Rev 1.0 of August 1998; Rev 2.2 of April 1999

Presented at the Homeokinetics 98, a conference held at the University of Connecticut, Storrs, July 24-26, 1998, in honor of the 80th birthday of Arthur Iberall, the creator of homeokinetics.

The Origins and Bifurcations of Algebra

by

Ralph H. Abraham, abraham@vismath.org
The Visual Math Institute, www.vismath.org
Santa Cruz CA, USA-96051-7920

*Abstract.*This paper is a fantasy on the origin of algebra around 820 AD at the Bayt al-Hikma (House of Wisdom) in Baghdad, and its later evolution as the abstract symbolic system we know today. Our approach is typical of dynamical historiography, that is, the application of dynamical systems theory (including the mathematical theories of chaos and bifurcations) to history. In this case we view cultural diffusion and bifurcation from the perspective of reaction-diffusion equations, that is, as if layers of culture were diffusing and reacting chemicals. This view is also characteristic of homeokinetics and the style of its creator, Arthur Iberall.

Dedicated to Arthur Iberall on his 80th birthday.

CONTENTS

1. Introduction
2. Writing systems
3. Number systems
4. Spiritual systems
5. Histomaps
6. The birth of algebra
7. The rise of symbolism
8. Conclusion
Bibliography

1. Introduction.

Dynamical historiography uses the concepts of bifurcation theory to analyze historical trends and their transformations. Three major bifurcations of world cultural history,

- from the paleolithic to the neolithic (that is, agriculture),
- the discovery of the wheel (and writing, and patriarchy), and
- the chaos revolution (now ongoing),

have been so analyzed in recent writings. (Abraham, 1994)

The origin of algebra, in comparison, is a minor bifurcation in the history of mathematics. However, due to its key role in the paralyzing epidemic of math anxiety in the United States, together with our conviction that concepts are most easily learned in historical order, we have selected it for a dynamical historiographical analysis.

Our analysis, in this case, uses (tacitly) the paradigm of the reaction-diffusion equation, which has been developed to model some of the mysterious natural phenomena of biological morphogenesis, such as:

- How does the leopard get its spots?
- How does an egg turn into a chicken?

This paradigm is the source of the peculiar metaphors we will use, namely, those of:

- chemical reactions, as for example, 2H + O -> H2O, and
- hydrodynamical diffusion or perfusion, as water through sand.

We are going to locate the origin of algebra in a chemical reaction between different layers of cultural reactants diffusing through the geocultural medium:

- the writing system (literatic) layer,
- the number system (arithmetic) layer, and
- the spiritual system (kabbalistic) layer.

Again, this approach is typical of homeokinetics. These three layers of signification of Hebrew and Arabic literature, in particular, are well known. (Schimmel, 1984, p. 90)

In fact, we wish to model these systems as a point in a geometric space, which is a map for the part of human collective consciousness where writing systems reside. Thus we are thinking of a cultural histomap having three groups of dimensions: one or two spatial dimensions, one temporal dimension, and one or more conceptual dimensions. The conceptual states, in this model, follow the rule of some unknown dynamical system. Cultural/conceptual/mental morphogens diffuse in this histomap, and natural evolution occurs gradually, and occasionally, through catastrophic bifurcations. All this is rather like the reaction-diffusion models used in mathematical biology to account for the formation of a leopard's spots, but we are interested instead in the formation of literacy and numeracy, the very foundations of human consciousness.

We will begin with a review of each of these three reactants, one at a time.

2. Writing systems

For the sake of background, we may recall that the second of the major bifurcations of world cultural history (the wheel, writing, patriarchy) took place about 6,000 years ago. The morphogenetic sequence of the several aspects of the bifurcation are unknown. One possible sequence, which I have favored in the past, has the pottery wheel first, then its adaptation as the cart wheel, which empowered the urban revolution, the chariot wheel giving advantage to militant patriarchy, and support for the priestly elite, and then writing systems. Another sequence, championed by Leonard Shlain, has writing before patriarchy. (Shlain, 1998) And David Diringer places writing above all in importance. He wrote that it *represented an immense stride forward in the history of mankind, more profound in its own way than the discovery of fire or the wheel.* (Diringer, 1962, p. 19) In any case, our story here begins with the creation of writing systems.

Our interest now is to extract enough data from the literature on the history of writing so as to be able to picture the diffusion of writing on a histomap. We will proceed by discussing a series of illustrations.

The earliest scripts of the Indus valley (still undeciphered), of Mesopotamia, and of Egypt, were originally pictographic. Eventually, all three writing systems evolved in the direction of syllabic, or even phonetic (that is, alphabetic) systems. We wish to distinguish gradual evolution from bifurcation.

Figure 2.1. The geographical pattern of the early writing systems, 3,000 BC to 1,000 BC.

This map shows the cluster pattern of the early writing systems, adapted from Diringer. Like agriculture, the early systems began in several closely spaced centers of the Old World. The most recent studies (Walker, 1987, Ch. 1) favor a slightly more diffuse pattern, centered on Mesopotamia. In any case, the data suggest a gradual diffusion and evolution, like a spreading stain.

Figure 2.2. The tree of writing systems, 3000 BC to early AD.

This space-time pattern shows the main bifurcations of the tree of writing systems, from earliest Sumer, before 3,000 B.C.E., to the modern alphabets of today. (Healey, 1990, p. 61) A connection of some importance for us, from Aramaic to the Brahmi of India, 7th C. B.C.E., is not shown. (Diringer, 1962, p. 144) We regard most of these bifurcations (simple branching) as gradual shifts, or subtle bifurcations. The most important catastrophic bifurcations would be:

- from pictographic pre-writing to pictographic writing (pictography)
- from pictographic writing to phonemic writing (phonograms)
- from phonetics to alphabetics

Figure 2.3. Development of cuneiform signs from 3,000 B.C.E.

Here three stages in the evolution of cuneiform signs are shown in parallel, horizontal rows. These illustrate the first two of the three bifurcations listed above. (Walker,1987, p. 10)

Figure 2.4A. Comparison of early alphabets.

Here we get to see the shapes of the letters of some of the early alphabets, along the main trunk of the tree, as they evolved from Phoenician around 1,000 B.C.E. (Healey, 1990, p. 29) This figure

suggests that between the leap from phonetics to alphabetics up to the present time, primarily gradual shifts have taken place. See also (Naveh, 1982, p. 23).

Figure 2.4B. The development of the English alphabet.

Here, for comparison, is a tabular view of the morphogenesis from the North-Semitic, 1,000 B.C.E., to the modern English alphabet. (Diringer, 1968, v.2, Fig. 22.1) A lesser, but still discontinuous, leap takes place with the addition of vowels in the Greek alphabet.

Figure 2.5. The development of the Greek alphabet.

Zooming in on the Greek branch of the tree, here we see the step-by-step changes during the 8th and 7th C. B.C.E. (Healey, 1990, p. 37) Here again, on a microscopic scale, we see the continuity in development of the first complete alphabet.

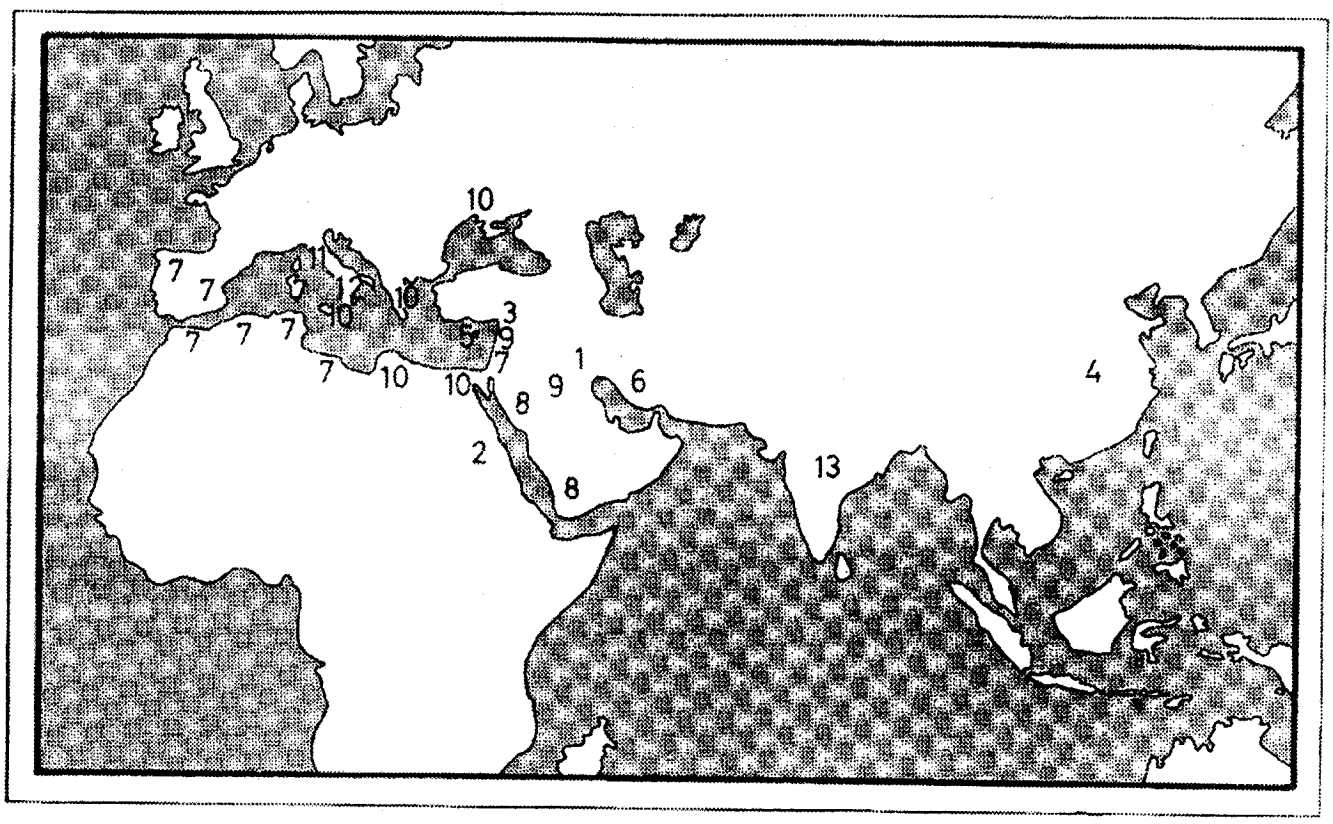
Figure 2.6. The development of the Arabic alphabet.

The Arabic alphabet began its development in late Roman times, and evolved into one of the most complete, and widely used, alphabets today.

In summary, we may say that long after pictographic pre-writing spread throughout the human-inhabited world, a new idea popped up in Sumer: cuneiform pictography. This was followed rather quickly by a bifurcation to cuneiform phonograms. After this new idea had spread widely, another new idea popped up in Canaan: the alphabet. In between these local catastrophic bifurcations, in addition to geocultural diffusion, there took place a gradual evolution, much like the wearing and polishing of pebbles in a river.

Figure 2.1. The geographical pattern of the early writing systems, 3,000 BC to 1,000 BC.

(Diringer, 1968, Map 1)
Yellow: Cuneiform
Beige: Hieroglyphics
Red: Alphabets
(dirmap1tif)



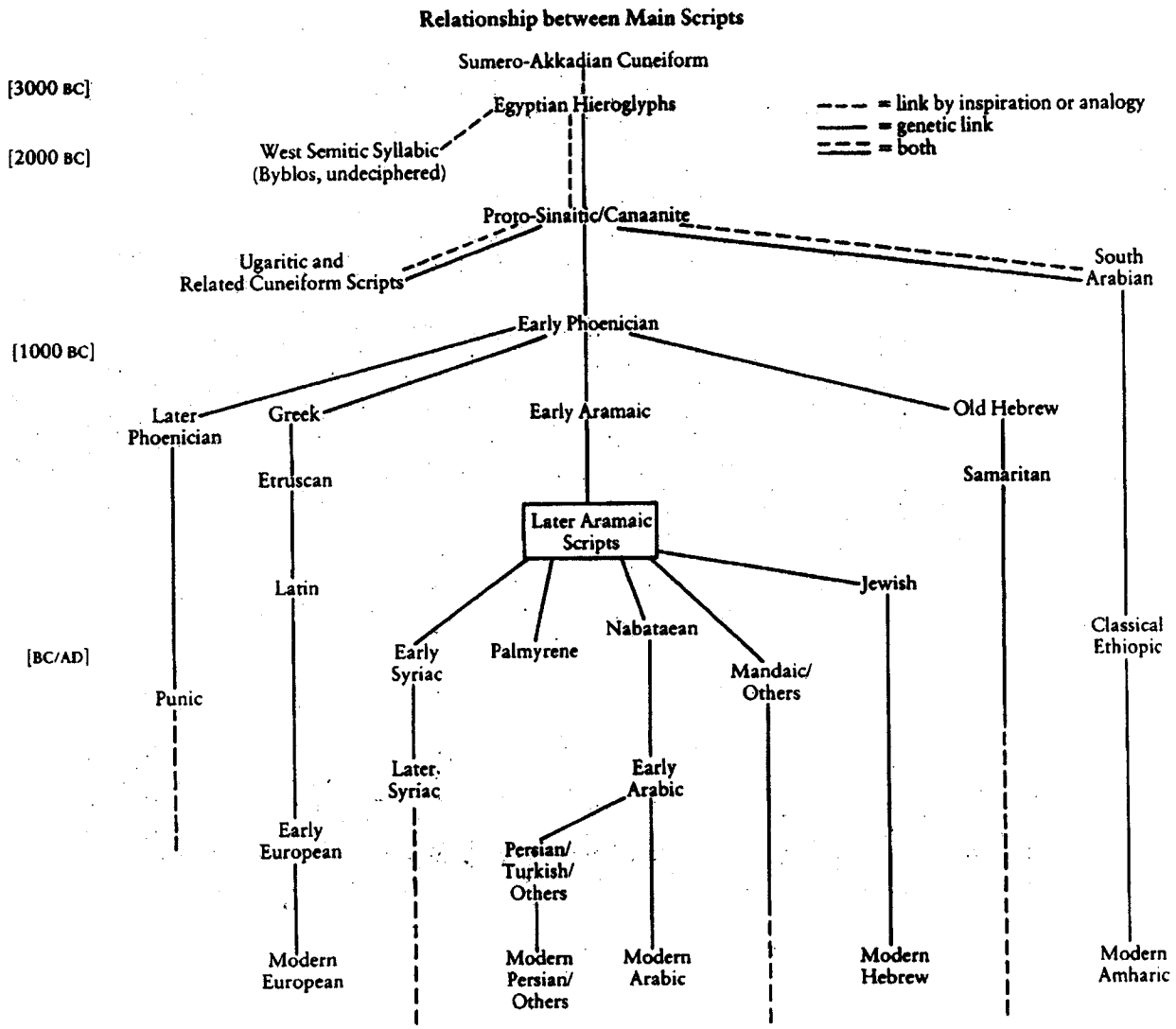
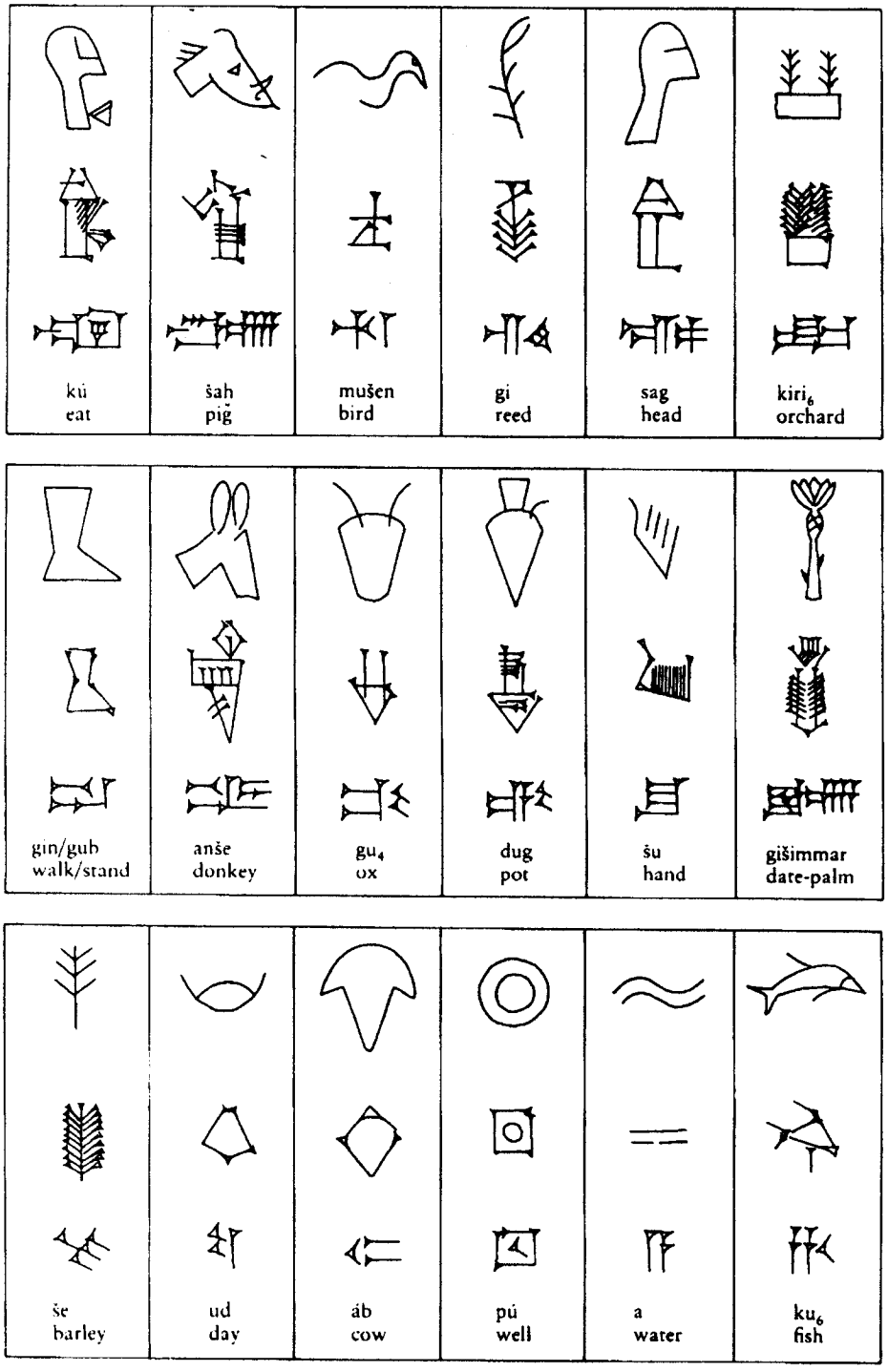
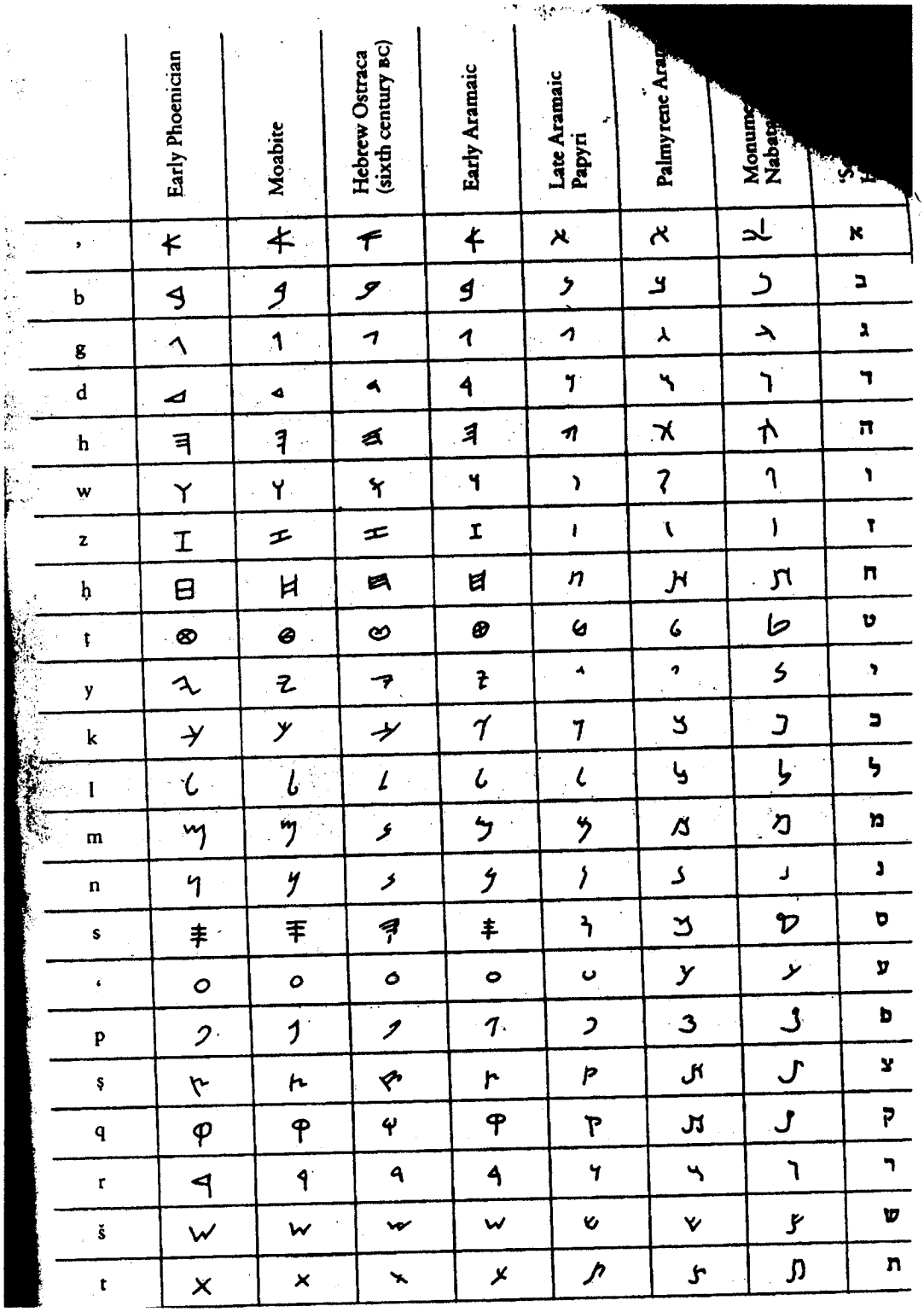


Figure 2.2. The tree of writing systems, 3000 BC to early AD.
(Healey, p. 61) (healey61.tif)

Figure 2.3. Development of cuneiform signs from 3,000 B.C.E.
(Walker, 1987, p. 10) (walker10.tif)





	Early Phoenician	Moabite	Hebrew Ostraca (sixth century BC)	Early Aramaic	Late Aramaic Papyri	Palmyrene Ara	Monum Nabat	
'								
b								
g								
d								
h								
w								
z								
ḥ								
ṭ								
y								
k								
l								
m								
n								
s								
'								
p								
ṣ								
q								
r								
š								
t								

Figure 2.4A. Comparison of early alphabets.
(Healey, Fig. 15, p. 29) (healey15.tif)

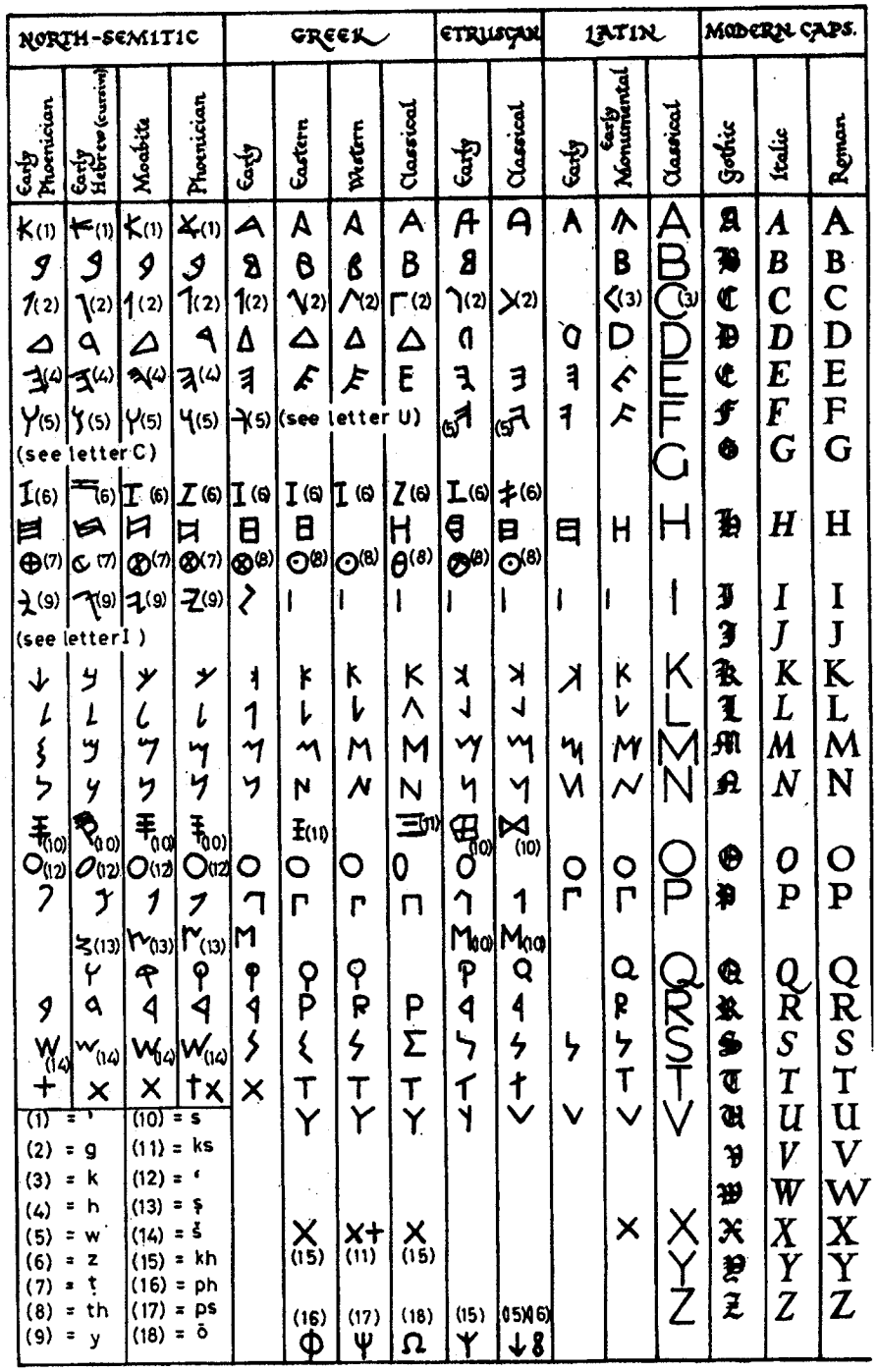


Figure 2.4B. Development of the English alphabet.
(Diringer, 1968, Fig. 22.1) (dir221.tif)

Figure 2.5. The development of the Greek alphabet.
(Healey, Fig. 27, p. 37) (healey27.tif)

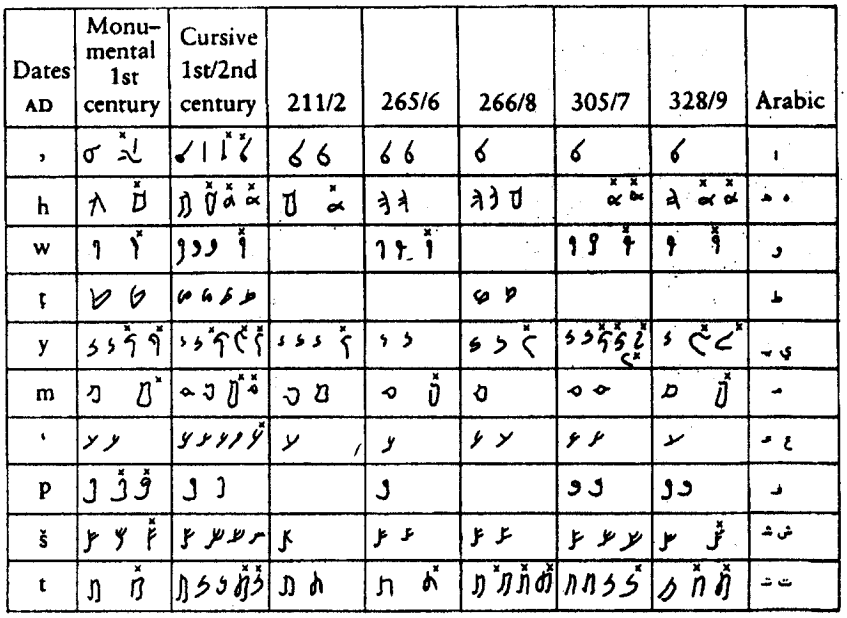


Figure 2.6. The development of the Arabic alphabet.
(Healey, Fig. 37, p. 54) (healey37.tif)

3. Number systems

Contrary to popular belief, it seems that number systems are older than writing systems. TAlly sticks have been found dating from deep prehistory. In fact, it seems to be firmly established that the first writing systems evolved out of number systems, in the context of inventories, accounting, urban administration, bookkeeping, calendar keeping, and the like.(Senner, 1989, pp. 8-9; Ch. 2)

Within the cuneiform writing system of Mesopotamia, a relatively advanced number symbol system was embedded. Shortly before the Common Era, a rather different system emerged in India, which evolved with impressive rapidity into the modern system. According to Diringer, the Indian numerals evolved from the Aramaic alphabet, which reached India in the 7th C. B.C.E. Again, with the Indian numerals, we see a cone-like pattern of cultural diffusion from India, this time with greater speed of diffusion.

Figure 3.1. Numerical notation, preceding the arabic numerals.

Here we see, line-by-line, a comparison of the number symbols of various cultures and writing systems, from 3,000 B.C.E. up to medieval times. They show considerable uniformity, perhaps due to their having a common source in finger counting. The Roman numerals, still much used, are rather typical. (Diringer, 1968)

Figure 3.2. Numbers after alphabets: letters as number symbols.

The cuneiform and hieroglyphic symbols were given an additional burden of number signification. Following the spread of the alphabet after 1,000 B.C.E., the letters of the alphabet inherited this double entendre. Here we see, in parallel columns, a comparison of number values for six writing systems. This double signification phase of cultural evolution is the basis for Hebrew *gematria*, or Islamic *jafr*, a kind of error correcting code in sacred texts, in which the number spelled out by a word is consciously combined with the literary meaning.

Figure 3.3. The development of the Indian numerals.

The Indian numerals developed from the Kharosthi, from 400 B.C.E., which resemble Roman numerals. Kharosthi was a writing system, which included these number signs. This writing system was succeeded by the Brahmi, around 300 B.C.E., which was influenced by the Aramaic alphabet, and is the parent of the syllabary used today in India. (Menninger, 1969, pp. 294-399) These number symbols of the Brahmi writing system evolved into the Gwalior numerals around 850 C.E., similar to the ones universally used today. By 876 C.E., the decimal place value system, with zero, had been established in India. (Joseph, 1991, pp. 239-243)

Figure 3.4. The Brahmi numerals in detail.

Here is a longer list of the numerals of the Brahmi system. (Menninger, 1969, p. 395)

Figure 3.5. The migration of Indian numerals.

Here we see a family tree of numeral systems, from the Brahmi of 300 BCE to Medieval Europe. In our view this is a gradualist evolution punctuated by subtle (that is, continuous) branching bifurcations. (Menninger, 1969, p. 418)

Figure 3.6. From Brahmi to modern numerals.

Here is another comparative sequence from the family tree of Indian numerals. (Menninger, 1969, p. 419

After viewing these illustrations, we are left with an impression of diffusion and reaction of a cultural/intellectual substance through a geocultural space-time matrix. The leopard spots are obtained by the major catastrophic bifurcations, like phase changes, which occur along the way. So far we have noted mainly four of these:

- from pictographic pre-writing to pictographic writing (pictography)
- from pictographic writing to phonemic writing (phonograms)
- from phonetics to alphabetics
- from alphabetic numerals to Indian numerals

three from the proceeding section on writing systems, and one more for numerals.

Note: The diffusions of literacy and of numeracy originate from quite different times and places.

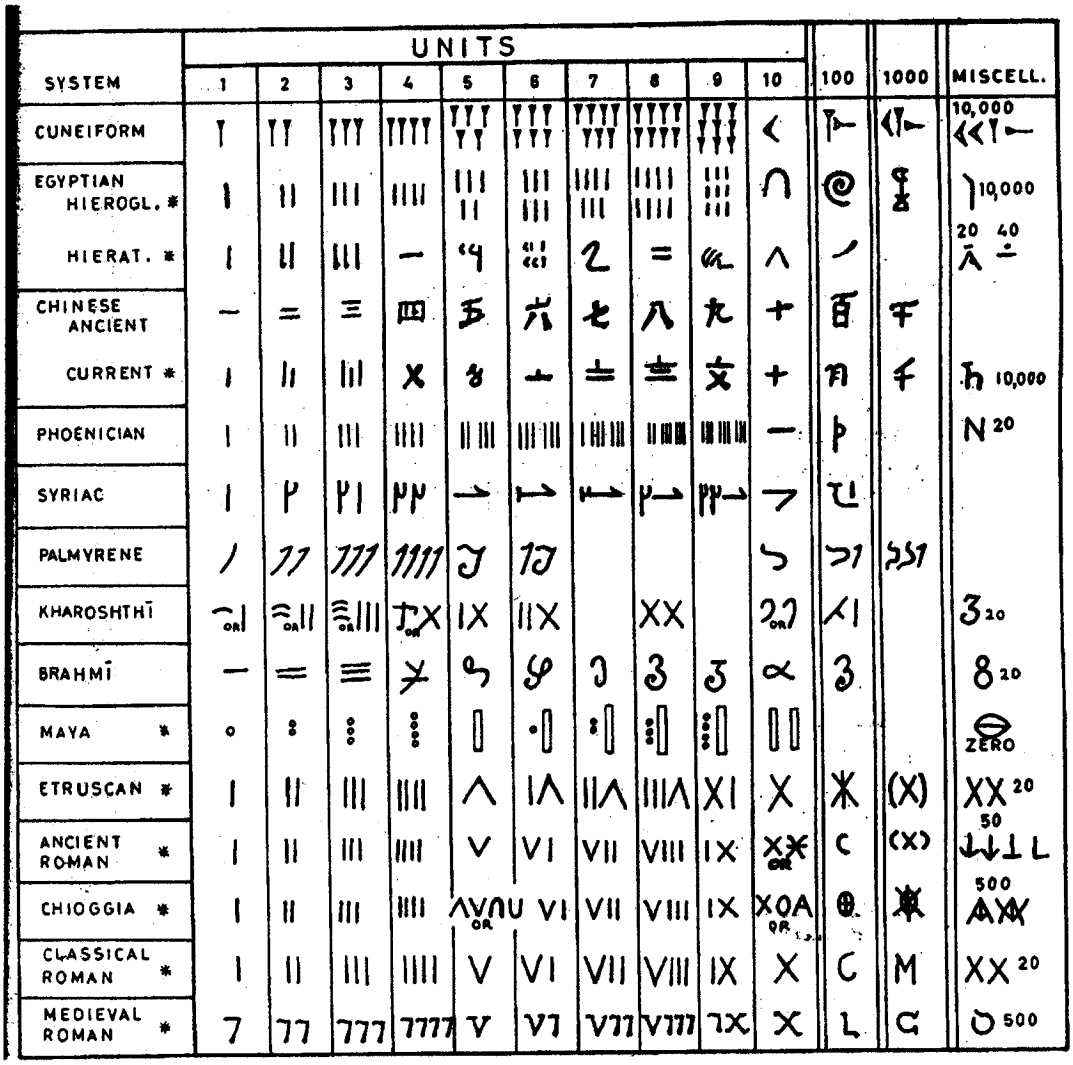


Figure 3.1. Numerical notation, preceding the arabic numerals.
(Diringer, 1968, v.2, p. 437)(diringerc10.tif)

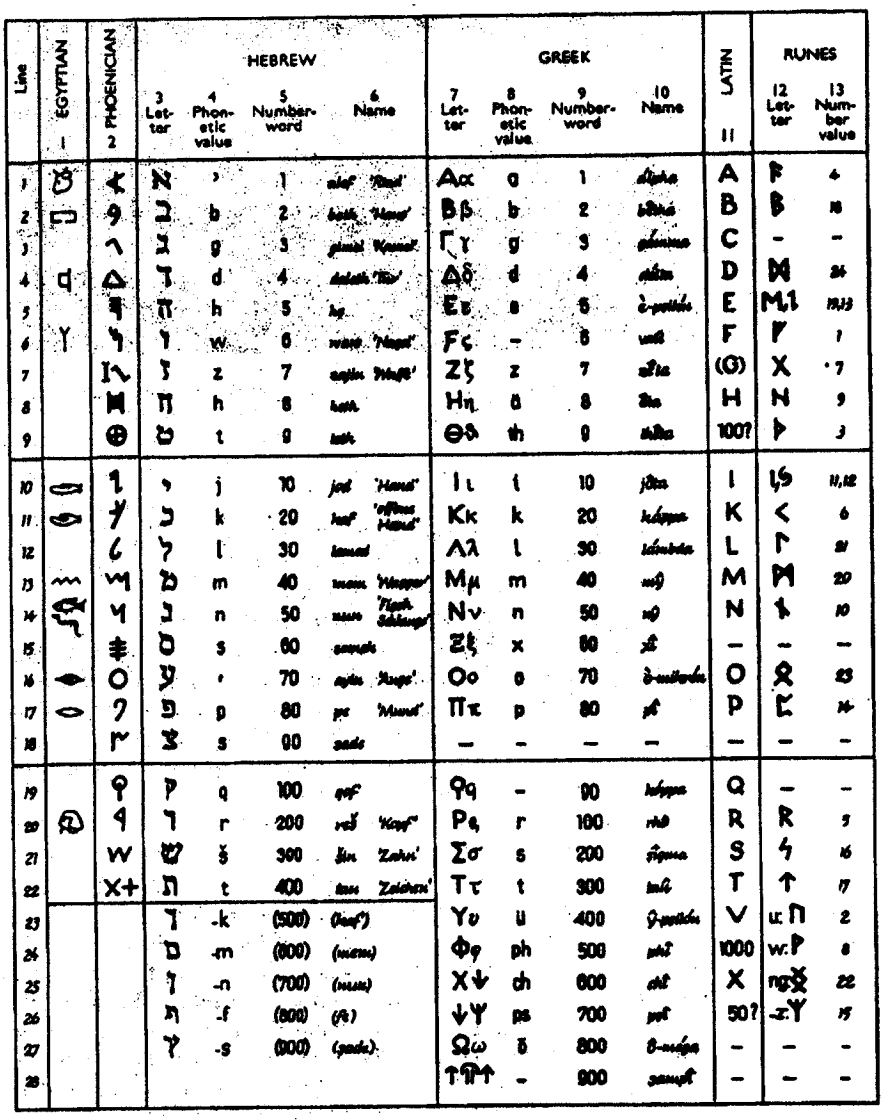


Figure 3.2. Numbers after alphabets: letters as number symbols.
(Menninger, Fig. 95, p. 265) (menninger95.tif)

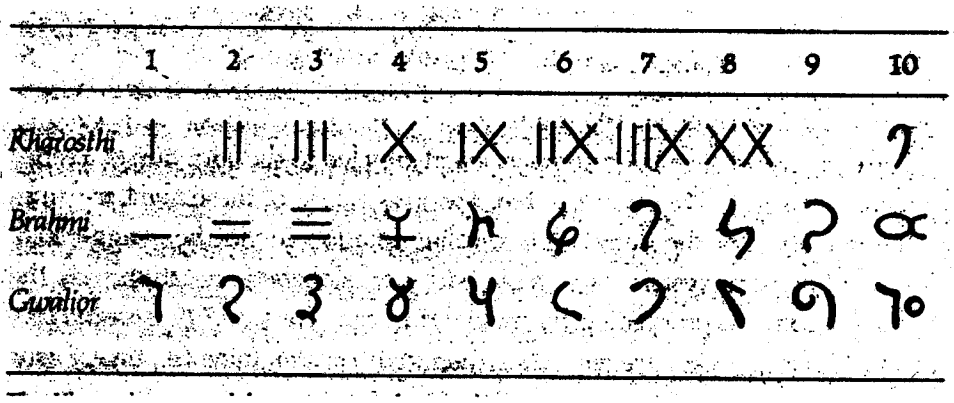


Figure 3.3. The development of the Indian numerals.
(Joseph, Fig. 8.2, p. 241) (joseph82.tif, 240 dpi)

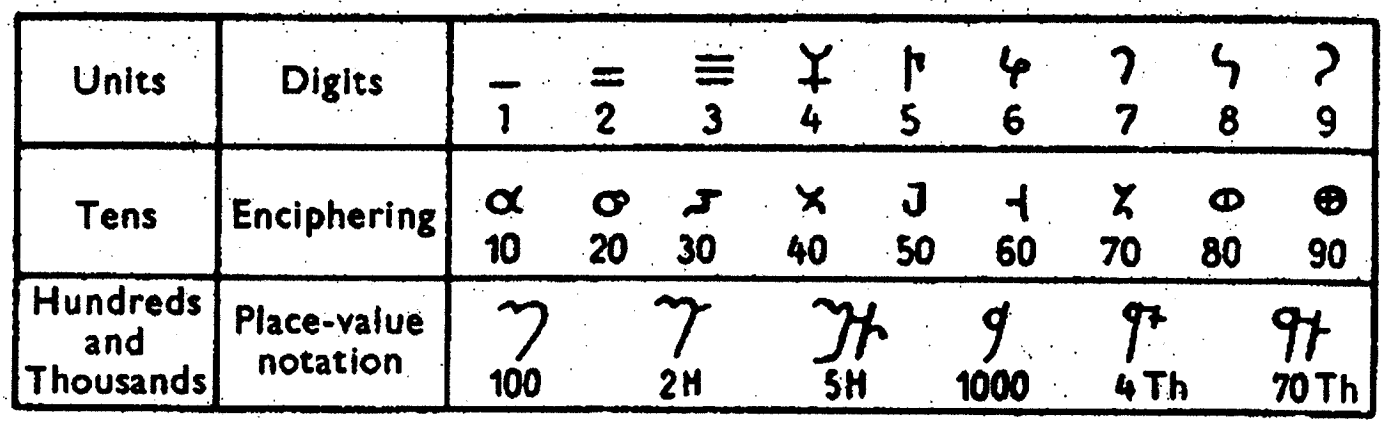


Figure 3.4. The Brahmi numerals in detail.
(Menninger, Fig. 223, p. 395) (menninger295.tif sic)

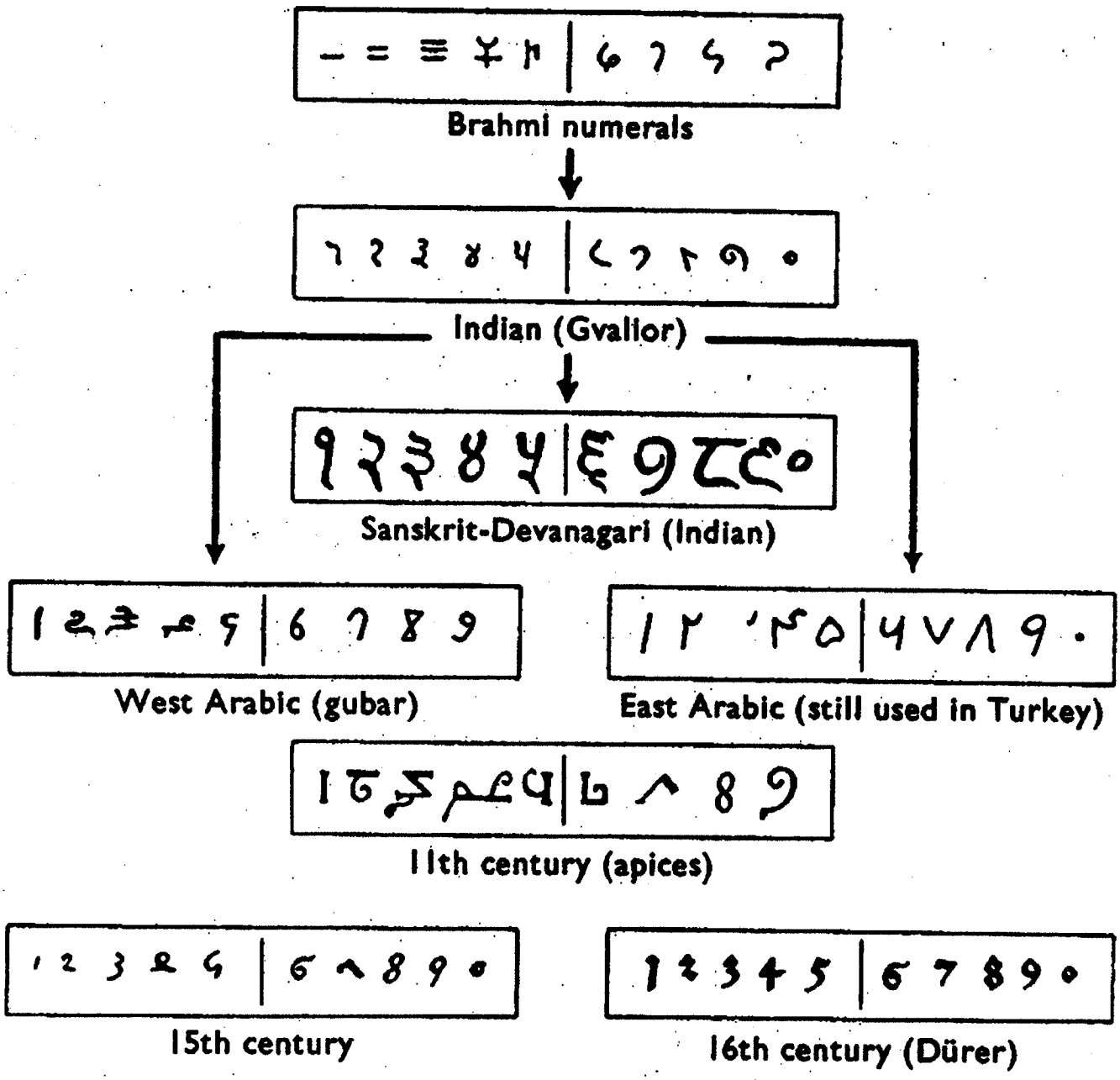


Figure 3.5. The migration of Indian numerals.
(Menninger, Fig. 239, p. 418) (menninger239.tif)

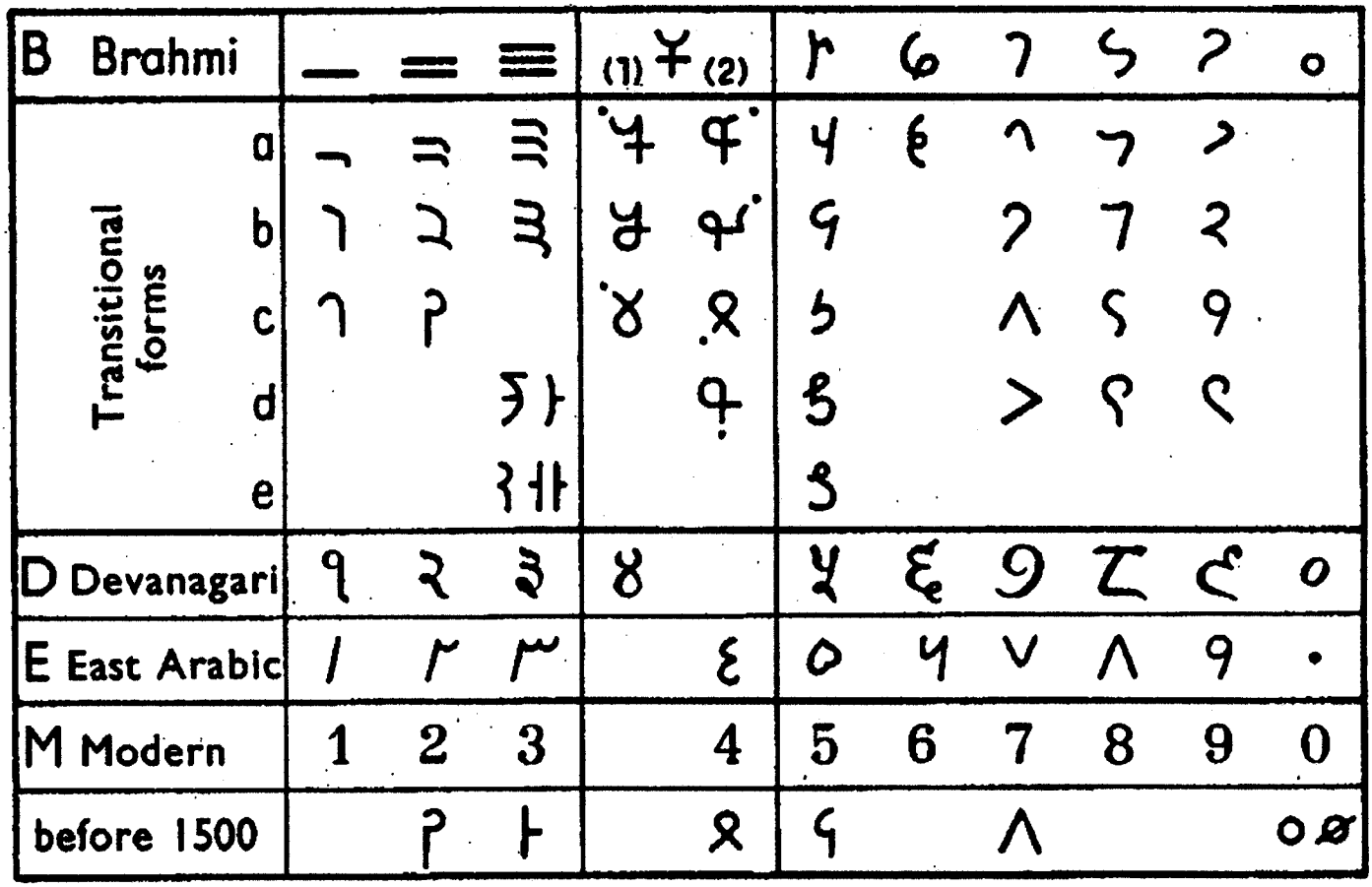


Figure 3.6. From Brahmi to modern numerals.
(Menninger Fig. 241, p. 419) (menninger241.tif)

4. Spiritual systems

It is well known that prehistoric pictographs all over the world carried supernatural associations: sacred, mystical, magical, mythological, spiritual, religious, cosmological, astrological, and so on. We may call such symbol schemes *spiritual systems*, and the art of their use, *sacred calligraphy*.

As pictographs evolved into prewriting and writing systems in Sumer, China, Maya, and elsewhere, some of these spiritual associations survived. Without doubt the apex of the art of sacred writing was achieved in ancient Egypt. And as phonemics evolved and simplified into syllabaries, alphabets, and number systems, the new symbols inherited these supernatural associations, along with the literary and/or numerical layers of signification. Most likely this spiritual layer is the oldest of the three.

We might speculate that as the diffusion of this cultural morphogen radiated outward from Egypt, it infused the early alphabets. While there is no evidence of sacred calligraphy in ancient Greece, there are strong arguments for a spiritual layer in some Presocratic texts (Bernal, 1987), and in the late antique Neoplatonic and Hermetic corpi. (Fowden, 1993)

However, the earliest historical record we have of spiritual practices based on the letters of the alphabet is relatively recent: the Kabbalistic practice involving the twenty-two letters of the Hebrew alphabet and the ten number symbols of the Indian number system. This literature begins with the *Sefer Yetsirah*, or *Book of Creation*, written somewhere in the interval 200 to 600 CE. (Scholem, 1941, p. 75) And we speculate that from this Hebrew source, and its sequel in the culture and literature of the Kabbalists of medieval Europe, evolved the spiritual practice of Latin and English calligraphy among medieval Christian monks, by 700 CE or so, and soon after, the spiritual practice of Arabic calligraphy within early Islam. Similar evolutions occurred in China, Japan, and elsewhere, and continue to this day.

The heart of this speculation is the idea that each letter or numeral signifies a spiritual state, meditation on the signification of a sign while drawing it produces that state, and the slow calligraphy of a word induces thus a sequence of spiritual states: a sort of mental Tai Chi or Sufi dance. Certain words, then, have been found by experience to ascend a ladder to heaven, as it were. This idea is explicit in the Jewish and Islamic mystical literature. (Schimmel, 1984,

Against this speculation, we have the conventional theory that calligraphy is simply a craft of writing in a monumental, decorative, or artistic style, evolved from metalworkers in Mesopotamia and Egypt. We might then contrast esoteric and exoteric calligraphy. The Old Syriac Gospels of the 5th century AD are regarded as outstanding examples of exoteric calligraphy. (Healey, 1990, p. 50)

In any case, the evolution of the well attested art of sacred calligraphy plays an important role in our story on the origins of algebra, as we shall see.

Figure 4.1. Latin/English calligraphy.

An initial P from the Lindisfarne Gospels. (Backhouse, 1981, p. 59) The Lindisfarne monastery was founded on an island off the East Coast of England around 635 AD, as a mission of Irish Christianity. The Lindisfarne Gospel was created by Eadfrith around 700, an early example of Latin calligraphy. It is written in a script known as *insular majuscule*, an early Irish Christian script, and is now a prized possession of the British Library.

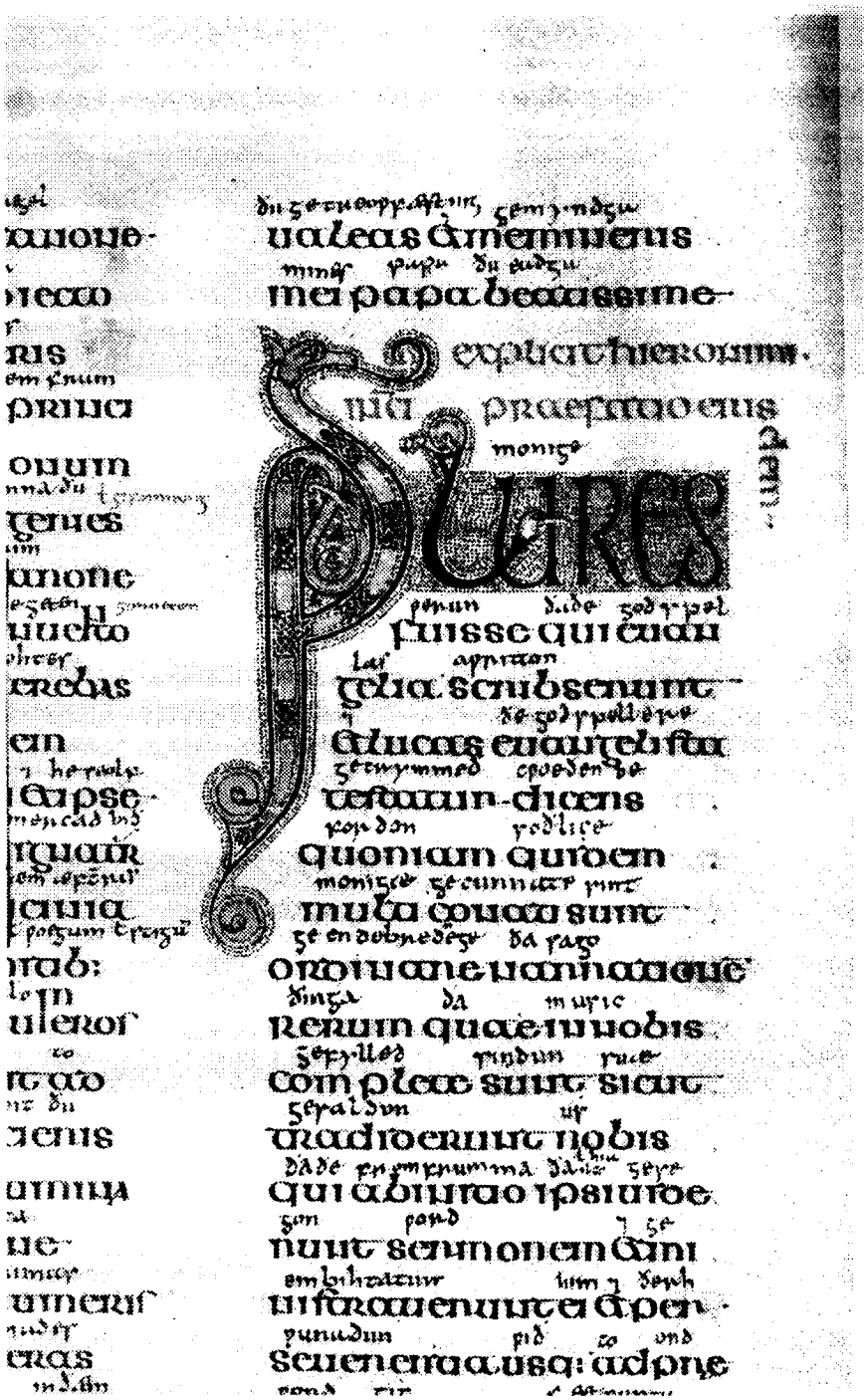


Figure 4.1. Latin/English calligraphy. An initial P from the Lindisfarne Gospels. From Backhouse, Fig. 36, p. 59. (lindisp.tiff, import at 200 dpi)

Around the year 1000, the Anglo-Saxon interlineal gloss was added by Aldred in the *insular miniscule* script, an early example of Anglo-Saxon calligraphy, and the first translation of the Four Gospels into an English language.

The text includes 15 special pages, elaborately decorated, called *great decorated pages*, and a number of *smaller initial pages*. Here we see one of the smaller initial pages, the initial P at the beginning of the Preface. Note the knots within the lines of the initial P, which derive from Babylonian metalwork. Such decorations are characteristic of the great decorated pages.

The Book of Kells, created around 800 in an Irish monastery, Kells or Iona, in insular majuscule. is regarded as the greatest example of Latin calligraphy. It now resides at Trinity College, Dublin. (Meehan, 1994)

If you want to try your hand at English calligraphy, authentic step-by-step instructions area available. (Baker, 1983)

Note that these masterpieces of medieval Christian literature employ splendid decoration apart from the calligraphy, as in the gresat decorated pages. Applied to the text, we find only decorative initials on the smaller initial pages. The main text is artistic, but not obviously spiritual: these belong to the exoteric calligraphic art.

Figure 4.2. Arabic calligraphy.

Arabic calligraphy began within Islam in the 9th century AD. As we have described above, the Arabic calligraphy of the Sufi tradition of Islam has preserved a living tradition of esoteric calligraphy for over a thousand years. But what does it look like? Here is an example of the *archaic kufic* style. Kufic, one of seven styles of Islamic calligraphy, was the dominant priestly script. (Khatibi, 1995, p. 94)

Figure 4.2. Arabic calligraphy.
(Khatibi, 1995, p. 94) (khatibi.tif)

5. Histomaps

We have now seen the birth, evolution, and diffusion of writing and number systems as space-time patterns on a histomap. The development of modern alphabets radiate from a birth-point in ancient Greece, and appears on the histomap as a downward expanding cone or horn. Within this cone, the alphabets evolve into the practice of calligraphy as a spiritual practice, as in the Lindisfarne Gospels, the Book of Kells, and the Kufic Koran.

At this time, say 700 AD or so, the alphabet is a palimpsest of three layers tightly attached. That is, each letter has three layers of meaning:

- the phonemic
- the numeric
- the spiritual

To view and compare the space-time patterns of these three processes of cultural diffusion, it will be convenient to make use of a standard coordinate grid. We begin the step-by-step development of this grid with a sequence of histomaps from Joseph.

Figure 5.1. The cultural diffusion of ancient math, according to the usual interpretation.

Here is a block diagram in which Joseph shows the flow of ancient math using only one dimension, time, which increases to the right. (Joseph, 1992, p. 4)

Figure 5.2. An improved diffusion path, the multicultural interpretation.

In this block diagram, Joseph is explaining an alternative diffusion path. No matter, what interests us here is the addition of second dimension, a one-dimensional geographical model has appeared as a vertical axis. (Joseph, 1992, p. 9)

Figure 5.3. The medieval transmission of mathematics on a two-dimensional geographic chart

Now the time axis has been suppressed, to make room for a second geographical dimension. This is a (not-to-scale) two-dimensional map of ancient Eurasia. (Joseph, 1992, p. 10)

Figure 5.4. All the foregoing diffusion paths, combined into a single figure.

Here, as in Figure 5.2, we have two dimensions, one each for space and time. But the orientation has been changed, so geographical space is horizontal, while time is vertical, and increasing downwards. Now we are getting somewhere: a true histomap. (Joseph, 1992, p. 14)

Figure 5.5. The coordinate grid for space and time.

We now abstract the coordinate grid of this histomap, uniformizing the geographical dimension, and choosing convenient endpoints for the time axis. China is unfortunately omitted. We will use this grid as a background template upon which to visualize cultural diffusion in three layers of signification: writing, numbers, and sacred codes.

Figure 5.6. Histomap of writing systems.

Writing has spread from a single center, Sumer, like an ink stain on a paper towel. Seen upon our coordinate grid, this appears as a nonuniform cone, opening downwards. A few milestones are named within the spreading stain.

Similary, we have viewed the birth, evolution, and discussion of number systems as a pattern on a histomap. The development of the modern number system from a birth-point in medieval India appears on the histomap as another downward radiating cone, the vertex displaced both in space and in time.

Figure 5.7. Histomap of writing and number systems

These two cone-shaped patterns have been drawn on the same coordinate grid, which is based on the histomaps of George G. Joseph. We now superimpose the two histomaps. Within the larger cone of the writing system is the smaller cone of the number system. Within this smaller cone, the alphabet has been liberated from the burden of representing numbers.

Figure 5.8. Histomap of writing, number and spiritual systems

Finally, we superimpose the third diffusing cultural morphogen, the spiritual. Within the largest cone of the writing system we now see two smaller cones:

- The number system cone radiates downward from the upper right.
- The spiritual system cone radiates downward from the upper left.

Within the smaller cone on the left, the alphabet has been burdened by an additional layer of meaning. Within the smaller cone on the right, the alphabet has been liberated from the burden of representing numbers. These layers of the histomap palimpsest are indicated by different colors in this Figure.

Notice that these two downward radiating cones meet in Baghdad in 820 AD. We will return to this point later, in our discussion of the origins of algebra.

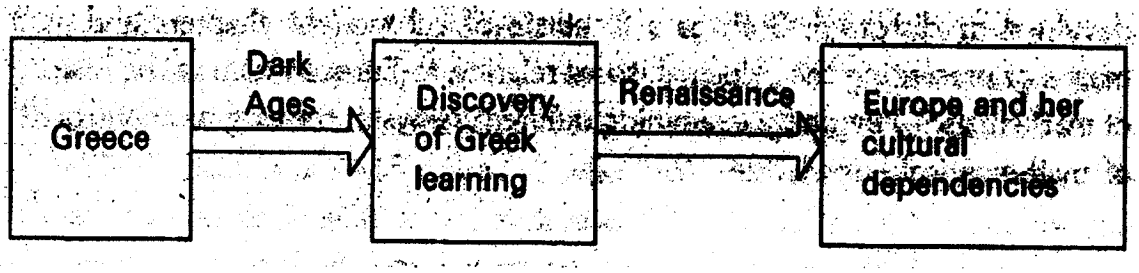


Figure 5.1. The cultural diffusion of ancient math, according to the usual interpretation. (Joseph, Fig. 1.1, p. 4) (joseph11.tif, 200 ppi)

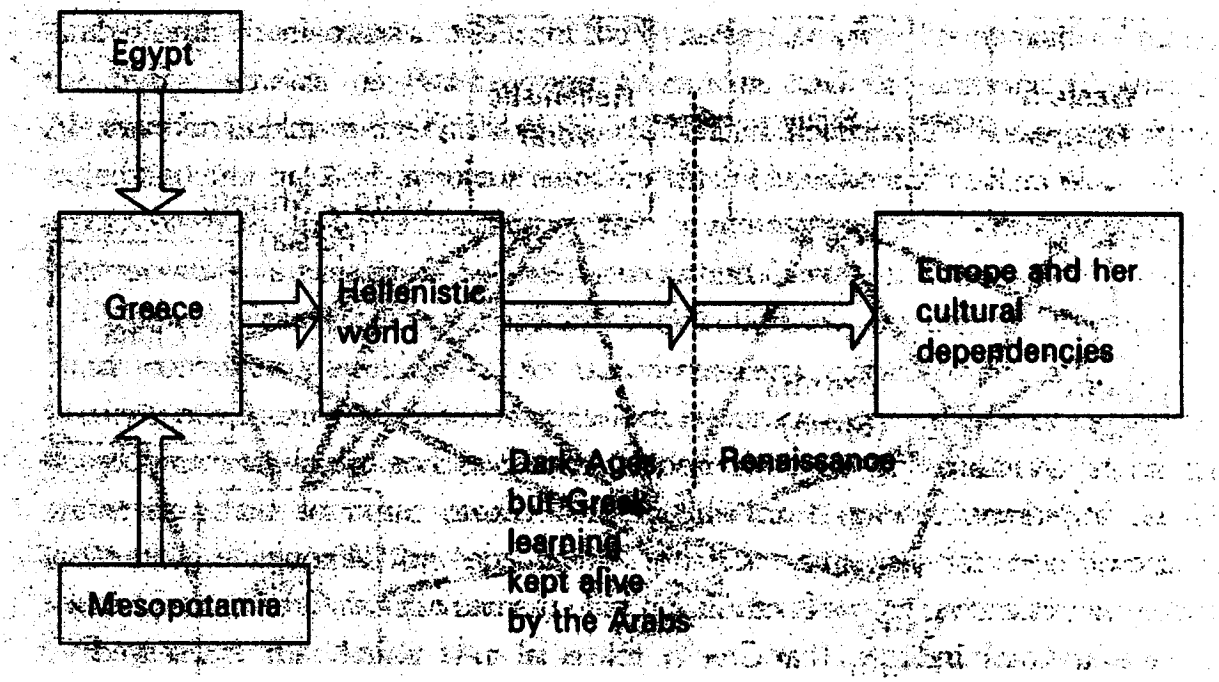


Figure 5.2. An improved diffusion path, the multicultural interpretation. (Joseph, Fig. 1.2, p. 9) (joseph12.tif, imp at 200 dpi)

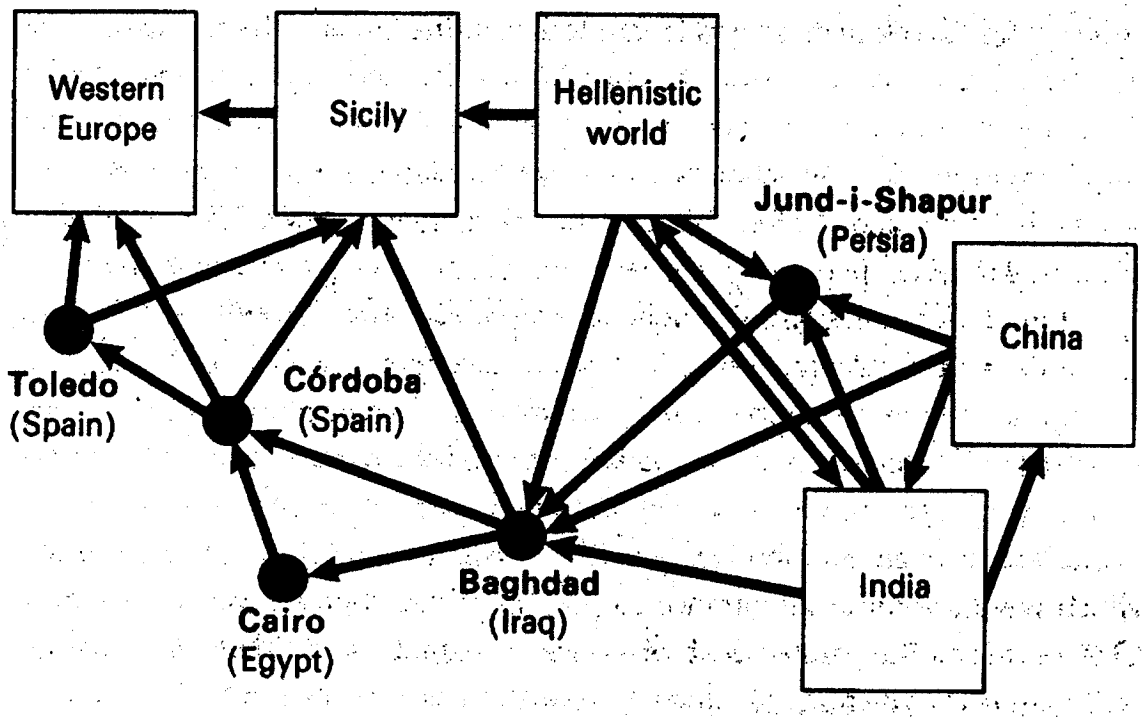


Figure 5.3. The medieval transmission of mathematics on a two-dimensional geographic chart. (Joseph, Fig. 1.3, p. 10) (joseph13.tif, 200 dpi)

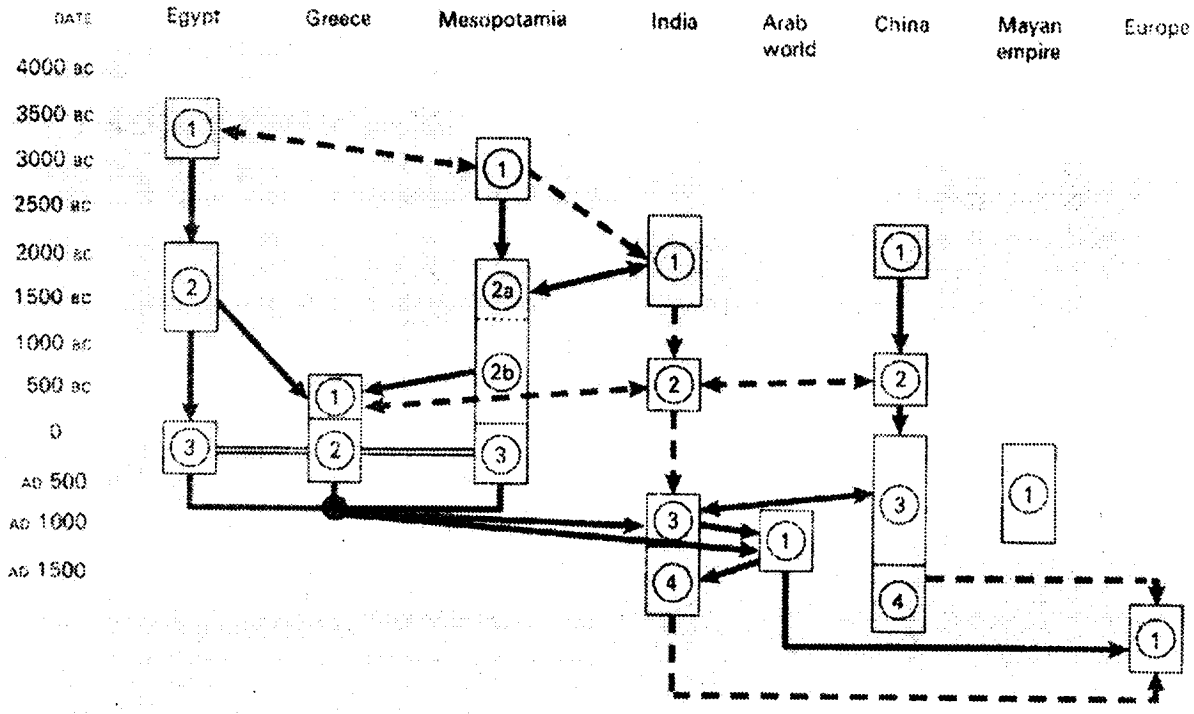


Figure 1.4 The spread of mathematical ideas down the ages.

Figure 5.4. All the foregoing diffusion paths, combined into a single figure. This is an archetypal histomap.

(Joseph, Fig. 1.4, p. 14) (joseph14.tif)

Figure 5.5. The coordinate grid for space and time, abstracted from the coordinate system of Joseph. The world is imaged as a one-dimensional continuum, divided into six cultural regions, the horizontal axis. Time, on the vertical axis, increases downwards. China and Maya are omitted to simplify the patterns of the following figures. Yellow indicates the region of prewriting.

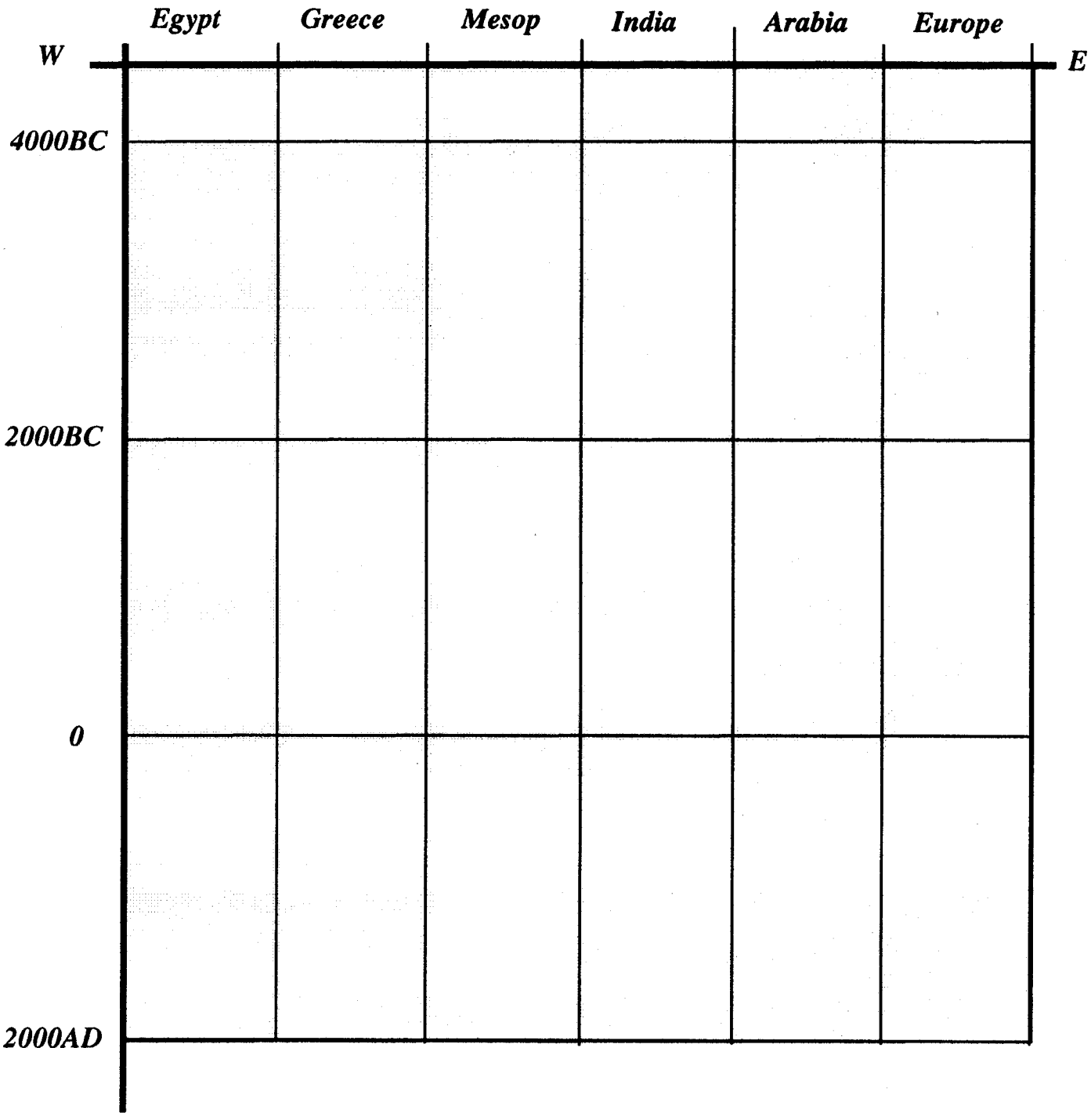


Figure 5.6. Histomap (space-time pattern) of writing systems.

- Yellow: prewriting
- Gray: prealphabetic writing
- Blue: alphabetic writing

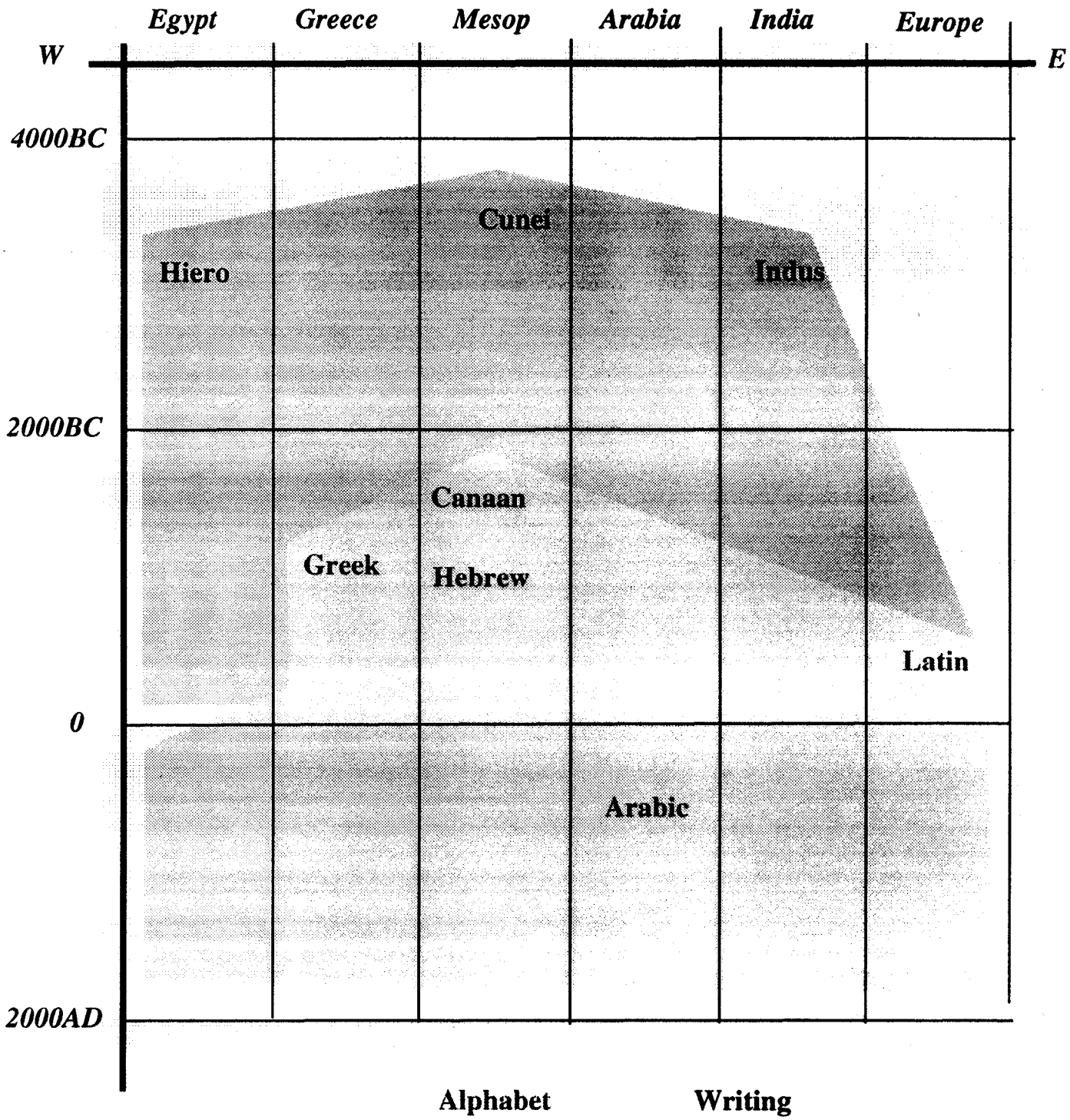


Figure 5.7. Histomap (space-time pattern) of writing and number systems.

- Yellow: prewriting
- Black: prealphabetic writing
- Blue: alphabetic writing
- Red: number systems (aljabr3.ai)

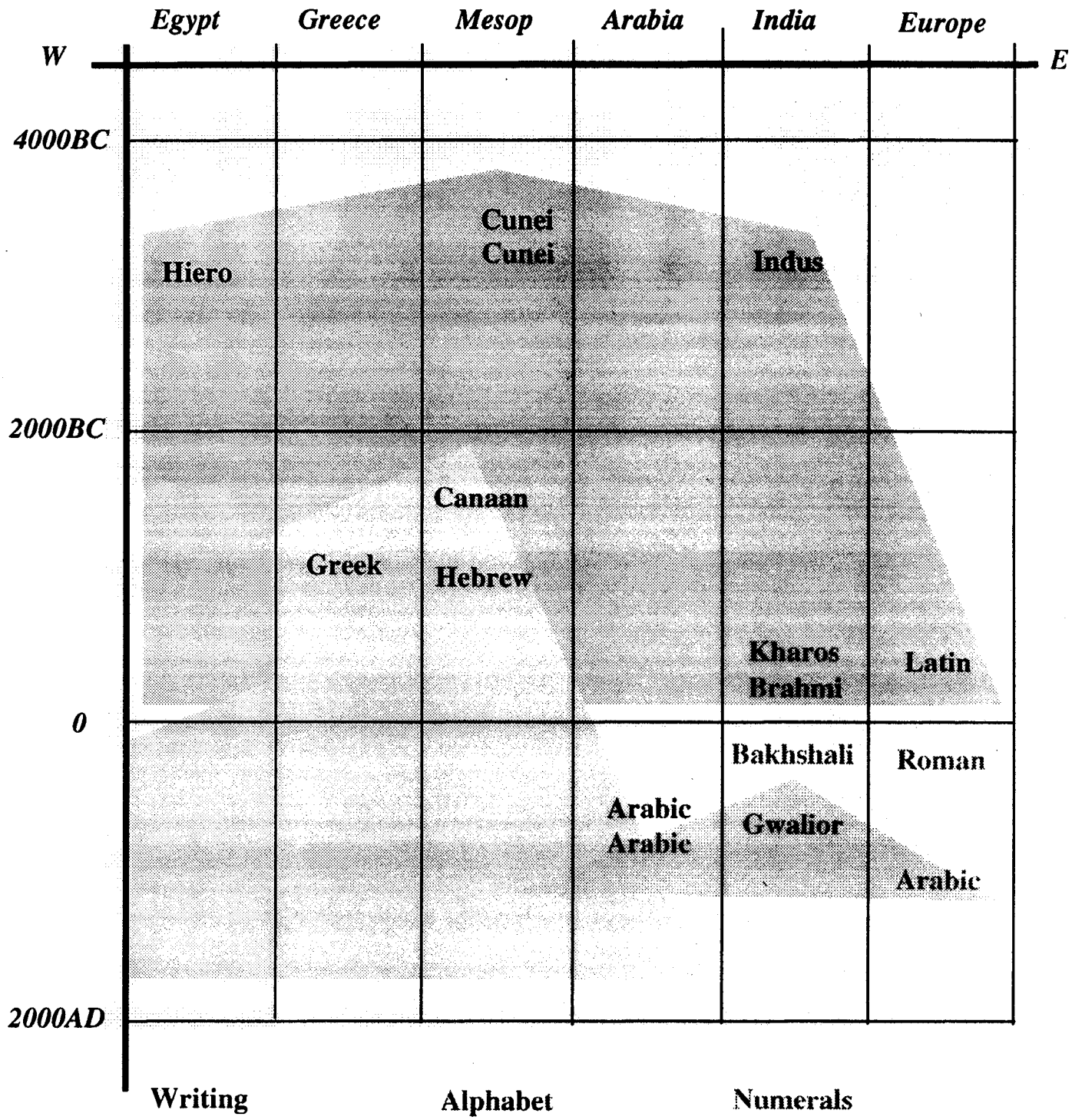
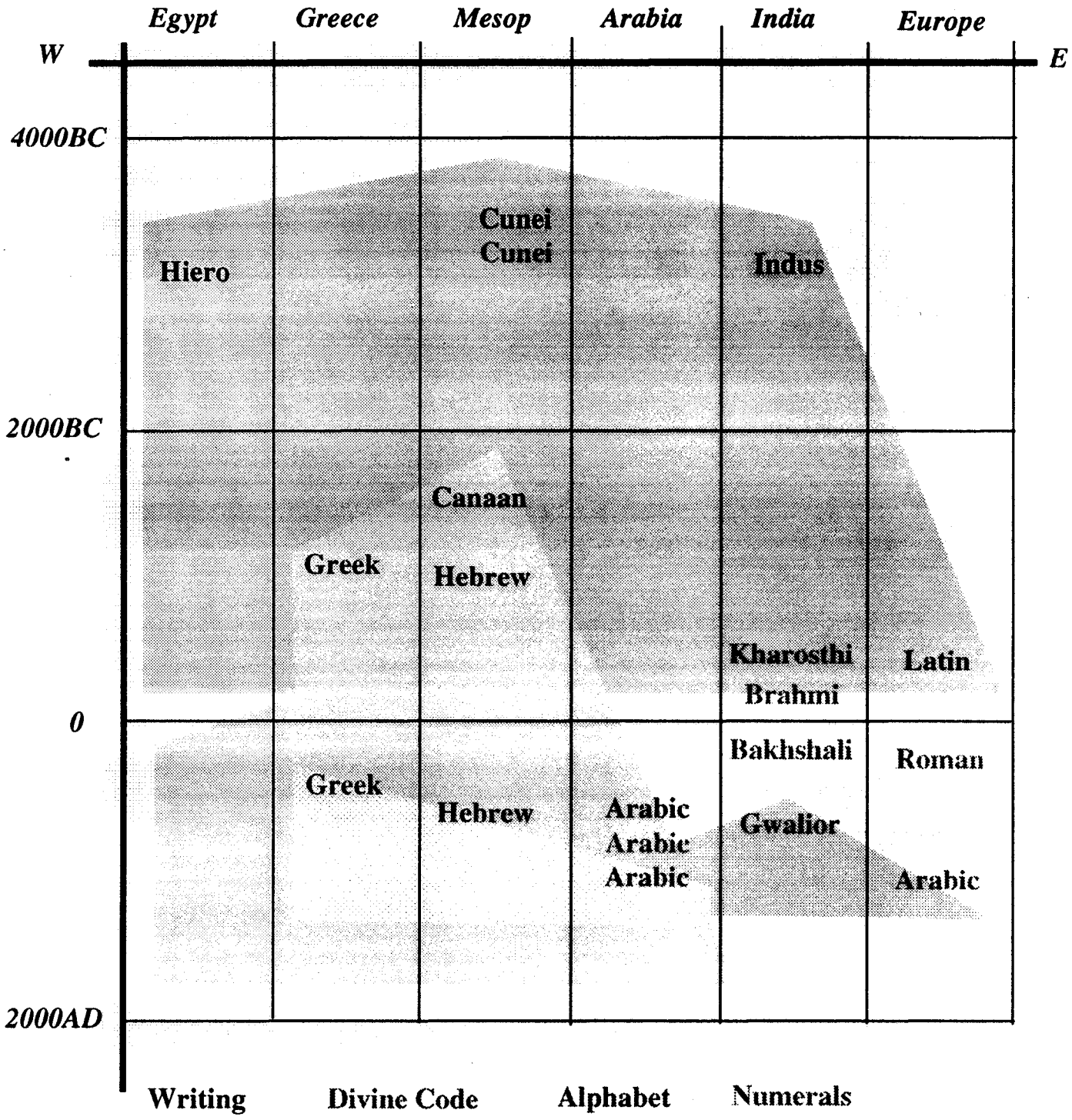


Figure 5.8. Histomap (space-time pattern) of writing and number systems.

- Black: prealphabetic writing
- Blue: alphabetic writing
- Red: number systems
- Green: divine codes (aljabr4.ai)



6. The birth of algebra

Abu Ja'far Muhammed ibn Musa al-Khwârazmî is generally acknowledged the father of algebra. He is called al-Khwârazmî, from which derives our word *algorithm.* In the early 9th century AD, he was the leader of the Bayt al Hikhma (House of Wisdom, an official institution of the Caliphate)) of Baghdad, a key figure in the intellectual development of early Islam. (Sayili, 1989) He wrote two books:

- *On the Hindu Numbers*, which survives only in a Latin translation, and
- *An Abridged Treatise on the Jabr and Muqabala Calculation* (*Al-Kitab al-Mukhtasar fi Hisab al-Jabr wa al-Muqabala*)

The latter was one of the first algebra texts in Arabic. Our word *algebra* comes from *al-Jabr* in this title. As if by coincidence, these two works reflect the coincident diffusion into Islam of the number system from the Hindus, and outward of the new algebra.

Together with the recent rise of trivalent Islamic calligraphy, two vectors converged on Baghdad in his time., as shown in Figure 5.8. Further, it is certain that he knew the recent translations of Euclid's geometric algebra into Arabic, to which he refers in his Abridged Treatise. To this fact we may add the surmise that he was aware of a vestigial survival of the original Babylonian geometric algebra, which no doubt was the basis for Euclid's version.

Our chemical analysis of the birth of algebra in this intersection of diffusing reactants boils down to this.The Hindu numbers dissolved the attachment of the symbols of the alphabet from numerics, thus liberating them for a new level of significance: the unknown variables and operations of geometric algebra. At the time of the arrival of Euclid's *Elements* in the home ground of the original Babylonian arithmetic, Euclid's solutions to the algebraic problems of Babylonia by means of geometric constructions, known as *geometric algebra*, stimulated the mathematicians of the Bait al Hikma to make use of this newly liberated symbol system for a fresh expression of the classics.

In summary:

- Ancient Babylonia had a tradition of geometrical algebra, that is, algebraic problems solved geometrically, by 2000 BC,
- Thales and Pythagoras learned this tradition during their travels, stimulating Greek geometry, around 600 BC.
- Greek geometry was organized by Euclid as a justification and proof of the constructions of Babylonian geometrical algebra, around 300 BC.
- Euclid's *Elements* reached Babylon and were translated into Arabic by 800 AD.
- At about the same time, the Indian number system arrived from India, and sacred calligraphy arrived from the Christians. The numerical layer was replaced by the spiritual in a significant cultural phase transformation.
- In this new cultural phase, al-Khwârazmî's efforts to understand Euclid, bolstered by a residuum in situ of the old Babylonian tradition, gave rise to algebra.

These vectors are indicated in the histomap of Figure 6.1.

7. The rise of symbolism

The algebra of al-Khwârazmî was not yet the algebra we know. It was still rhetorical, that is, the variables and operations were denoted by words, rather than by the signs and symbols which characterize algebra today. It is interesting to note the step-by-step evolution of the modern symbols. The plus and minus signs appeared in Germany around 1500 AD, that is, about three centuries after the arrival in Europe of the Hindu numbers. A few decades later, the equal sign appeared in England. And in 1637, it all came together in the *Discours* of Descartes, with modern notations for variables, exponents, operations, and so on. (Cajori, 1928)

This process may be regarded as an enlargement of the number system, and further reduction of the burden placed on the writing system by mathematics. With the diffusion of the several cultural morphogens into the fresh soil of Europe, a burst of chemistry brought forth the Renaissance. The ancient and medieval developments of geometry, arithmetic/algebra, astronomy/astrology, medicine/alchemy, Kabbalah/magic were reborn. And kinematics/dynamics was born. Another spot for the leopard, in the morphogenetic sequence of the human life/consciousness story.

8. Conclusion

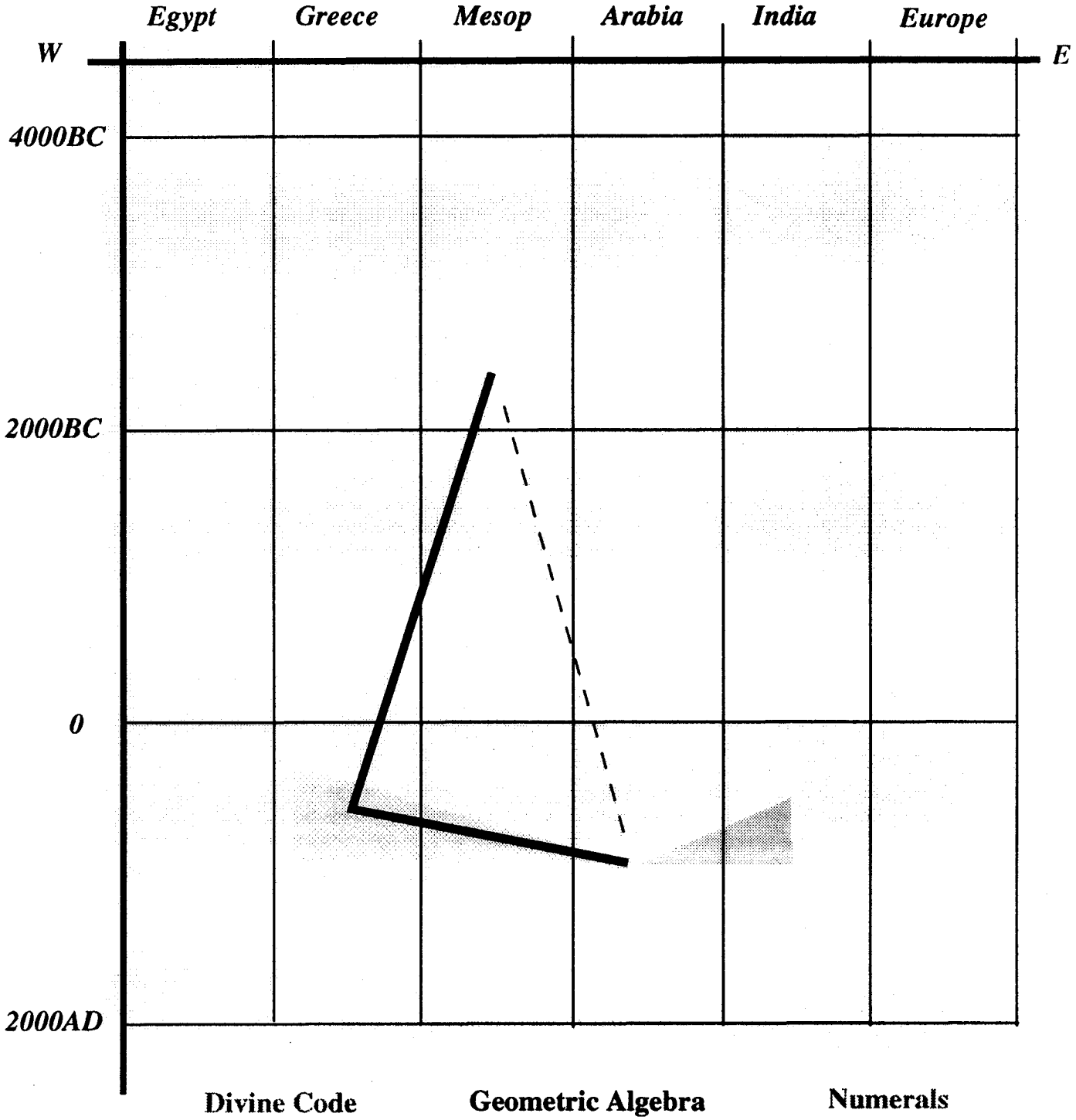
There is little doubt about the concurrent arrival in the new and spirited cultural milieu of Baghdad, early in the 9th century, of:

- Arabic alphabet literacy,
- the Hindu numerals,
- sacred (esoteric) calligraphy,
- Euclid's *Elements.*

The connection between this concurrence and the birth of algebra in the same time and place as a cultural/chemical reaction is plausible perhaps, yet not conclusive. One objection immediately comes to mind: why did we not have this explosion in India, where the liberation of the Devanagari syllabary from its numerical burden came a century or two earlier?

Our understanding of this involves the idea of the numerical layer as a sort of electrochemical insulator or barrier between the phonemic and the spiritual layers. With the melting or dissolving of the barrier, the spiritual significance of the characters came into catalytic interaction with the alphabet, and the abstraction of the divine attributes of the spiritual layer flowed into the space vacated by the numerical layer of signification, elevating it into an abstract, or algebraic, significance. And the spiritual layer was much stronger in medieval Islamic, Christian, and Jewish culture than in India. Also, the cultural memory of ancient Babylonian geometrical algebra might have been the critical factor.

Figure 6.1. Three vectors arrive in Baghdad, 820 AD.



Bibliography.

Abraham, Ralph, *Chaos, Gaia, Eros*, San Francisco: Harper-Collins, 1994.

Develops the ideas of dynamical historiography and applies them to world cultural history.

Abraham, R., Bifurcation of the Kalihari !Kung, 1992.

A dynamical analysis of the transformation of the Bushmen from stone age to neolithic culture around 1974.

Abraham, R., Chaos and the millennium, 1994. Preprint.

Bifurcation theory reviewed, and applied to the Chaos Revolution now underway.

Backhouse, Janet, *The Lindisfarne Gospels*, Oxford: Phaidon, 1981.

xx

Baker, Arthur, *Celtic Hand, Stoke by Stroke: Irish Half-Uncial from "The Book of Kells"*, New York: Dover, 1983.

How to do it manual for creating your own *Book of Kells*.

Bernal, Martin, *Black Athena: the Afroasiatic Roots of Classical Civilization*, 2 vols., New Brunswick, N.J.: Rutgers University Press, 1987, 1991.

xx

Cajori, Florian, *A History of Mathematical Notations*, La Salle, Ill.: Open Court Pub. Co., 1928/1951.

xx

Diringer, David, *Writing*, New York: Praeger, 1962.

This is the masterful, concise, and complete story of writing, from pictograms to alphabets, with amazing illustrations.

Diringer, David, *The Alphabet, A Key to the History of Mankind, 3rd edn.*, New York: Funk & Wagnalls, 1968.

Two volume expansion of the alphabet development story.

Fowden, Garth, *The Egyptian Hermes: a Historical Approach to the Late Pagan Mind*, Princeton, N.J.: Princeton University Press, 1986/1993.

xx

Healey, John F., *The Early Alphabet*, Berkeley: University of California, 1990.

A short and up-to-date version of the alphabet story.

Joseph, George Gheverghese, *The Crest of the Peacock, Non-European Roots of Mathematics*, London: Penguin, 1992.

A unique history of mathematics from a multicultural perspective.

Katz, Victor, An Introduction...

Well balanced and current text on the history of mathematics.

Khatibi, Abdelkebir, and Mohammed Sijelmassi, *The Splendour of Islamic Calligraphy*, London: Thames and Hudson, 1996.

xx

Meehan, Bernard, *The Book of Kells: An Illustrated Introduction to the Manuscript in Trinity College Dublin*, London: Thames and Hudson, 1994.

Menninger, Karl, *Number Words and Number Symbols, A Cultural History of Numbers*, New York: Dover, 1969, 1992.

xx

Naveh, Joseph, *Early History of the Alphabet: An Introduction to West Semitic Epigraphy and Paleography*, Jerusalem: Magnes Press, 1982.

xx

Rosen, Frederic, tr., *Al-Khwarazmi's Algebra*, Islamabad: Pakistan Hira Council, 1989.

The classic translation of Rosen of 1831 is here reprinted and updated.

Sayili, Aydin, Introduction, in (Rosen, 1989).

This essay is a recent and very well informed account of Al-Khwarazmi's role in the development of algebra.

Schimmel, Annemarie, *Calligraphy and Islamic Culture*, New York: New York UNiversity Press, 1984.

Complete explanation Islamic esoteric calligraphy and the Sufi spiritual values of the letters of the Arabic alphabet by an adept.

Scholem, Gershom G., *Major Trends in Jewish Mysticism*, New York: Schoken, 1941/1946.

Walker, C. B. F., *Cuneiform*, London: The British Museum, 1987.

xx