HUMAN FRACTALS:
THE ARABESQUE IN OUR MIND

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The rise of fractal geometry as a new branch of mathematics is intertwined with paradigm shifts in the sciences. First, the physical sciences were impacted, then the biological, and now, the social sciences. What are we to think of the diffusion of fractals into cultural studies? Here, in response to Marilyn Strathern's important contribution to this volume, "One-legged Gender," we review the fractalization of anthropology since Donna Haraway's "Cyborgs" of 1985.

THE FRACTAL WAVE

Although the mathematical ingredients of fractal geometry have been evolving for a century or so, we owe the development of this important new branch of mathematics to the genius and courage of one person, Benoit Mandelbrot. In a series of books and papers beginning with *Les Objets Fractals: Forme, Hasard et Dimension* in 1975, he has not only given the subject its definitive mathematical form, but also pioneered many of its most important applications.[1] Because of its novelty, its codependence with computers, or for some other reason, many respected mathematicians have viciously attacked fractal geometry, and Mandelbrot personally, and this has contributed to a public discomfort associated with the words *fractal, geometry,* and even mathematics. In my view, however, fractal geometry is an exciting and important new chapter in the history of mathematics.[2]

Another confusing factor in this context is the mistaken identification of fractal geometry with chaos theory, another exciting and important new chapter in the history of mathematics. Although fractal objects do sometimes display an aspect of spatial chaos, this is very secondary to their fractal nature. The coast of Britain is fractal, but not spatially chaotic. On the other hand, the main objects of chaos theory (attractors, separatrices, and bifurcations) are fractal, but this is secondary to their chaotic nature. The irrational torus is chaotic, but not at all fractal[3]

What then is a fractal object? In a phrase of Freeman Dyson quoted by Mandelbrot himself,

Fractal is a word invented by Mandelbrot to bring together under one heading a large class of objects that have [played]... an historical role... in the development of pure mathematics... structures that did not fit the patterns of Euclid and Newton. These new structures were regarded... as *pathological...* kin to the cubist paintings and atonal music that were upsetting established standards of taste in the arts at about the same time... The same pathological structures... turn out to be inherent in familiar objects all around us.

By definition, fractal objects have fractal dimension. According to Mandelbrot, they are *broken, irregular, fragmented, grainy, ramified, strange, tangled, wrinkled.* These wrinkled structures may extend over space, over time, or over both: fractal space-time patterns.[4] For our purposes, a single example will suffice to characterize a fractal: the sandy beach.

THE SANDY BEACH

In Mandelbrot's classic text, the second chapter is titled: *How long is the coast of Britain?* I will describe the sandy beach in the two-dimensional context of a map. Thus, the ocean and the land are mostly two-dimensional. Before

fractal geometry, the map showed the boundary between the ocean and the land as a smooth curve: a one-dimensional coast. But now, thanks to Mandelbrot, we may zoom in on the coast, and see that it has very small islands, even pebbles, in a densely packed structure. Zooming in again, we see grains of sand on the beach, and in the ocean close to the beach. All this is the coast: it has a fractal dimension. Land penetrates into the ocean in a frothy structure of sand, ocean penetrates into the land in a frothy structure of water in the wet sand. Not only is the coast a fractal, with a dimension more than one but less than two, but it is a fractal region: the coastal zone. The ocean and land are not divided by the coast in a binary fashion: they interpenetrate in a fractal geometry. The fractals of chaos theory (attractors, separatrices, and bifurcations) are all of the sandy beach variety.

MATH AND SOCIETY

At the dawn of modern anthropology in 1871, Edward Burnett Tylor speculated, in *Primitive Culture*, that speech may have originated *among mankind in the savage state*. He then went on to conclude:

> From the examination of the Art of Counting a far more definite consequence is shown. It may be confidently asserted, that not only is this important art found in a rudimentary state among savage tribes, but that satisfactory evidence proves numeration to have been developed by rational invention from this low state to that in which we ourselves possess it.

And since 1871, rational invention has provided us with transfinite arithmetic, the incompleteness of formal mathematics, topology, the classification of finite groups, chaos theory, fractal geometry, and numerous other fabulous mathematical discoveries. Does the evolution of mathematics from primitive counting to computer graphics follow, lead, or accompany, the evolution of cultural history?

Flinders Petrie found, in excavating ancient Egypt,

that mathematics held a commanding lead in the sequence of shifts comprising a canonical revolution of culture.[5] This possibility has dominated my own directions, in the pursuit of mathematical research and application, over the past decade or two. My early experience in the successful revolution of physics by chaos theory in the 1960s and early 1970s gave me a powerful optimism: I had seen the power of new mathematics for change. In the 1970s and early 1980s, I worked toward a similar transformation in the biological sciences. This came more slowly, due in part to a math anxiety and avoidance reflex which unfortunately has poisoned our society, yet we are now experiencing the chaos revolution of biology. The future of the social sciences, however, is hard to foresee. For example, in May of 1992 I was invited to UCLA to give a workshop on chaos theory in the Economics Department. During the first lecture, riots began in the streets outside. The lectures were postponed for a week or so. After resuming, I asked the audience — a large group of professional economists, professors, and graduate students from several continents — if they thought the new mathematics they were learning and applying in their theoretical models could be useful in mitigating the economic problems underlying the riots. Unanimously, they shouted "No! Our work is only theoretical. It is a kind of game. It cannot be applied." Well, I disagree. I believe that the diffusion of chaos theory and fractal geometry into the social sciences is essential to our future evolution, just as the Art of Counting was essential to the Origins of Culture. It is significant that chaos theory has already entered the field of literature.[6]

FRACTAL PEOPLE

Now I will briefly describe some fractal concepts in the context of cultural studies, in temporal order. These include all the examples I have seen, but there must be many others.

Donna J. Haraway, "A Manifesto for Cyborgs: science, technology, and socialist-feminism in the late twentieth century," 1985.

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In this long essay, the author (an historian of science) analyzes the *cyborg*, an integral being who is part human, part machine. Without explicit reference to fractal geometry, her vision is essentially fractal.[7] In fact, she writes: *Cyborg 'sex' restores some of the lovely replicative baroque of ferns and invertebrates...* thus connecting implicitly with fractal geometry. She describes three crucial boundary breakdowns: human/ animal, animal-human/ machine, and physical/ non-physical. She extends these examples to a long list of *fractured identities* — self/ other, mind/ body, culture/ nature, male/ female, etc. — of political significance. This pathfinding analysis leads the way to a fractal method (the sandy beach) for the deconstruction of all binaries, and the reconstruction of self-images (and scientific categories) as fractal identities.

Ron Eglash and Peter Broadwell, "Fractal geometry in traditional African architecture," 1989.

In this provocative seven page research report, the authors (scholars of the history of consciousness and computational mathematics) study the fractal structure of traditional architecture and city planning, in a tribal village in Mali. The fractal nature of the arabesque style is also noted[8] This is the first explicit application, to my knowledge, of fractal concepts to social theory.

Will McWhinney, "Fractals cast no shadows," 1990.

In this highly original twelve-page essay, the author describes the border between good and evil as a fractal boundary. This model is applied to the problem of the management of evil, in two paradigms: holism and arabesque.[9] This is the first explicit application, to my knowledge, of fractal concepts to the human psyche. The fractal concept applied is that of the sandy beach.

Marilyn Strathern, "The mediation of emotion," 1990.

Borrowing from Haraway, the author (an anthropologist) introduces the concept of a person who is neither singular nor plural. Again, the fractal concept was not explicitly applied.[10] A significant development here is the application of the fractal concept of self-similarity across scales. This is applied to the field of information faced by the ethnographer: data of individuals, societies, histories and myths, etc. The view of a culture (or an individual mind) from the perspective of fractal geometry is new and important: the field of information within which we live is a sandy beach.

Roy Wagner, "The fractal person," 1991.

Here, in an essay of fifteen pages, the concept of a fractal boundary in the psyche (as in McWhinney, 1990) is explicitly applied to the works of Haraway (1985) and Strathern (1990). Inspired particularly by an early draft of Strathern (1992), Wagner develops these ideas further,

applying them to the boundary between big-men and great-men systems.[11] He sees the fractal identity of the individual within a relational network as an aspect of Melanesian society.

Marilyn Strathern, *Partial Connections*, 1992.

In this recent book, the fractal concept of self-similarity across scales is extensively applied to the complexity and quantity of anthropological materials: cultural data, ethnographic recordings, etc., as in Strathern (1990).

Marilyn Strathern, "One-legged Gender," 1993. The last few pages of this paper, published in this volume, return to these fractal concepts, in a further evolution from Haraway (1985), Strathern (1990), Wagner (1991), and Strathern (1992). As in her earlier works and in Wagner, the fractal concept of self-similarity across spatial scales is applied and developed. Beyond this extension of her earlier work, the sandy beach aspect of fractal geometry is applied to *gender*. This carries further the work of Haraway (1985) on the fractal deconstruction of binaries.

The fractalization of the gender binary — so fundamental to social structures throughout the animal kingdom — is radical and difficult. Strathern carries it off successfully in this piece, completing a new milestone in the sequence begun in Haraway (1985). We now have a model application of the new mathematics of fractals to anthropology, which may profitably be repeated and extended in future works, enriching both anthropology and mathematics and advancing the paradigm shift now underway in the social sciences.

WHAT NEXT?

To many pure mathematicians, especially those to whom fractal geometry itself is not mathematics but heresy, these applications of new mathematical ideas to anthropology will seem anathema, vulgarization, fractal evil itself. In my perspective, however, they are the first steps of a major paradigm shift, a critical renewal arriving in timely fashion, of an entire area of cultural studies. Let us encourage this trend, which could be advanced spectacularly by a new generation of students well-trained in mathematics as well as in a social or human science. If so, a long lost partnership between mathematics and cultural history and evolution may be restored, jump-starting a social transformation to a sustainable civilization of peace, diversity, and understanding, such as the Garden of Eden of the Goddess envisioned by Marija Gimbutas in the prehistoric partnership society of Old Europe. And in this jump-start, the fractal view of the human mind and the social field of information,

pioneered by Donna Haraway and Marilyn Strathern, is a critical step off the sandy beach of Pythagoras, Plato, and Euclid, and into the post-Pythagorean sea of Mandelbrot. For the future of cultural studies, this is a great leap into space.

NOTES

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1.   For the most recent comprehensive treatment, see Mandelbrot (1982).
2.   For Mandelbrot's view, see his Foreword: fractals and the rebirth of experimental mathematics, in Peitgen (1992).
3.   For a visual introduction to these ideas, see Abraham & Shaw (1992).
4.   For the fractals of music and speech, see Voss (1988) , and Eglash (1991).
5.   See Petrie (1912).
6.   See Hayles (1990).
7.   See Haraway (1985) , also reprinted as Ch. 8 in Haraway (1990).
8.   See Eglash (1989).
9.   See McWhinney (1990), as well as the Epilogue of McWhinney (1992).
10. See Strathern (1990), and also Strathern (1992).
11. See Wagner (1991).

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