reaction-diffusion equations.

* *Reaction-Diffusion Systems.* An unusual example of serial coupling is provided by the reaction-diffusion model for biological morphogenesis, introduced by Fischer in 1930 (see Section 5 for more history). Given a spatial domain or substrate, D, and a biochemical state space, B, the state space is an infinite-dimensional manifold, F, of functions from D to B. The reaction-diffusion equation may be regarded as a simple dynamical scheme of vectorfields on F, depending on a control space, C. Meanwhile, the spatial substrate is actually composed of biological cells, considered identical in structure. As the reaction-diffusion scheme, the master in this context, determines instantaneous states of biochemical (morphogen, or control metabolite) concentrations in the substrate, $f : D \rightarrow B$, the cell at a fixed position in the domain will extract the values of this function at its location, $f(d)$. This is a point of B, which may be regarded as the control space for another simple dynamical scheme, modeling the dynamics within the standard cell. Let $g_d(f) = f(d)$. Then g_d is the static coupling function from master to slave. But there are many slaves, each distinguished by its own location, hence coupling function. The directed graph is thus a radial spray, or star, of slaves of a common master, as shown in Fig. 9. If in addition each cell may be a source or sink of biochemical (metabolite) controls, then each connection is a bicoupling.

* *Neural Nets.* Neural nets may be regarded as a special case of CDS network. The dynamical nodes are all identical schemes, in which a simple scheme with one dimensional control and state spaces has a single basin of attraction (with a static attractor). The control value adjusts the location of the attractor, and there are no bifurcations. The planar response diagram has an inclined line as attractrix. The coupling schemes are all identical as well, and are simple amplifiers, $g(e,s) = es$. All nodes interconnected, as shown (Fig. 10). The intelligence of a neural net depends on the matrix of controls, $E = (e_{ij})$. This strategy, called *connectionism,* may be extended directly to any CDS.

* *Biological organ example.* Organs typically contain many different types of cells. In the unusual case that there were only one type of cell, one could imagine a model for the organ consisting of a single cellular dynamical system. This is the case with Zeeman's heart model. An explicit cellular dynamical model for the organ will require an explicit model for the standard cell, which (with luck) may be found in the specialized literature devoted to that cell.

However, if there are two distinct cells, then each will give rise to a distinct cellular dynamical model. The model for the organ will then consist of a coupled system of *two cellular dynamical systems,* one for each cell type. More generally, the organ model will consist of a *complex dynamical system,* comprising a network of distinct cellular dynamical models, one for each of the distinct cell types, visualized (intermixed) in a common physical substrate.

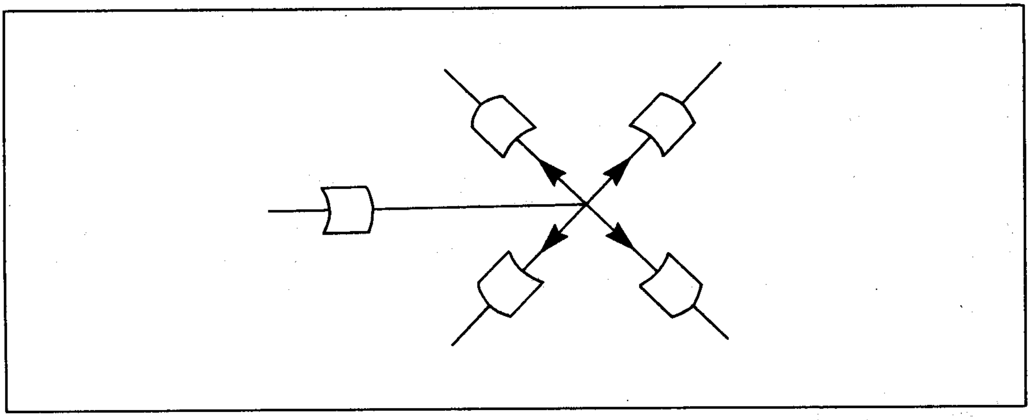

Figure 9. A star complex

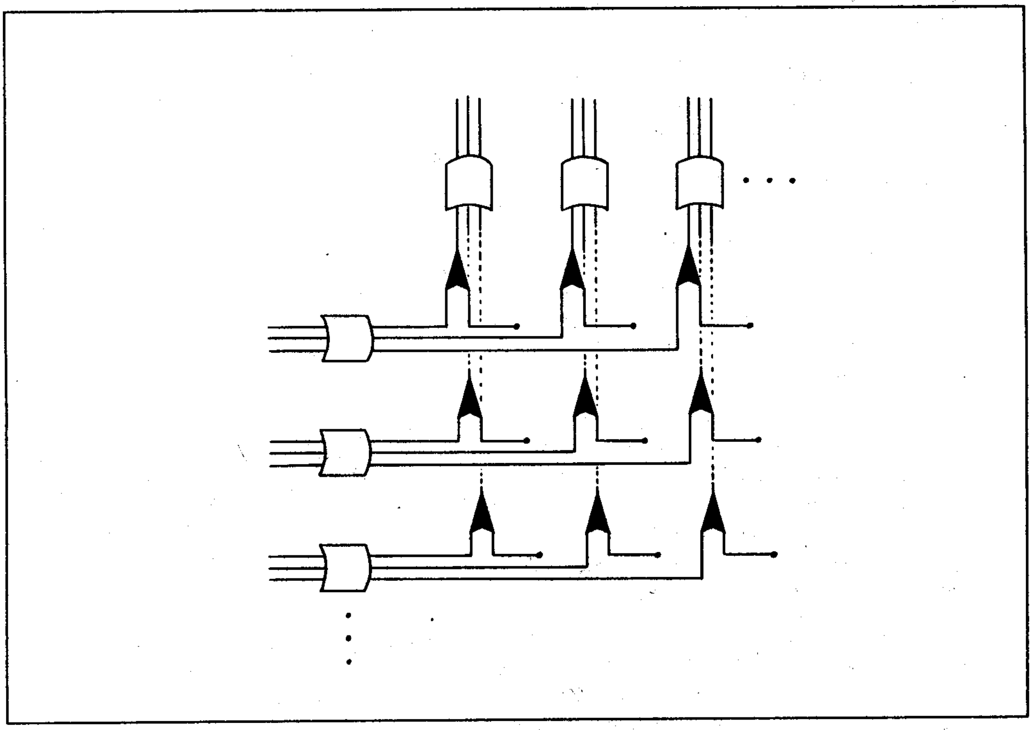

Figure 10. Neural Net

Moreover, even if there is only a single cell type in the organ (for example, a liver cell) a network of cellular models may nevertheless be required. For there are usually at least two important compartments in the organ: the intracellular space, and the extracellular space. The concentration of control metabolites or humoral substances (such as the pacemaker substance in Zeeman's heart model) in the extracellular space contributes a second cellular dynamical system to the model. This second system arises through the discretization of the nonlinear Fickian diffusion equation for the perfusion of metabolites through the organ. Even if the substance in the two compartments is the same (for example, cortisol in the adrenal cortex), there will be two distinct cellular systems in the organ model. The dynamics of the extracellular substance will be modeled by a (discretized) reaction/diffusion system, while the intracellular dynamics may be modeled by reaction kinetics alone.

8. Simulation methods

After the strategies of complex dynamical systems have been used in an application, the resulting model is simply a large dynamical scheme. That is, a system of coupled ordinary differential equations with free control parameters, or partial differential equations of evolution type (parabolic or hyperbolic) must be explored experimentally. The goal of the exploration is to obtain the response diagram, which is the useful outcome of the qualitative modeling activity. As the exploration of the response diagram is an unfamiliar goal for simulation, we review here some of the strategies used.

* *Orbit Methods*. When the dynamical scheme consists of a modest number of ordinary differential equations of first order, simulation by the standard digital algorithms (Euler, Runge-Kutta and so on) or analog techniques provide curve tracing in the bifurcation diagram. A large number of curves, for various values of control parameters and initial conditions, may reveal the principal features of the diagram. Monte Carlo techniques are sometimes used to select the control parameters and/or initial states.

* *Relaxation Methods*. When partial differential equations (reaction-diffusion, hydrodynamic, plasma, liquid crystal, solid state, elastodynamic, and so on) are part of the model, they may be treated most naturally as dynamical systems by discretization of the spatial variables. Thus, the infinite-dimensional state spaces are projected into finite-dimensional approximations. Finally, these may be treated by orbit methods, to obtain a bifurcation diagram with loci of attraction and separation. This is essentially the relaxation technique of Southwell (method-of-lines).

* *Dynasim Methods*. Whether small or large, ordinary or partial, the exploration of a bifurcation diagram by analog, digital, or hybrid simulation is extremely time intensive. A considerable gain in speed may be obtained with dynasim methods. (Abraham, 1979) Here, special purpose hardware traces a large number of orbits in parallel. Having thus found all the most probable attractors at once, time is reversed and the basin of each is filled with its own color. This process is repeated (perhaps in parallel) for different values of the control parameters. When dimensions are large, new techniques of visualization may be needed. (Inselberg, 1985)

* *Distributed processing*. For the simulation of a complex dynamical model, the static coupling schemes may be implemented by look-up tables, or fast arithmetic. It is the dynamical schemes which are FLOP intensive. It makes sense, if distributed processors are available, to devote one to each dynamical node. Thus, the architecture of the simulation device is identical to that of the complex dynamical model, and similar to that of the target natural system. Message passing traffic may be decreased by the following trick, if the model is loosely coupled. This means that although an output may be changing rapidly, the node it controls is only slowly sensitive to the rapid changes. Thus, occasional updates of control may be transmitted in place of rapid ones. Further, if all current states are broadcast on a schedule to the static nodes and controls of the rapid integration routines running in each processor, the node processors may (if they can afford the time) make predictions of the next broadcast. A cheap predictor, such as Euler

integration, may be used to change the local controls linearly with each local time step, in ignorance of the real values at the neighboring nodes.

* *Numerical methods for cellular systems.* The destiny of a cellular dynamical model is a computer program for qualitative simulation. Although we may expect someday a theory of these models, it may not replace simulation as the dominant method of science, but only supplement it. Thus, we need a technology of numerical methods adapted to these large-scale simulations. Beyond brute-force integration of thousands of identical copies of the standard dynamical scheme with differing (and slowly changing) values of the control parameters, lookup-table methods might be employed for acceleration or economy. In any case, massively parallel hardware and software will be needed, along with new methods of monitoring large numbers of state variables. Color graphics is the method of choice at the moment, and we may imagine a color movie projected upon a model of the physical substrate of the organ as the monitoring scheme.

The current state of the art seems to be simple experiments with standard cells culled from the literature of the physical sciences, such as the Duffing pendulum, the cusp catastrophe, and so on. From these experiments, we may try to recognize some functions of natural intelligence, such as memory, perception, decision, learning, and the like, as in neural net theory.

9. Global modeling

Now we discuss the adaptation of the techniques of complex dynamical systems theory to the modeling of large-scale economic systems in contact with environmental factors. Recent developments in the mathematical theory of complex and cellular dynamical (CD) systems and their simulation give new promise to the social sciences, especially, economics. Some proposals for CD economic system simulations (see, for example, Abraham, 1990A; and Abraham, 1990B) are based on this technology.

Biospherics is the synthesis of the biological and earth sciences (biogeography, atmospheric science, climatology, oceanography, geology and the like) into a unified understanding of planetary ecology and physiology (see Snyder, 1985; and, Lovelock, 1990). It is increasingly important to study biospherics, and CD models are promising here as well. The adoption of a common modeling strategy for biospherics and for economics enables their combination into a single massive model, in which environmental factors are coupled to the economy. In this section, we apply the techniques of cellular erodynamics to the problem of building a spatially distributed model coupling biospherics and economics.

* *Spatial economic models.* Spatial economics denotes a theory of spatially extended economic systems, in which transportation times and other geographic factors may be considered (see Puu.) We consider now a global model, in which the entire globe is covered by more-or-less uniform plaques, of a size such that even the smallest country has several plaques within its borders. We chose an economic model for the dynamics of a regional economic system, as for example in Chapter 3 of (Goodwin, 1991). By appropriate choice of parameters, adapt a copy of the model to each plaque. Finally, couple each local regional model to each of the other regions, with appropriate coupling functions. The result is a cellular dynamical model for the global economy: a spatial economic model. Simulation of the model will be most natural on a massively parallel supercomputer.

* *Biospheric models.* Spatially distributed models for various aspects of the biosphere currently exist in various laboratories. In particular, those aspects intervening in the climate --- such as atmospheric dynamics and chemistry (including the greenhouse effect and the ozone hole), solar radiation, ocean currents --- have been extensively modeled. Other aspects, such as those of the hydrologic cycle --- forest transpiration, ground water, top soil --- and others, such as toxic wastes, have models in development, either in laboratories or in consumer-level computer games such as SimEarth (Maxus Software, Santa

Clara, CA). The synthesis of these distinct CD models into a complex CD model for the biopshere is a relatively simple matter, as they are devised in the common modeling environment of complex dynamical systems theory.

* *Erodynamic models.* Consider now two CD models, one for the global economy, the other for the biosphere. Suppose that both are made of cells for the same regions. All that is neccesary now is to couple the two CD models into a single system. This could be done most simply by coupling, in each region, the economic cell and the biospheric cell. This coupling is the subject of current research in the new field of environmental economics. More general coupling would allow the influence of the biospherics of all regions upon the economics of each region, and vice versa. Such coupling, in the style of connectionist neural nets, might be created by an evolving and learning system. In any case, the result is a monolithic CD model for the spatially distributed economic biosphere, a cellular erodynamic model.

10. Conclusion

While this simple prescription for a world model could be made immediately, perhaps little could be learned from it. This is because the associated mathematics, the theory of massive CD systems, is still in its infancy. Along with the advances in massively parallel computers and the arts of scientific visualization, this theory may be crucial to our future.

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