# The Canon of Lespugue

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Dedicated to Ernest G. McClain

### Abstract

Linear measurements taken from the Venus of Lespugue, a 25,000 year old sculpture, closely match the diatonic scale of the Vedic Aryans, also known as the Dorian mode of the ancient Greeks.

### 1. Introduction.

In *Pacific Shift* (Thompson, 1985), world cultural history is divided into four periods called cultural ecologies. Associated with each is a characteristic mathematical style, or mentality. These, with approximate beginning dates, are:

- $\star$  4,000 BC, the riverine cultural ecology, with the arithmetic mentality
  - \* 500 BC, transcontinental, geometric
  - \* 1700 AD, oceanic, dynamic
  - \* 1972, biospheric, chaotic

While the theory of dynamical systems and chaos theory are new branches of mathematics, arithmetic and geometry are ancient, and reach back into remote antiquity. Nevertheless, the cognitive style of prehistoric cultures is arithmetic, while the geometric style was an innovation of the ancient Greeks, sometimes credited to Thales or Pythagoras.

As mathematics has evolved from astronomy and music, we may regard archeoastronomy and archeomusicology as the foundation stones of archeomathematics. And these two foundation stones were eventually combined in the idea of the music of the spheres, in which the zodiacal constellations were mapped onto the tone circle of the twelve-tone chromatic scale. This development goes back to the Pythagoreans, 400 BC, at least. (See Part III of Godwin, 1987. See also Mountford, 1920.)

This paper is an exercise in archeomusicology, demonstrating the existence of the arithmetic mentality long before the beginnings of the riverine cultural ecology.

### 2. Archeomusicology

As the findings which we report here assume a high degree of musical and arithmetical sophistication on the part of preliterate humans before 23,000 BC, we may begin with a review of some recent archeomusicological discoveries which support this assumption.

## A. Fragment of Flute, 40,000 BC

Currently the oldest known musical instrument, this is a segment of the femur bone of a cave bear, dated 43,000 to 82,000 B.P. It has two complete holes and two partial holes, one at each end of the broken fragment. It was found in 1997, at a Nean-derthal campsite, by paleontologist Ivan Turk of the Slovenian Academy of Sciences, and has been analyzed by Canadian musicologist Bob Fink. Its tuning corresponds to *Mi-Fa-Sol-La* of the diatonic scale. For more details, see: <a href="http://www.webster.sk.ca/greenwich/FL-COMPL.HTM">http://www.webster.sk.ca/greenwich/FL-COMPL.HTM</a>.

For a skeptical view of this interpretation, see *Science News*, 153:14 (April 4, 1998) p. 21.

## B. Complete Flutes, 7,000 BC

Six complete flutes, and fragments of 30 others, have recently been recovered from burials at Jiahu, in Henan Province, China. The site is firmly dated from 7000 BC to 5700 BC (Zhang, 1999). Made from legbones of the red-crowned crane, the complete flutes have 5, 6, 7, and 8 holes.

The best preserved flute has seven holes, and has been spectroscopically analysed. It is apparently tuned to a six-tone or seven-tone Chinese scale. It is playable, and part of *The Small Cabbage*, a Chinese folk song recorded on this 9,000 year old instrument, may be heard at: <a href="http://www.nature.com">http://www.nature.com</a>.

### 3. The Quadrivium

The quadrivium —arithmetic, geometry, astronomy, music— is generally credited to Pythagoras, and the "Greek miracle" of the 6th century BC. The details of the musical arithmetic of the several scales, in various intonations, has been recovered by various authors. The encoding of this ancient numerology in the dialogues of Plato has been analyzed (Brumbaugh, 1954). The numerical methods of the Pythagoreans have been graphically reconstructed (McClain, 1978). On the other hand, Pythagoras was known (according to ancient accounts) to have traveled and studied in Egypt, Babylonia, and India. The discovery of the musical arithmetic of Pythagoras encoded in the poetry of the Rig Veda pushed back the dating of this sophisticated mathematics by a millennium (McClain, 1979).

We have applied the methods of musical arithmetic to measurements of the Venus of Lespugue, a paleolithic sculpture of about 23,000 B.C., and found them to fit very closely the diatonic scale basic to the theory of the Rig Veda and the ancient Greeks.

## 4. The Monochord or Canon of Pythagoras

The monochord is a one-stringed musical instrument, like the ektar of India. Also known to the Pythagoreans as the canon or otherwise in ancient Greece as the pandoura (Levin, 1994, p. 61) the monochord was used as a musical instrument by the ancient Egyptians since 1570 BC (Levin, 1994, p. 71). To the Pythagoreans, it was a scientific rather than a musical instrument. That is, it was constructed primarily for the purpose of experiments in the physics of sound. According to legend, the last words of Pythagoras were an admonition to his disciples to continue their researches on the monochord (Levin, 1994, pp. 71, 96).

In these studies were discovered the relations between the harmonic intervals of music theory and the ratios of arithmetic. This discovery is usually attributed to Pythagoras in the 6th century BC, but is attested in historical evidence only after 300 BC. See David R. Fideler in Guthrie (1987, pp. 24-28, 47, 327-328) and Levin (p. 96). Repetition of these experiments on the monochord or canon are described in detail in numerous texts ancient and modern, and are highly recommended as a basis for understanding the canon or Venus of Lespugue. See especially Fideler in Guthrie (1987, pp. 24-28), Levin (1994, pp. 143-147), and McClain (1978, pp. 169-175).

## 5. The Hindu-Greek Diatonic Scale

The relationship between number and tone common to the Vedic Aryans, Sumerians, Hebrews, and Greeks, involves a reference scale of integers used for defining ratios between the numbers 1 and 2. Usually, these integers are products of the prime numbers 2, 3, and 5. Also, the reference numbers corresponding to tones of a musical scale should be closed under reciprocation. Details are found in many musical texts, but McClain is particularly clear on this theory. For example, some reference scales found in the literature use the whole number intervals: [30, 60], [72, 144], [360, 720], and so on. Bypassing many complications, we choose one of these for our purposes, [72, 144], which is the smallest reference scale which may be interpreted as a scale of string lengths on a monochord for the diatonic scale. McClain calls this range of integers the Davidic set. The heptatonic scale found by the usual Pythagorean method —in rising order

(decreasing string lengths, increasing tones)—is thus: Lengths: 144, 135, 120, 108, 96, 90, 80, 72 Tones: D, eb, f, G, A, bb, c, D'.

Ratios: 15/16, 8/9, 9/10, 8/9, 15/16, 8/9, 9/10 This is the Hindu-Greek Diatonic Scale, also known as the Dorian Mode of ancient Greece and as Ptolemy's Diatonic Syntonon, and is shown in Figure 1. (McClain, 1976, Chart 1, p. xxi, and Chart 3, p. 13) (Levin, 1994, p. 77.)

# CHART 1 Foundations of Mathematical Harmonics

Equivalent representations of the basic Hindu-Greek scale (Ptolemy's Diatonic Syntonon).

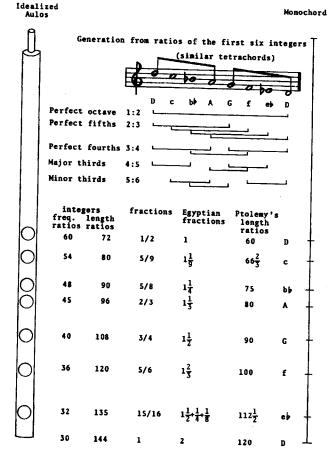


Figure 1. The aulos and monochord, tuned to the Hindu-Greek scale. (From McClain, 1976, Chart 1.)

#### Notes

- \* The pattern of rising intervals —whole tones (T) and semitones (S)—in the Hindu-Greek Diatonic Scale, or Greek Dorian Mode, is: STT,T,STT. The first note is taken above as D, but could be any note. This sequence agrees with the Bhairavi Thaat, one of the ten modes of North India (See Batish, 1989).
- \* The Greek Phrygian Mode, also known as the Christian Modus Primus and as the Modern Dorian Mode, has the pattern TST,T,TST (McClain, 1976, p. 61). This sequence may be obtained from the

preceding by one rotation, that is, starting the scale one note higher. It agrees with one of the Modern Minor Modes, and with the Kafi Thaat of North India.

- \* The Greek Lydian Mode has the pattern TTS,T,TTS (see Lauer, 1989). It may be obtained from the preceding by one rotation. It agrees with Modern Major Mode, and with the Bilaval Thaat of North India.
- \* All these Greek modes are usually presented in Pythagorean ratios involving powers of the first three primes only: 2, 3, and 5. Considering the ancient aulos, or Greek flute, as studied from actual surviving examples, the Early Greek Dorian Mode is found in variants involving ratios of powers of the first six primes: 2, 3, 5, 7, 11, and 13 (Schlesinger, 1939), (Lauer, 1989, p. 202).
- \* The patterns, STT,T,STT, etc., are of the form: tetrachord, whole tone, tetrachord. Using only wholetones and halftones, there are only three tetrachords which fit into this pattern and add up to an octave. Therefore, there are nine possible heptatonic (7-tone) scales made according to this pattern. Of the nine, only three are symmetric, that is, have the same tetrachord sequence in both position. And these three are the Greek Dorian, Phrygian, and Lydian Modes.
- \* According to the rules of Aristoxenos only intervals S and T are allowed in Greek modes, with S exactly twice, while other combinations are found in the (older) scales of North India. Also, S is allowed between the two tetrachords, but there should not be TTTT in the scale, nor in its extension to two octaves. (Mountford, 1920) The seven Greek modes satisfying these rules are (in order of rotation):

Mixolydian: STT,S,TTT Syntonolydian: TTS,T,TTS (symmetric)

Phrygian: TST,T,TST (symmetric) Dorian: STT,T,STT (symmetric)

Lydian: TTT,S,TTS lonian: TTS,T,TST Aeolian: TST,T,STT

• There are no other modes following these rules.

## 6. The Venus figurines

The most surprising aspect of our discovery is the extreme antiquity of the figurine, the Venus of Lespugue. This is a sculpture carved from mammothivory, 6 inches (147 millimeters) high, from the Gravettian-Upper Perigordian culture, dated about 25,000 B.P., found at the Des Rideaux site at Lespugue, Haute Garonne, France (Graziosi, 1960; Leroi-Gourhan, 1967; Marshack, 1972; Gimbutas, 1989). The ages of our historical scales have been traced back by McClain only to early Sumer, around 6,000 B.P.

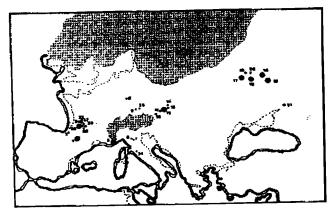


Figure 2. Map showing the distribution of paleolithic goddess figurines (from Gamble, 1982).

About 40 similar figures, called paleolithic Venus figurines, or Aurignacian Venuses, have been found since 1895 in sites of Europe and west Asia, as shown in Figure 2. Theories of their meaning abound in the literature of prehistory since 1967. A very interesting recent summary has been given by Gimbutas (1989), Leroi-Gourhan, (1967, p. 90), Gamble (1982). The most studied exemplars are those of Lespugues, Kostienki, Dolni Vestonice, Laussel, Willendorf, Gagarino (2 cases), and Grimaldi. Outline drawings of these, enclosed in rhombi, or lozenges, are shown in Figure 3, and details of the Venus of Lespugue are shown in Figures 4 and 6.

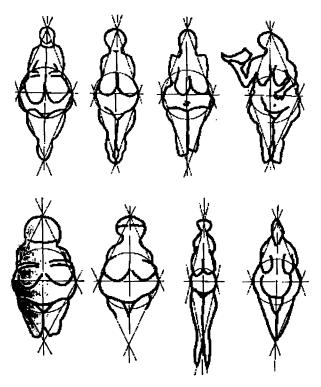


Figure 3. Famous Venus figurines, enclosed in rhombi (from Leroi-Gourhan, 1967, p. 92).

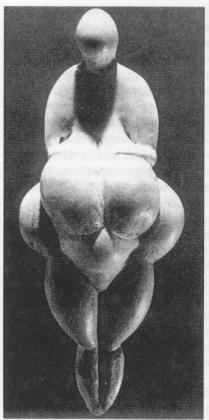


Figure 4a. Front view of the Venus of Lespugue (Marshack, 1985, Fig. 6, p. 11). Marshack says (p. 10): "The famous Venus of Lespuque, 25,000 BC, was excavated in France and appears somewhat later than the Cro-Magnon skeletons (Fig. 6). We have, again, an image of great sophistication, containing an almost modern, 20th century shape and form."

See also Gim-butas (1989, Pl. 5, frontis.; also Fig. 252, p. 163). Caption to Gimbutas' Plate 5.

This extraordinary mammoth- ivory sculpture conflates breasts and buttocks into a zone of eggs circling the figure's middle. H. 14.7 cm. Gravettian-Upper Perigordian (Des Rideaux), at Lespugue, Haute Garonne, France; c. 23,000 BC (breasts in front were found damaged; here they are shown in reconstruction.).

On p. 370 we find a reference to Marshack (1979). The caption for Gimbutas' Figure 252 is identical, except the date is given as 21,000 BC, and the reference is to Marshack (1972). Following the original publication by Saint-Perier in 1922, the oldest reference we have on hand is Graziosi (1960), p. 48) which says:

This statuette, carved from a fragment of mammoth tusk, is 147 mm. high; the breasts, stomach, and left side (forearm and thigh) are considerably damaged, but the abrasions are not such as to prevent a perfect reconstruction of the sculpture; the opposite, or right side being intact, Saint Perier's work of reconstruction was reduced to a purely mechanical task, and the resulting figure can be studied almost as though it were the complete original.

As evidence for this judgement, Graziosi shows the photographs of the statuette before and after reconstruction, in his Figures 3a and 3b. Figure 3b in Graziosi shows an object clearly identical to that of Gimbutas, although seen from a slightly different angle. Graziosi does not give an explicit date for the figurine, simply calls it Aurignacian-Perigordian, that is, in the range 26,000 to 22,000 B.C.

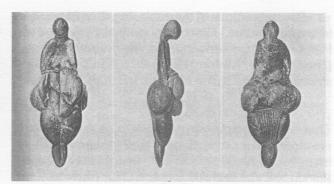
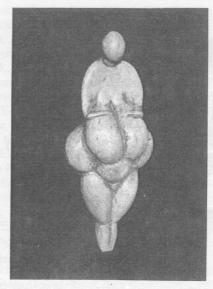


Figure 4b. The Venus of Lespugue, before reconstruction (Graziosi, 1960, Fig. 3a).



**Figure 4c.** The Venus of Lespugue, after reconstruction (Graziosi, 1960, Fig. 3b).

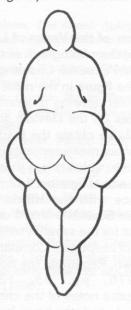
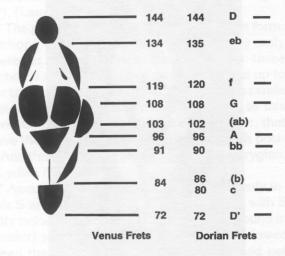


Figure 4d. Outline drawing of the Venus of Lespugue, front view.

The proportions of the enclosing rhombi provide interesting ratios, like those studied by Villard de Honnecourt for drawings of the human figures. (Bowie, 1959; Kayser, 1946) Of the eight figurines shown in Figure 3, the ratios of width to height are:

0.39, 0.33, 0.37, 0.37 0.44, 0.44, 0.20, 0.28

These seem too closely grouped to be random, and are close to the proportions of the Pythagorean triple triangles: 5-12-13, 12-35-37, and 44-117-125, of the cuneiform tablet Plimpton 322 from ancient Babylonia, dated about 1700 B.C. McClain has given a tonal interpretation of Plimpton 322 (McClain, 1978, p. 124).



**Figure 5.** The Canon of Lespugue alongside the Greek Dorian Mode.

7. Measurements of the Venus of Lespugue

We oriented the photograph of the Venus, front view, as shown in Figure 4d. Choosing horizontal lines (frets) across the figure in the most obvious places (using a computer drawing program) and measuring the distances on the Davidic scale of lengths, from 144 to 72, we obtain the results of Figure 5. The accuracy of our measurements is within one half of a reference unit, which is less than a millimeter in the scale of the actual figurine, 147 millimeters. The correspondence with the Hindu-Greek (Greek Dorian) Diatonic Scale is within 1 unit, as shown in Figure 5, except for the seventh tone. This result we call the Canon of Lespugue. Compare with Figure 1, held upside down. Regarding the note symbols, see McClain (1976, pp. 33, 125, 127). In Figure 5, two extra notes of the chromatic scale in just intonation, b and ab, have been interpolated among the Dorian frets for comparison. Comparing the two scales, we may note two main differences.

First, note that the subdominant perfect fourth, G, and the dominant perfect fifth, A, occur in both scales. The subdominant fret of the Venus scale coincides with the top of the cusp between the breasts. The dominant fret of the Venus scale coincides with an incised line at the top of the pubic triangle. In between, in the Venus scale, we have noted a line defined by the bottom of the breasts, corresponding to ab, the tritone. This most discordant note does not occur in the Dorian scale. We may interpret this line as a fret for occasional ornamentation, and not part of the Canon of Lespugues: an extra fret.

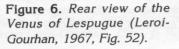
Secondly, the seventh note of the Canon of Lespugue at coordinate 84 is audibly flat, relative to the seventh note of the Greek Dorian Mode, c, at 80. It is also slightly sharp relative to the b of the chromatic scale in just intonation, so we might denote this b+ or c-. Of the seven notes of the diatonic scale, this is the only one which differs significantly from the Greek Dorian Mode. An interval greater than a whole tone is suggested, in contradiction to the (later) rules of ancient Greece.

We may compare three scales (the Dorian, the Venus, and the aulos measured by Schlesinger) in Davidic length coordinates:

Dorian:144, 135, 120, 108, 96, 90, 80, 72 Venus: 144, 134, 119, 108, 96, 91, 84, 72

Aulos: 144, 131, 118, 105, 92, 86, 79, 72 Note that the 7th note of the Venus, at 84, is closer to the 6th note of the Aulos. Otherwise, the Venus is closer to the theoretical Dorian than is the aulos of ancient Greece!

Further support for our musical interpretation of the Venus of Lespugue is provided by the incisions on the back of the figurine, which look very much like the strings of a lute, gathered to a single peg or nut, as shown in Figure 6. An alternate interpretation of the incisions on the backside of the Venus of Lespugue, as a string skirt, has recently been offered by anthropologists (Angier, 1999; Barber, 1991, p. 40).





#### 8. Conclusion

We are faced with the need to upgrade our estimation of the cultural advancement of paleolithic people. To the great arts of the Gravettian period, including the fabulous paintings of Lascaux and the Venus figurines, we must now add an advanced musical arithmetic previously ascribed to Pythagoras, the Vedic Aryans, or the Old Babylonians. This supports the suggestion of McClain that advanced arithmetic preceded writing, and demands a rethinking of prehistoric mathematics and music. It would seem useful to proceed with measurements of the other figurines and artifacts of Paleolithic times, in the style of Alexander Marshack, in search of further clues to the quadrivial pursuits of prehistory.

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### References

- Abraham, Ralph, *Chaos, Gaia, Eros*, San Francisco: Harper, San Francisco, 1994.
- Angier, Natalie, "Furs for Evening, but Cloth was the Stone Age Standby," New York Times, Dec. 14, 1999, pp. D1-2.
- Barber, E. J. W., Prehistoric Textiles: The Development of Cloth in the Neolithic and Bronze Ages, Princeton, NJ: Princeton University Press, 1991.
- Batish, Shiv Dayal, Ashwin Batish, Ragopedia, Volume One: *Exotic Scales of North India*, Santa Cruz, CA: Batish Publications, 1989.
- Bowie, Theodore, ed., *The Sketchbook of Villard de Honnecourt*, 2nd ed., Bloomington, IN: Indiana University Press, 1959.
- Brumbaugh, Robert S., *Plato's Mathematical Imagination:*The Mathematical Passages in the Dialogues and Their Interpretation, Bloomington: Indiana University Press, 1954/1964.
- de Nicolas, Antonio T., Four-Dimensional Man: The Philosophical Methodology of the Rig Veda, Bangalore: Dharmaram College, 1971.
- de Nicolas, Antonio T., *Meditations Through the Rig Veda*, York Beach, ME: Nicolas Hays, 1976.
- de Nicolas, Antonio T., Avatara: The Humanization of Philosophy Through the Bhagavad Gita, York Beach, ME: Nicolas Hays, 1976.
- de Santillana, Giorgio, and Hertha von Dechend, *Hamlet's Mill*, Boston: Gambit, 1969.
- Gamble, Clive, "Interaction and Alliance in Paleolithic Society," *Man*, 17, 1982, pp. 92-107.
- Gimbutas, Marija, *The Language of the Goddess*, San Francisco: Harper, 1989.
- Graziosi, Paolo, *Paleolithic Art*, New York: McGraw-Hill, 1960
- Godwin, Joscelyn, *Harmonies of Heaven and Earth: The Spiritual Dimensions of Music*, Rochester, VT: Inner Traditions, 1987.

- Godwin, Joscelyn, ed., Cosmic Music, Musical Keys to the Interpretation of Reality: Essays by Marius Schneider, Rudolf Haase, Hans Erhard Lauer, Rochester, VT: Inner Traditions, 1989.
- Guthrie, Kenneth Sylvan, *The Pythagorean Sourcebook and Library*, Grand Rapids, Ml: Phanes, 1987.
- Kayser, Hans, Ein Harmonikaler Teilungskanon, Basel: Schwabe, 1946.
- Leroi-Gourhan, Andre, *Treasures of Prehistoric Art*, New York: H. N. Abrams, 1967.
- Leroi-Gourhan, Andre, The Art of Prehistoric Man in Western Europe, London: Thames and Hudson, 1968.
- Levin, Flora R., *The Manual of Harmonics of Nicomachus the Pythagorean*, Grand Rapids, MI: Phanes, 1994.
- Marshack, Alexander, The Roots of Civilization: the Cognitive Beginnings of Man's First Art, Symbol, and Notation. New York: McGraw-Hill, 1972.
- Marshack, Alexander, *Ice Age Art*, New York: American Museum of Natural History, 1979.
- Marshack, Alexander, Hierarchical Evolution of the Human Capacity: The Paleolithic Evidence, New York: American Museum of Natural History, 1985.
- McClain, Ernest G., "The Myth of Invariance: The Origin of the Gods, Mathematics and Music from the Rig Veda to Plato," Boulder, CO: Shambala, 1976/1978.
- McClain, Ernest G., *The Pythagorean Plato: Prelude to the Song Itself*, York Beach, ME: Nicolas-Hays, 1978.
- McClain, Ernest G., Meditations though the Quran: Tonal Images in an Oral Culture, York Beach, ME: Nicolas-Hays, 1981.
- McClain, Ernest G., "Tonal Isomorphism in Plato and the I Ching: Brumbaugh as Cultural Anthropologist," in: Plato, Time, and Education: Essays in Honor of Robert S. Brumbaugh, Brian P. Hendley, ed., Albany NY: State University of New York Press, 1987.
- Mountford, J. F., "Greek Music and Its Relation to Modern Times," *Journal of Hellenic Studies* 40 (1920), pp. 13-42.
- Schlesinger, Kathleen, The Greek <u>Aulos</u>; a Study of its Mechanism and of its Relation to the Modal System of Ancient Greek Music, Followed by a Survey of the Greek Harmoniai in Survival or Rebirth in Folk-Music, London: Methuen, 1939.
- Schneider, Marius, Cosmic Music: Musical Keys to the Interpretation of Reality, Rochester, Vt.: Inner Traditions, 1989.
- Thompson, William Irwin, *Pacific Shift*, San Francisco: Sierra Club Books, c1985.
- Thompson, William Irwin, Coming into Being: Artifacts and Texts in the Evolution of Consciousness, New York: St. Martin's Press, 1996.
- Werner, Eric, *The Sacred Bridge*, New York: Columbia University Press, 1959.
- Zhang, Juzhong, Garman Harbottle, Changsui Wang, and Zhaochen Kong, Oldest playable musical instrument found at Jiahu early Neolithic site in China, *Nature* 401:366-368 (1999).