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EXPERIMENTAL MECHANICS, THE FIRST FORTY YEARS

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Abstract

Since the first work of Fermi, Pasta, and Ulam in 1952, computation has played a small but stimulating role in the explosive development of conservative mechanics. Here is a brief survey of the highlights of the early experimental work in Hamiltonian mechanics, done by the pioneers of computational mathematics on digital computers.

Contents

1 Introduction

In the history of dissipative dynamical systems theories and experiments have been in coevolution since Archimedes, with experimental methods progressing from real machines (eg, Galileo, 1600) to analog machines (eg, Vanevar Bush, 1940) to digital machines (eg, John von Neumann, 1947.)

In the history of conservative mechanics, experiments have played a smaller role, perhaps because dissapation is always present in real and analog machines. Thus, we may date the beginning of experimental (conservative) mechanics with the revolutionary simulation of the nonlinear spring lattice bu Fermi, Pasta, and Ulum beginning in 1952. This early work may now be viewed as providing experimental evidence for Kolmogorov's conjecture, made (coincidentally?) about the same time.

Ten years later, with KAM theory and chaos newly established, a new experiment by He'non and Heiles in 1963 provided excellent computer graphics of chaotic behavior in a simple plane symplectomorphism. And after another decade of so, simulations of model solar systems (such as those of Jack Wisdom, W) began to pile up evidence of KAM chaos in our own solar system. We now turn to brief reviews of these three pioneering experimental programs.

2 Fermi, Pasta, and Ulam, 1952

In the summer of 1952, Fermi, Pasta, and Ulam planned the first simulation of the conservative vibrating string. Integrating from an initial configuration defined by a sine wave, they expected to observe .ul thermalization, that is, the diffusion of the spectrum of the displacement curve from a single frequency to a continuous spectrum (and ergodicity), as indicated by the generic nonexistence of first integrals. They planned to measure the rate of thermalization.

Instead, during the simulations carried out in 1953, they observed the long persistence of shapes, locked into just a few of the lowest Fourier modes, as we would now expect because of KAM tori. They discretized the string as 16 (later 32 and 64) moving points, plus the two fixed endpoints. As the work continued into 1954, Fermi died unexpectedly. The results were mentionned in a Los Alamos report published in 1955, creating a storm of research which persisted for some twenty years, until the connection of the observed phenomena were firmly connected to the Kolmogorov conjecture, originally presented at the International Congress of Mathematicians of 1954. Could there possibly be an experimental influence in the evolution of Kolmogorov's conjecture? Letters from Fermi to Russia in 1953-54 have been cited in this connection. [2]

3 Heńon and Heiles, 1963

In 1963, the astrophysicists Heńon and Heiles studied the motion of a star in a cylindrically symmetric gravitational field. They were interested in the possibility of a third integral of motion, in addition to the energy and angular momentum, due to some puzzling observations. Later, in 1973, Heńon did find an analytical third integral, but in 1963, they chose to study their model galaxy in simulation, using a Poincare' surface of section. The model was similar to that of FPU, but their expectation was the opposite. However, what they discovered was a bifurcation from invariant curves to ergodic behavior. They correctly interpreted their results as the KAM bifurcation of an area-preserving surface transformation.

4 Chaos in the solar system, 1973

Since the original controversies on the ice ages beginning in 1787, many studies have tried and failed to extract dominant periods from the paleoclimatic records. [6] Thanks to the advent of chaos theory, this is no longer a concern of scientists, the rocks are just showing that they have a chaotic memory. But is this compatible with Hamiltonian mechanics? Simulations of the n-body problem adapted to the data of our own solar system were first reported in 1973. After 20 years of work with the number of planets, n, increasing from 4 to 10, the verdict is in: the planets (and the moon) are indeed chaotic. In recent work on the *digital orrery* Gerald Jay Sussman and Jack Wisdom reports that a simulation of 100 million years by symplectic integration is enough to observe the signs of chaotic behavior. [5]

5 Conclusion

One of the aspects of classical mechanics which was overlooked in our text *The Foundations of Mechanics* was the experimental aspect. In this little review, we have pointed out three areas in which simulation has been important in the develpment of theoretical mechanics. All relate to the KAM theory of chaotic motion, and both the theory and the experiments shared the same epoch of time. Some historical interaction between these developments cannot be ruled out.

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7 References

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