Rev 1 of Feb 1998, from talks/1995/vienna2 (edited transcript)
Presented at the Third Vienna Architecture Congress, November 3, 1995:
Conventional Thoughts on the Chaos of Europe

The Mathematics of Chaos and the Urban Revolution

by

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Abstract. An earlier epoch of chaos came to an end with the urban revolution, millennia ago. And now, chaos reappears in a revolution of its own. Can chaos theory rectify the negative aspects of the urban revolution and help us create the future of Europe? I think so, and here's why.

CONTENTS

1. Introduction.

By *the mathematics of chaos*, I mean *chaos theory* — a popular name or pseudonym for a new branch of mathematics properly known as *dynamical systems theory*. It may or may not be about the usual concept of chaos, as used for example in the title of this conference. The meaning of this word, changed in a process of mythogenesis over thousands of years, is the subject of my book, *Chaos, Gaia, Eros*.

By the *urban revolution* I mean a sequence of three events, the main social transformations of our planetary civilization during these past 12,000 years:

- Firstly, the *agricultural revolution*, when hunting and gathering tribes became sedentary and built cities for the first time, such as Jericho.
- Secondly, the *patriarchal revolution*, with the inventions of the wheel, written language, and history, in Sumer, 6000 years ago. Along with the wheel came the possibility of separating the village from the farmland and the animal corrals, hence the terrific explosion of cities all over the old world.
- Thirdly, the major social transformation in which we are presently enmeshed, and which I like to call the *chaos revolution*, from which a planetary society is emerging.

Many people hope that the chaos revolution will undo the damage that was done in the patriarchal revolution of 6000 years ago, most especially its hierarchical structure, its emphasis on order, its addiction to violence, its suppression of women, and so on. This is a lot to hope for, but it may be possible to achieve some of it.

In my view, mathematics, which is a beautiful, sacred, and absolutely essential ingredient for the survival of our species on this planet, has been destroyed in our culture. It has been destroyed and negated in our schools. After the school experience, one becomes mathematically less, rather than more capable. Everyone is born with natural mathematical capabilities, which are generally damaged in school.

This situation could be easily changed were it not for the extraordinary inertia of the school system, and the negative impression of mathematics most people have, as a result of the school system. Math anxiety and the math avoidance reflex are so deeply imbedded in our society that it's almost impossible to retain one's mathematical capabilities through school. Even so, it might be possible for a person to make a fresh start in math development whenever mathematics develops a new branch, which it rarely does. A new branch of math would be rejected by the school system, which wants to maintain the status quo. After all, its real mission in our society seems to be to retard evolution. This is what is happening now with chaos theory. However, a fresh start could be made after school, as it were. That is, the rejection of the new mathematics by the school system gives each of us individually the opportunity to make a fresh start, to revive our innate mathematical skills with self-study of the unspoiled new branch.

I propose to discuss three things here: the history of mathematics, the mathematics of history, and the relationship between the mathematics of chaos and the urban revolution.

2. The history of math

I'm going to describe mathematics now in three epochs: ancient, modern, and postmodern.

Our understanding of mathematics took its definitive form in ancient Greece with the Pythagoreans, and culminated with Euclid. Euclid's *Elements* was such a successful textbook of mathematics at the time that it superceded all others. They were discarded, and in their place we have Euclid's *Elements*, a wonderful book devoted to sacred mathematics. Its true meaning has become arcane, nowadays few people can learn from it. But it has been the most successful mathematics textbook in history. It was still used when I went to school, and I hope it's still used somewhere today. Every great mathematician was trained by it. It was written in Alexandria shortly after the time of Alexander the Great, and since the burning of the Alexandrian Library, the book has traveled through culture after culture. In every culture through which it traveled, it triggered a major cultural transformation. It reached Europe in the 13th century.

In Euclid's *Elements*, mathematics has four branches which comprise the *quadrivium*, which was half of the classical curriculum in schools throughout Europe and the Middle East during the Middle Ages. The quadrivium includes arithmetic, geometry, music, and astronomy. Arithmetic included number mysticism. Geometry meant Euclidean geometry, which until very recently was considered to be the only geometry. Music meant not the theory of music as we know it, but the theory of musical scales, the tuning of stringed instruments, and so on. Astronomy meant the dynamics of the solar system, encompassing the tables of positions of the wandering planets as they cycle the zodiacal belt, calendars, and clocks.

Of these four branches of ancient mathematics, three are prehistoric, and one is relatively new. Geometry was only about 2000 years old, as far as we know, in the time of Pythagoras and Euclid. Among the oldest geometric objects attesting to an advanced knowledge of geometry are Cheops' Pyramid and Stonehenge. In this context of megalithic monuments we see the traditional relationship between sacred geometry and architecture. Since ancient times these four branches of mathematics remained virtually unchanged, and Euclid's textbooks remain the best source for them.

Modern mathematics, beginning in the time of Newton, presented ideas that were new to the entire enterprise of the human mind. Since Newton, these ideas developed over two centuries with no fundamental changes until about a century ago, when the new, postmodern math arrived: *dynamical systems theory, catastrophe theory, chaos theory, fractal geometry, fuzzy logic*. These are truly new branches of mathematics. You could major in math at most any university and not hear a word about them. The modern subjects derived from Newton dominate the graduate program, the preliminary examinations, the qualifying examinations, thesis, and so on. Most of mathematics, and currently most of the activity of the international research community of mathematics is devoted to the final outstanding problems of the ancient and modern times.

3. The chaos revolution

The reason I think the chaos revolution is so important is that the mathematics of Euclid is an integral part of our culture, which has separated itself substantially from nature. Mathematics is first

of all a study of beautiful abstract objects projected from a divine sphere. It has been found very useful in practical ways, in architecture, engineering, medicine, industry, economics, and other important areas. But mathematics, in the tremendously successful program of basing the sciences on a coherent theory, has consistently failed to model nature, because of its restriction to the Euclidean realm and thus the exclusion of the chaotic, fractal, and fuzzy.

The advent of chaos theory in mathematics has impacted the sciences in a revolutionary paradigm shift. It has boosted the art of mathematical modeling so as to be able model nature at a higher degree of complexity. Mathematical modeling can now attain the minimum amount of complexity necessary to begin to get an idea of how nature works as a whole system. In showing the complexity of an ecosystem, for example, the language of nature itself enters mathematics for the first time. And the importance of this is its potential for undoing the harm caused by the lack of this capability over these past few millennia.

Many people consider chaos theory a fad, and I admit that it has had an amplified wave of popularity which may be unjustified. Nevertheless I believe that the chaos revolution, simply said, is the biggest thing to come down the pike since the wheel.

4. The math of history

Prehistory is older than history. The memories of the life of our family on this planet is older than the writing down of it. Even before writing there was mathematical history, that is, historiography based on mathematics, or mathematical models for history. Within the sciences we have mathematical models for the electromagnetic field, the solar system, the gravitational field, the nucleus of the atom, and so on. We have mathematical models in all the physical sciences, the biological sciences, and in the social sciences too. But the mother of all math models, the synthesis of all and everything, is the mathematical model of history itself, beginning with the Big Bang if there was one, and so on.

And as a matter of fact, a mathematical model for history was among the earliest of all mathematical models, so historiography is not a new enterprise. It's a living tradition. In *Chaos, Gaia, Eros*, I devoted a chapter to the history of history, sketching the life and work of many historiographers over a span of centuries. The most basic idea of all this historiography is to divide our full story into epochs, to divide these epochs into eras, eras into chapters, chapters into sections, sections into subsections, and so on, according to a certain point of view. When there's a shift in the point of view, then history has to be rewritten from scratch.

Let me describe the ideas of one of the historiographers of this long tradition. I think that these ideas may help us to try to understand what is happening today, which may be not just the disintegration of planetary society and the fall of the world economy, but actually a tremendous opportunity for the architecture of the future.

The exemplary historiographer I want to discuss is Sir Flinders Petrie. He was a surveyor in England who became excited about the pyramids in Egypt when he read that they had astronomical orientations. He thought that, as a surveyor, he had the skills and tools to check this out, to settle once and for all whether it was possible that the ancient Egyptians could have known enough about mathematics and astronomy to accurately orient their temples to the cardinal directions or celestial alignments.

So before the turn of this century, Flinders Petrie set off for Egypt with his surveying instruments. Once there, he developed a veritable obsession with ancient Egyptian culture. He became one of the first field archaeologists; he excavated much of ancient Egypt, creating many of the techniques used by field archaeologists today. He also sorted out the detritus of ancient Egyptian civilizations — books written on papyri, pottery shards, household implements, and so on — into different styles, creating an historiography of ancient Egypt. He determined the moment in which these styles underwent sudden transformations, and classified ancient Egyptian culture into eight strata, which he named and dated.

He then constructed a universal theory for major social transformations, based on what he observed. In this theory, transformation on the mathematical level came first. It then triggered a transformation on the scientific level, then in the arts, and then in technology. When it finally came to economics there would be an escalation of wealth, then a collapse of the culture, and a new epoch would begin. He believed that this same sequence of transformations and stages, a kind of domino theory of social transformation, would extrapolate to other cultures, particularly to those of old Europe.

This domino theory is very hard to really test for historical transformations of the past, for example the Italian Renaissance, because we are not there, and the historical data that we have has already been filtered through the lens of people who may have had a different paradigm of social transformation in mind. In fact, the analysis of the Italian Renaissance as a cultural shift gave birth to modern historiography, in the work of Jacob Burkhardt. Of course there have been many social transformations, larger and smaller. Between any two larger transformations there may be many smaller ones, and in between any two of those, many smaller ones, and so on. Thus, there seems to be a fractal structure to history: no matter how powerful a microscope you look through, you may never see a continuum. This fractal view of history has been pioneered by Terence McKenna.

From this perspective, I see our own history as having three primary social transformations: the agricultural revolution, the patriarchal revolution, and the chaos revolution. We are living through one of them, and we have the best chance to check out the Flinders Petrie domino theory of social transformation by applying it to our own, the chaos revolution. And here we can ask questions — based on our own experience, or by reading newspapers — about the significance, for example, of mathematics in this social transformation. This is an interesting case, as here we are at an international conference on chaos and architecture!

Whether Flinders Petrie is right or wrong, chaos theory, a new branch of mathematics, is now playing a crucial role, a nuclear role, in a major social transformation, a paradigm shift of huge proportions. At the same time, chaos theory provides a sophisticated mathematical model for history, a new historiography, and our best tools for viewing current events as the behavior of a massively complex dynamical system. Environmental aspects, ecological systems, the perspective of the biosphere, the Gaia hypothesis and so on: all are mathematically based. Chaos theory plays a critical role in the transformation now underway, and at the same time it's a tool for the interpretation of what's happening. This is a unique feature of our present social transformation: it provides us with its own system of self-reference. In other words, a major social transformation, consciously held, that at the same time is perceiving itself. Here we have the emergence of a new viewpoint, which, when pointed at itself, bootstraps consciousness up to a new level.

5. Chaos theory

A *dynamical system* is a mathematical structure in which rules are given for motion. When rules are given that confine or regulate or inform the motion or the dynamics of a moving system, when the rules don't change although the system does, then what one wants to know is: what will happen in the long run? That's what chaos theory is all about. No matter where you start, if you follow the rules, you will end up in a certain region doing a certain dance, which is called an *attractor.* If you start in a different place and follow the rules, you may end up in another region doing another dance, another attractor. What informs us most in this theory is a map that shows which initial positions end up in which final attractors. That map is called the *attractor-basin portrait* of the dynamical system. Generally these are found by extensive computer experiments, recorded in a computer graphic. And that's why the chaos revolution is happening now, in combination with the computer revolution, which empowers us to view dynamical systems in a completely new way.

Now when the rules of the game change, the attractor-basin portrait changes. The portrait changes when the rules change, and of course the rules are constantly changing. We would really like to know what kind of changes are possible in the attractor-basin portrait of a dynamical system, and those changes are called *bifurcation.* This is what dynamical systems theory has mostly provided for us up to this point, when it's only 100 years old. It's an encyclopedia, a taxonomy of attractors and bifurcations. Once you have this taxonomy, you have made a start in what we could call *dynamical literacy.*

My most radical claim is that if you would be the architects of the future — well, you are the architects of the future — you must have dynamical literacy. Chaos theory gives us guidance in understanding our history and in creating our future. Attractors and bifurcations are space-time patterns, a new kind of sacred geometry.

6. Fractal geometry

Fractal geometry is truly non-Euclidean geometry. It shows us things that Euclidean geometry can not: clouds, mountains, waterfalls, aspects of wild nature captured in a mathematical model for the first time. This is really fantastic news.

One of the things which has helped to create these new branches of mathematics, and contributed to their popularity, is computer graphics. Computer graphics, you see, makes mathematics visible. Mathematics has always been visible to mathematicians, but has only now become visible to everyone. We see phenomenally beautiful objects in books and magazines which were previously visible only to a few shamans, acid heads, or meditators, who were somehow capable of seeing these things internally. Now, thanks to computer graphics, these images can be viewed by everybody. And I think that most people who see these images recognize them as objects familiar from their own mental sphere, or as abstractions from nature.

Many people who have seen these images, artists especially, have understood them better than scientists or mathematicians. There was a wonderful show of chaos in the arts in Graz some years ago. Musicians, sculptors, and painters participated, artists who appreciated the meaning of the new mathematics. They had the courage and the ability to master the technicalities, and were able to translate into art what had not yet been mentioned in schools. Here we have an example Flinders Petrie domino theory: first in mathematics, then in the arts. And then come the sciences, technology, economics, and finally, the collapse of the culture.

Fractal geometry deconstructs binaries, dichotomies. It deconstructs the simple (Euclidean) boundary between two different regions. For example, we have an exemplary fractal that is known to all: the sandy beach. The sandy beach has land on one side, and water on the other side, but when you go down on the beach you see that there is water on both sides of the boundary, and also sand on both sides. A sandy beach is a model fractal. Fractals are ubiquitous in dynamical systems. Chaotic attractors are fractals, and so are (usually) the boundaries between basins in an attractor-basin portrait. The language of fractal geometry is an essential part of dynamical literacy.

7. The urban revolution

And there, in a nutshell, is the mathematics of chaos and fractals. Now for the relevance of chaos and fractals to the city of the future, and to our civilization in the future.

In the very concept of city we have the edge of town, especially in a fortified city. We have the city within, and the suburban, rural, or wild landscape without. The clean division between these is an ancient (Euclidean) geometrical line.

But we have thousands of years of experience with the disadvantages of the fortified city. Despite the fact that you have gardens on your roofs and balconies outside your windows, there's still too large a difference between the city and nature, inside and outside. The replacement of Euclidean geometry by fractal geometry as the sacred geometry of architecture provides for the deconstruction of this dichotomy, this binary opposition of the city and nature. It offers a geometrical basis for eliminating the disadvantage of cities as we know them.

Figure 1 shows an example of fractal geometry in the context of the urban revolution. This is an aerial photo of a village of the Dogon tribe in Mali. Fractal geometry has been used in these African villages for millennia, and a thoughtful analysis has shown that the fractal geometry of this city was intentional. (Eglash, 1989) The aspect of fractal geometry called *self-similarity* — circles of circles of circles and so on — was derived by the Dogon from the idea that the house is a union of people and the village is a union of houses and so on. (Duly, 1979) The image of the human body is used on many different scales in the architecture and city plan. And because of this sacred geometry of the Dogon, their villages are essentially and intentionally fractal. Bounding these villages there is a sandy beach rather than a Euclidean fortified wall: no inside and no outside, no separation of city from nature. And this may be the basis for a new urban revolution.

8. Conclusion

Wanted, an anti-urban revolution: that which undoes the damage of the urban revolution of 6,000 years ago? Chaos theory, fractal geometry, and Dogon city planning to the rescue! And this is why I think it unlikely that you can succeed as architects of the future without first becoming dynamically literate. I challenge you to resume your mathematical education!

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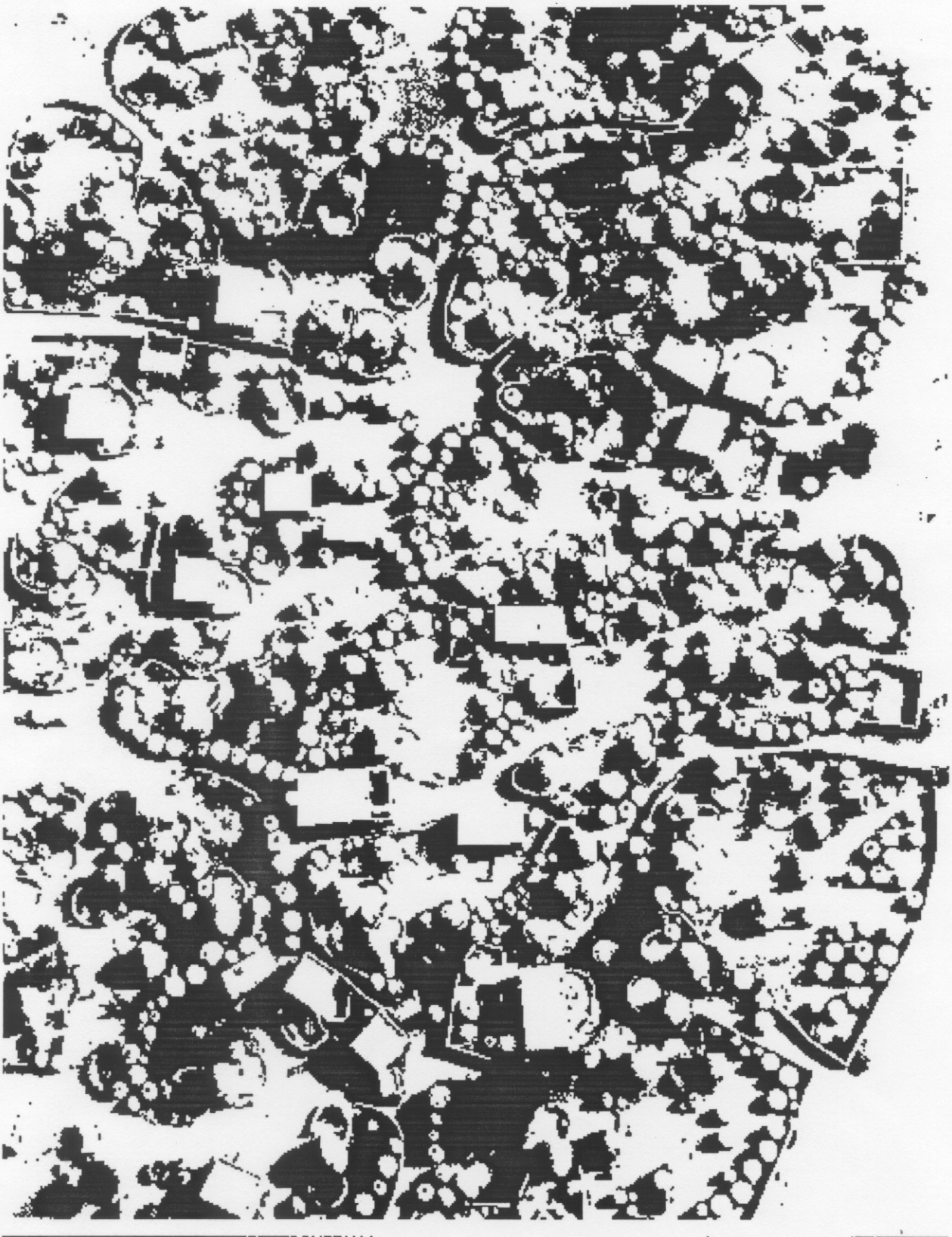

Figure 1. The village of Mali Labbezanga.