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# MIMI AND THE ILLUMINATI

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ABSTRACT. Unpon the invitation of the Lindisfarne Association and the Cathedral Church of St. John the Divine in New York City, a concert of audiovisual mathematics/music took place in the Cathedral Church on 17 October, 1992, at about eight o'clock in the evening. Three computers and three persons cooperated in the performance. This is the story behind the event (and its video recording).

## 1. Visual music and math.

Here is a little background on our artistic heritage and orientation.

### Visual Music.

is a traditional art medium with an extensive, if little known, history. One can imagine that it played a basic role in the arts and rituals of the cave cultures of the European epipaleolithic, and the early civilizations of the Anatolian neolithic. The cave sanctuaries and rituals of Minoan Crete, and the derivative mystery schools of Ancient Greece may have continued this artistic tradition, along with its mystical religious associations. The performances of Father Castel in Paris of color music created with candlelight and cloths were famous in the eighteenth century. In more recent times, visual music reappeared in the Theosophical Revival at the turn of this century. Aleksandr Scriabin intended *Prometheus: The Poem of Fire* (1910) to be accompanied by a light show. Alexander Wallace Rimington performed live color music in London at about this time. Claude Bragdon and Thomas Wilfred in New York created keyboard instruments for the live performance of color music in the 1920's. In the same period, color music compositions in the form of animated films were laboriously made by Arnaldo Ginna and Bruno Corra in Italy, and Oskar Fischinger in Germany. In 1936 Fischinger moved to California, where he continued his work, influencing John Whitney and others of a major group of color music film artists in America. The live performance of color music survived in the work of Mary Hallock Greenewalt in Philadelphia in the 1940's (she invented the rheostat for her organ), Charles Dockum in New York and California in the 1950's, and of course in the Hippie culture of the 1960's.

Traditionally, masters of visual music, such as Father Castel or Mary Hallett Greenwalt, gave performances in silence on instruments of their own creation. In

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the film works of visual music by Oskar Fischinger and by James and John Whitney, mathematics was used as a source of visual musical form for the first time. In the analog video synthesizers and fluid cymatic light shows of the 1960s, visual music - in partnership with aural music - ascended to new heights of popularity and creativity. Our work is further research in this direction, catalyzed by rapid advances in computer science and the mathematics of chaos.

**Visual Math**

is a new development empowered by the computer revolution. Computer graphics has boosted math into an art medium, making mathematical objects visible, and revealing the incredible beauty of the mathematical landscape. In the branch of mathematics known as *chaos theory*, massively parallel models called cellular dynamata, or *CDs*, are routinely observed (running on massively parallel supercomputers) as animated color images. Both spatial and temporal chaos abound in these morphogenetic natives of the mathematical landscape.

**MIMI** *(the Mathematically Illuminated Musical Instrument)*

was first conceived in a talk by Ralph Abraham at the International Synergy (IS) Institute in Los Angeles on October 7, 1984. Actual construction of the first model began in July, 1985, with support from the IS Institute. While this instrument is not yet complete, simulations by supercomputers have been circulating on videotape since 1989. The Cathedral performance in the Fall of 1992, the premiere MIMI concert, was an ensemble of three computers and three humans.

In this concert, the ILLUMINATI TRIO played visual music based on CD models in the supercomputer, and coordinated aural music based on the same models. The aural component is enlarged by an obbligato cello line. The evolution of the CD models themselves is affected by gestural input from the three performers. Beyond the exploration of computer/visual math/music, we are fascinated by the resonance between the two modes. We are extending the domain of Pythagorean harmony, following John Whitney, into an audiovisual and mathematical landscape

**2. THE PERFORMERS.** (A) The audio performer is Ami Radunskaya, Professor of Mathematics (ergodic theory of dynamical systems) at Rice University in Houston, Texas. Ami has worked extensively at the Center for Computer Research in Music and Acoustics at Stanford University and the Center for New Music and Audio Technology at the University of California at Berkeley. Two of her pieces will soon be released on compact disks on the Centaur Label. She is also responsible for the overall composition of the pieces of this performance.

(L) The linking performer is Ralph Abraham, Professor of Mathematics (chaos theory, computation, and applications) at the University of California at Santa Cruz, founder there of the ~~Visual Math Project and Computational Math Program~~, VMI and author of *Dynamics, the Geometry of Behavior.* and Chaos, Gaia, Eros.

(V) The visual performer is Peter Broadwell, of 3DO. Peter studied computational mathematics in the Visual Math Program at the University of California at Santa Cruz, where he helped create the software of the Computational Math Lab, earning the degree of Master of Arts in Computational Mathematics in 1988. He was for several years (including the time of the concert) a member of the Technical Staff at SGI, in Mountain View, California, where he created the IRIS software we

spell

are now using. He is presently Software Artist at The 3DO Company in San Mateo, California.

**3. The Cathedral concert setup.** We used three instuments, one for each performer.

The audio (A) instrument is a Yahama TG77 synthesizer, controlled by the IRCAM/Opcode program MAX running on an Apple Mac II. The program MAX receives, processes, and sends MIDI messages. It is set up to respond to the V instrument (which generates the animation) and also to commands sent from the L instrument, and sent from performer (A) with her electric cello, and Zeta MIDI controller.

The video (V) instrument is a IRIS 4D/440VGXT, a 200 MIPS graphics workstation on loan from the manufacturer, Silicon Graphics, Inc. (SGI). It runs programs from the research frontier of chaos theory, the mathematical theory of nonlinear dynamical systems, cellular dynamata, or CDs, and iterated endomorphisms of the plane, or *endos*. Two CDs were featured in the Cathedral concert: the Brusselator and the Logistic Lattice, and two endos : the twisted logistic and the Henon. All of these are classics of the recent research frontier.

These mathematical systems have parameters which may be manipulated by performer (V). The SGI IRIS workstation ~~creates~~ the ~~digital video stream in real~~ time, that is, as you watch. Performer (V) sends ~~commands to the~~ IRIS with ~~Lightning~~ wands, made by Buchla and Associates of Berkeley, California. mouse inboard

The ~~linking instrument (L)~~ connects the two instruments. This third computer, an SGI IRIS INDIGO graphic workstation, also on loan from SGI, uses algorithms from chaos theory to communicate features of the video (V) to the audio (A) system. The algorithms are controlled by performer (L) by means of LIGHTNING wands.

**Hardware.**

Gestural input device - Lighting. Of course the most powerful machines won't express anything if you can't give them some input. We wanted the input to be intuitive, and choose a gestural input device called "Lighting" and manufactured by Buchla and Associates of Berkeley CA, USA. It is designed by and for musicians and interfaces to the computer using the MIDI protocol. We only used a few of the many gestures that Lighting is sensitive to. In particular we mapped all hits (quick percusive like changes in direction) to be point triggers. We also continuously monitored the left and right hand positions and the state of the switches that are on each wand.

repeats

Machine 1 - named Morph, an Iris Indigo from Silicon Graphics, input coordinator. The MIDI signals from the Lighting were fed into an IRIS Indigo R4000 Elan. Here they were bundled up and merged with mouse input from performer G (peter). The mouse is not as expresive as the Lighting but for simply picking initial conditions and occational parameter changes it worked alright.

repeated

Machine 2 - named Orbit, an IRIS 4D/440VGX, image generator. Morph would then send the bundled up inputs over Ethernet to Orbit, and Silicon Grpahics 4D/480 VGX. This machine was responsible for calculating as fast as it could the next image. It would then display that image as an RGB signal from its framebuffer. It also kept track of where the interesting trigger points had been dropped and sent the calculated values from those points back to Morph.

repeated

Machine 3 - named Point, a Macintosh II, sequence controller. The Macintosh was connected to Morph via MIDI, and ran the MAX musical control language. In MAX, incoming time series were processed and mapped to musical parameters in the form of MIDI control, pitch, or velocity values.

## 4. Mathematics in Music.

New

As in the visual arts, or perhaps even more so, mathematics has played an enormous role in the creation of musical forms at every level. Music has, in fact, been defined as "structured sound" (see R. M. Schaefer *The Art of Sound*), and mathematics provices the richest vocabulary for the description of structure. The contributions of music to the understanding of mathematics are perhaps more obscure, but nevertheless present throughout history. The Pythagoreans made a religion of numbers and their relationships, and the audification of small integer ratios. The creation of "just" harmonic relation, for example octaves, fifths, and fourths, contributed to the understanding of the mechanism by which sounds themselves were produced - the creation of standing waves in a string, for example. With the advent of computers, and in particular of digital sound, the relationship between numbers and music has become much less subtle and much more intimate: digitally encoded sound is, after all, a list of numbers. The ear is capable of perceiving changes in these lists of numbers received at a rate of 40,000 numbers per second. It is this large band-width that makes the audification of certain mathematical processes in ntriguing: perhaps the ear can detect patterns which are too long for the eye to process in one pass. In fact, a heart specialist will sometimes "listen" to an electrocardiogram to detect periodicities and abnormalities that the eye could not: the data is treated as a time-domain representation of an audio signal, and is run through a DAC (digital-to-analog converter) so that the physician can hear it: repeating or periodic patterns can then occasionally be perceived as pitches.

Although the work described here was conceived of and presented as an artistic rather than a scientific enterprise, the same set-up can be used to shed light on the mathematical structures themselves. With the visualization and audification of an evolving mathematical structure occurring simultaneously, the observer can process more dimensions simultaneously, and hence has more hope of discovering evidence and justification for any theoretical conjuectures which may propose themselves. This bidirectionality - the music as an audification and hence a clarification of the mathematics; and the mathematics as a musical source - is to ~~me~~ us one of the more intriguing aspects of this work.

It is important to note that music as "structured sound" does not necessarily mean "ordered" sound. The structures in question could be stochastic as well as deterministic. In fact, the particular family of systems discussed exhibit the full spectrum of behaviour, from the ordered to the chaotic. The creation and dissolution of order is one of the parameters that is "played" during the pieces.

ref: "musical dice"

The use of random processes as a compositional tool goes back at least to Mozart, who composed a waltz which would now be described as a one-step Markov process: each measure was a state of the process, and the probability of another given measure occurring next depends on the current measure. The choice of which measure would actually follow a given measure in performance was to be made by rolling an appropriate (6-sided) die. This waltz of Mozart's can be used to

introduce the concept of a mapping from state space to musical parameter space. The "state space" of a process is the set of all possible outcomes of that process. For example, the process which consists of repeated tosses of a coin outputs heads or tails, so its state space is the set { heads, tails }. For Mozart's waltz, the process (performance) outputs measures one at a time, hence the state space is the set of all possible measures. He composed and labeled these measures with the
2 number 1 - 92. For a Markov process, the set of all possible sequences of states and their relative probabilities is described by ***transition probabilities***: the likelihood of moving from a given state to another. In Mozart's score it is shown that there are only 6 measures which are allowed to follow any given measure, and one out of these 6 measures is to be chosen "randomly", that is with equal chance of any of the six occurring. Thus the transition probability of going from measure A to measure B is zero, if B is *not* one of those 6 measures, and is 1/6 if it is.

Abstractly, we could describe this process by giving only these transition probabilities, and by saying we are dealing with a state space of 92 discrete elements. The performance could then be described by a *mapping*, or function, from this state space to *musical space*. In the waltz, the mapping associates a number between 1 and 92 with one measure of music, or a short sequence of pitches with their durations. Notice that this process, though random, does not produce a random sequence of pitches. All of the pitches are predetermined by the composer. In fact the ordering of pitches on a small scale, the pitches within one measure, is also predetermined. This is significant since the direct mapping of numbers to pitches, which is the naive approach to the audification of a mathematical process, should be considered as only one of many possibilities, and yet it is the most commonly heard. The application of mathematical structures in real-time, whether these structures are random or deterministic, can be applied at any or many of the levels in the musical hierarchy. Exploration of the range of possibilities is one of the goals of the MIMI project.

In our pieces, the visual and audio realizations of the mathematical processes are bound together by virtue of their common origin, but this connection could be either emphasized by synchronizing noticeable feature of the sound and visuals, or it could be de-emphasized by using mapping which incorporate varying amounts of delay. Both extremes are displayed in the two examples ~~on the video~~, as well as a range of performer inter-action and improvisation.

## 5. The pieces.

### Morphic Resonance.

The piece is governed by the Brusselator, a two-dimensional lattice of oscillators introduced by Ilya Prigogine as an evolution of Alan Turing's 1952 model for chemical morphogenesis. An initial pattern chosen by performer L sets off an evolutionary sequence of images, computed in real time by the super-computer, subject to parameters chosen by performer V.

This CD (cellular dynamata) evolves relatively slowly, and it suggests slowly evolving sonic material. Although individual spots vibrate with swiftly changing colors, the overall image is made up of slowly spreading regions of color, sometimes made up of many colors, sometimes more or less monochrome. An attempt to represent these features sonically resulted in the use of an averaging over many samples rather than requiring a perceptual change with each numerical value.

Information-theoretic entropy is a measure of complexity which was adapted from information theory in the 50's and applied to dynamical systems. Roughly speaking, it measures the average amount of information required to specify the next observation. If the process is completely predictable, no information is required at all, since it is known ahead of time what will happen next. Hence, *predictable* processes have zero entropy. Since this entropy is determined by the infinite future, it is impossible to actually compute it on the fly from observations of the output of a given process. However, an extimate can be made by counting the number of different sequences of digits appearing in a string of a fixed length, and divind the logarithm of this number by the length of the string:

$$H_N(P) = 1/N \log(\text{the number of strings of length N})$$

The higher the entropy, the more complex the stream of numbers.

The MAX patch for this piece estimates the entropy of the process by looking at sequences of length 127. This number, which we refer to as the *frame entropy*, is then fed to another subroutine which creates chords. The higher the frame entropy, the more dissonant the chord will be. This is accomplished by adding to the chord pitches which appear later in a specified harmonic sequence. As soon as one frame entropy is computed, the next series of 127 numbers is fed in, and a new frame entropy is output to the main patch. In MAX this computation does take some time, and there is some lag between "what you see" and "what you hear".

The overall harmonic progression is predetermined, but the rate at which it progresses is also affected by the complexity of the incoming numbers through the main patch. The timbres are singing voices, and they provide an accompaniament to an obbligato cello line which follows the evolving harmonies.

**Fruit Fly Fandango.**

The second ~~selection~~ example ~~on the video provides an example of~~ illustrates audification on a larger scale: this cellular dynamata is distinguished by abrupt, almost periodic changes in the time-series of a given lattice point. In order to hear this aspect of the evolution of the system, amplitude was chosen as the dominant perceptual musical parameter. The underlying process is a two-dimensional array of logistic functions connected by Laplacean coupling on the torus.

Performer L again selects two time-series for transmission to the Macintosh. Each of these time-series is visualized by mapping one variable to a color spectrum. The same variable, which we'll denote by $\theta$, is mapped to one of four amplitudes by rounding it off to the nearest quarter, and using the map:

$$P : \theta \mapsto \{\text{rest, p, mf, } f\} \quad ,$$

where

$$0.0 \mapsto \text{rest}$$

$$.25 \mapsto \text{p}$$

$$.50 \mapsto \mathrm{mf}$$

$$.75 \mapsto \mathrm{f}$$

The second variable is mapped to a pitch set which is selected by choosing one of five scales. A large jump between successive values causes a change in instrumental timbre. The rapidity of the evolution of the system, along with the abrupt numberical changes are seen as vibrating colors, and heard as rhythms which often seem to repeat but eventually evolve on a larger time-scale. When a new initial condition is selected by performer V it is obvious in the visualization as a discontinuity: the appearance of a new and unrelated image. One hears this discontinuity as an abrupt change in the melodic patterns.

## 6. Summary.

These examples illustrate a few of the possibilities in the audification of cellular dynamata. The choice of mappings from state-space to musical parameter space is made according to the process which is running, and to the region of interest in parameter space. The mapping can involve an averaging of information, to capture the complexity of the process rather than individual trajectories. On the other hand, when one or two trajectories are of particular interest, a mapping whose domain allows aural distinction between a wide range of numerical values, such as pitch or timbre, is preferable. Since the ear easily places pitches in a linear ordering, this is a simple way to audify a relatively slowly changing, linearly ordered sequence of numbers. If it is only the distinction between different values that needs to be emphasized, then a subspace of timbre space can be used effectively: different instruments from some "orchestra" or different values of timbre modulation, for example. For widely and rapidly varying processes, using pitch or timbre as the range of the mapping results in a jarring, confusing sonic representation. Using amplitude as the target space instead allows the ear to hear longer sequences as units, and to use rhythmic comparisons to pick out patterns in the evolution of the structure in terms of these longer units.

These are but a few of the possibilities, and these examples should be viewed as preliminary experiments. The use of continuous controllers which allow navigation through parameter space in real time encourages exploration with a variety of mappings even for one system. When particularly beautiful areas of parameter space are discovered, and when appropriate audification mappings have bben determined, a score can be written with whatever performer interaction is desired, be it as repeated initialization of values, movement in parameter space, control of the mappings themselves, or the addition of other musical lines. The rich structures of endomorphisms and cellular dynamata, most of which are based on models of real biological or chemical systems, can then be enjoyed by audiences without the abstraction and special vocabulary of mathematical formulae.