Chapter 10. The chaos and fractals of Paris

Introduction

Chaos theory and fractal geometry leaped to public attention as soon as they could, that is to say, soon after the arrival of computer graphics. Chaos theory began with the mathematical discovery of the homoclinic tangle by Henri Poincaré (1854-1912) in Paris, in December of 1889. (Barrow-Greene, 1997, p. 69) Almost 30 years later, Gaston Julia (1893-1978) also in Paris, found Poincaré's chaotic object in a much simpler context. Further research showed that this chaotic object, now known as the *Julia set*, was a fractal.

None of this became visible, however, until the analog computer graphics of Yoshisuke Ueda, then an electrical engineering graduate student in Kyoto, who was the first to view a chaotic attractor, in 1961, and the digital computer graphics of Benoit Mandelbrot, the fractal geometer who coined the word *fractal* around 1975 and made the first computer-drawn images of the Julia set around 1977.

Meanwhile, Frantisek Kupka (1871-1957), a Czech painter working in Paris, made hand-painted images very similar to these fractals as early as 1910: truly a *bolt from the blue*! Soon, chaos appeared in the musical compositions of Erik Satie (1866-1925) in Paris. We will begin with some brief background remarks on Poincaré, Julia, Kupka, and Satie separately, and then go on to speculate on possible communications between them.

Chaos, Fractals, and the Julia Set

The mathematical line of development goes from Poincaré to Julia to Mandelbrot.

Poincaré

Jules Henri Poincaré (1854 - 1912) earned the Ph.D. in mathematics from the Ecole Polytechnique in Paris in 1879. His talent was quickly manifest, and he became Professor at the University of Paris in 1881. He is now considered one of the great mathematicians of all time for his contributions in several traditional branches of mathematics, as well as his pioneering work in the creation of algebraic topology. Also to his credit is the discovery of chaotic dynamics in 1889, which cast off the anchor of the deterministic paradigm of the sciences, initiating a paradigm shift which culminated only recently in chaos theory, a new branch of mathematics.

Poincaré wrote over 500 papers, and several books, mostly high-level works of technical mathematics. However, he also published many papers in popular journals, and four nontechnical books, before his premature death at the age of 58.

The revolutionary artists of the modern art movement were greatly stimulated by the paradigm shifts of the sciences. Popular accounts by Bertrand Russell and Arthur Eddington were widely influential in England. (Waddington, 1969, p. 100)

Shortly afterward, the four popular books of Poincaré were widely read by artists and intellectuals in France, and had an effect on the development of the modern art movement in Paris. (Henderson, 1983, pp. 36, 97) (Shlain, 1991, pp. 195, 431) These books are:

- *Science and Hypothesis* (first Fr. ed., 1902),
- *The Value of Science* (1904),
- *Science and Method* (1908), and
- *Last Thoughts* (1913).

English translations of the first three of these were republished in one volume, *The Foundations of Science*, in 1913. Among the influential ideas are: non-euclidean geometry, the fourth dimension, x-rays, spatial perception, chance and probability, etc.

From our perspective today, we are particularly interested in Poincaré’s discovery in December of 1889 of the homoclinic tangle, the first example of a chaotic dynamical system. (Barrow-Greene, 1997,

69) About this object, he wrote in Volume 3 of his technical text, *Methodes Nouvelles* of 1892:

> One must be struck by the complexity of this shape, which I do not even attempt to illustrate. Nothing can give us a better idea of the complication of the three-body problem, and in general of all problems of dynamics for which there is no uniform integral.
> (Mandelbrot, 1983, p. 414)

Figure 1. The homoclinic tangle of Poincaré. Drawing by Y. Ueda, from Abraham & Shaw, Part 2, p. 106.

Note: This last condition is a generic property. That is, almost every problem of dynamics has this complexity: chaotic behavior. The first drawing of the homoclinic tangle was made by the mathematicians George Birkhoff and Paul Smith in the 1930s.

The full significance of this discovery became widely appreciated only recently: Laplacian determinism is doomed. (Smale, 1980) But Poincaré certainly understood this. For in *Science and Method* (in Chapter 4 of Part I, titled “Chance”) he wrote:

> A very small cause that escapes our notice determines a considerable effect that we cannot fail to see, and then we say that the effect is due to chance. (Peterson, 1993, p. 167)

In the wake of Poincaré’s discovery, chaos theory and fractal geometry gradually took shape. From all this history, we wish now to pull just one thread: the *Julia set*.

Julia

Paul Montel, professor of mathematics in Paris, was a contemporary of Poincaré. Among his students was Gaston Julia (1893-1978). Julia took up a problem left behind by Poincaré’s death in 1912, and discovered the homoclinic tangle in a much simpler context (a quadratic map from the plane to itself) than that of Poincaré (the restricted three-body problem of celestial mechanics). Birkhoff in the US pursued a similar line of research. In 1918, at age 25, Julia published a very long paper on his tangle, later known as the Julia set, which won him instant mathematical fame. His version of the complicated set, like Poincaré’s, was very difficult to visualize. In 1925, the slow process of visualization of the Julia set began, in a simple sketch by Cremer, a mathematician of Berlin. (Peitgen, Jurgens, and Saupe, 1992, pp. 138-139) This process languished until computer graphics came into the hands of Benoit Mandelbrot.

Mandelbrot

Benoit Mandelbrot (b. ca 1925) was a student at the University of Paris, and studied under Julia. Beginning around 1967, he formalized earlier work on strange sets, now known as *fractals*, and named and created fractal geometry. Around 1979, Mandelbrot created very detailed images of the Julia set, for

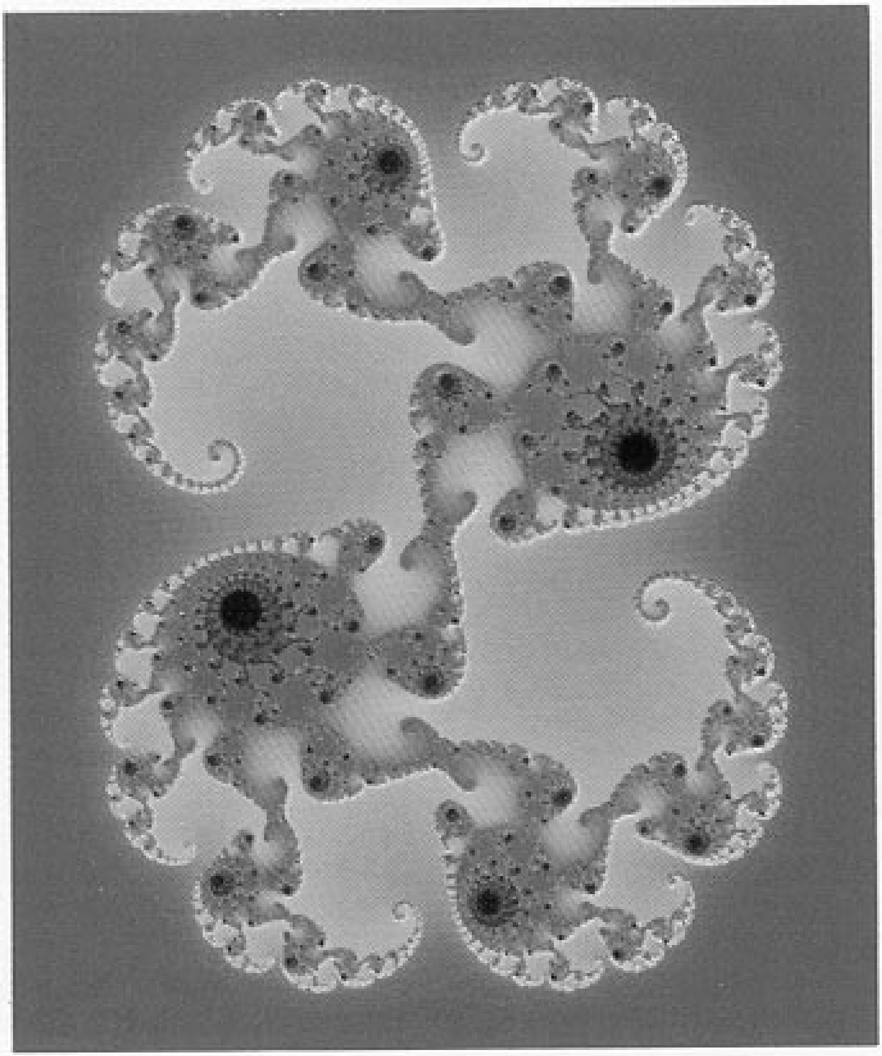

Figure 2. Julia set. From Devaney, 1992, Pl. 8.

many different examples of the family of plane mappings set by Julia. He recognized it as a fractal, that is, as belonging to the small overlap of chaos theory and fractal geometry, two new branches of mathematics which owe much to computer graphics. These fractal images from chaos theory are the best known fractals, although they represent only a very regular kind of fractal. We can say that the Julia set came into popular consciousness after 1980. However, by 1910, it was already well known to one person, Frantisek Kupka.

Kupka

Frantisek Kupka (1871—1957) was born in Eastern Bohemia, the eldest of five children. He learned drawing from his father, but his early drawings were destroyed by his stepmother. By age 27, he was apprenticed to a saddler who also worked as a spiritualist. Kupka developed a talent for leading séances, and supported himself as a medium after becoming a painter a few years later. He studied painting in Prague, Vienna, and Paris, and began exhibiting in these cities in 1899. Of note:

- 1892, Kupka encountered theosophy, became vegetarian.
- 1895, Kupka read extensively in philosophy, including Kant, Shopenhauer, Bergson, and Nietzsche.
- 1905, Kupka exhibition a great success. He began attending lectures in physics, biology, and physiology at the Sorbonne, and worked in the biology laboratory there.
- 1910, Kupka destroyed most of his paintings, and began a new phase. This is the year of Kupka's first abstract paintings, including the *Amorpha* series.
- 1911, Kupka attended meetings with a group of artists and writers every Sunday morning at the home of his neighbor, Jacques Villon.
- 1912, Kupka continues to attend the meetings. A mathematician, Maurice Princet, also attended. Kupka's first abstract paintings were exhibited in Paris.

We now see the first precognition of fractal and chaotic images in the Amorpha series of 1910 - 1912. (Andel, 1977, p. 85) (Waddington, 1970, p. 22)

Satie

Erik Satie was on the forefront of musical performance at the time of Poincaré's discovery of chaotic dynamics. He was involved with Sar Péledan, the high-priest of the Rosecrucian and Chaldean confraternity, and composed for it, in 1890. By 1900, he was incorporating American Ragtime styles in his

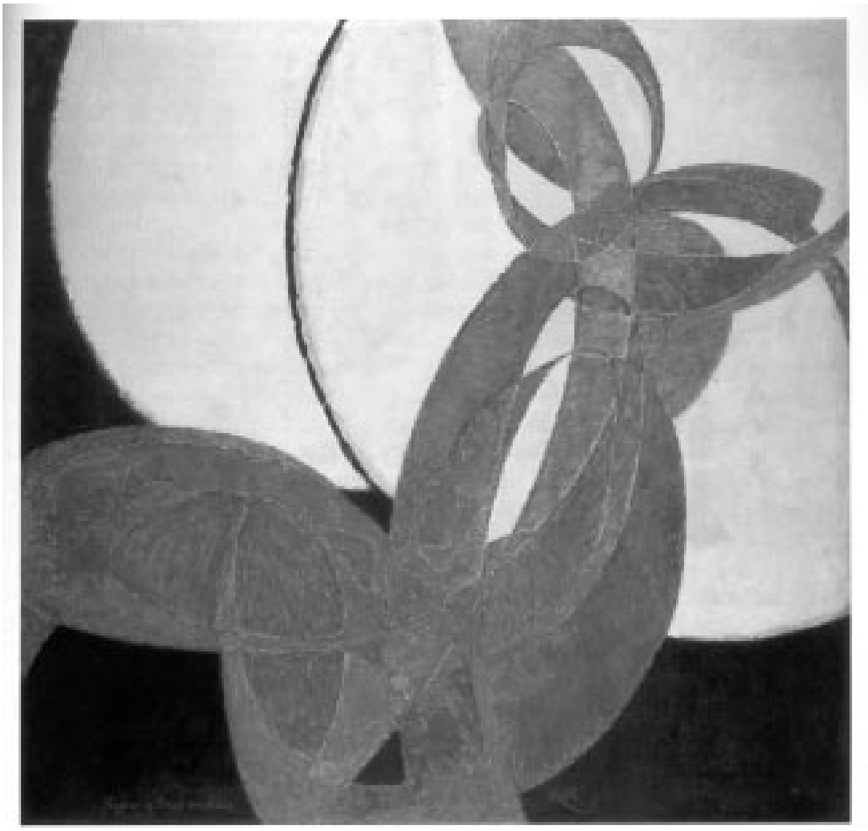

Figure 3. Kupka, Amorpha, Fugue in two colors, 1912, #68 from the DMA catalog. (Andel, 1996, p. 129)

compositions, an early form of Jazz. He was an eccentric composer in Paris by 1905.

He meet Jean Cocteau (1891-1963) — French poet, dramatist, and leader of the literary avant-garde — in 1915. Serge de Diaghilev (1872-1929) directed a Russian ballet company which was popular in Europe before the First World War. Cocteau was a close friend of Diaghilev. In 1915, Cocteau was commissioned by Diaghilev, then in Rome, to write a ballet. The idea for the ballet came to him in April of 1915, during a holiday from the front, after hearing a piano duet by Satie and Ricardo Viñes. Cocteau enlisted Léonide Massine, the most daring of choreographers, Pablo Picasso, and Satie to work as a team on this project. Work began in the Spring of 1915, the scenario by Cocteau, choreography by Massine, and sets and costumes by Picasso, were created in Rome, while Satie began working in the spring of 1916 on the music at a cafe in Paris.

The result, the realist ballet *Parade*, was produced at the Théatre du Châtelet in Paris on May 18, 1917, just as the Dada movement was getting underway. Revolutionary and shocking, it manifest Cubism in music, choreography, sets, and costumes. The audience rioted. Critics called the musical score — it called for revolvers, sirens, clappers, typewrites, lottery-wheels, Morse code keys, and so on — a 'mere din'. Progressing from order (an introductory fugue) to syncopation (inspired by Ragtime) to chaos (a frenzied dance finale), the musical score now suggests the bifurcation diagram of the logistic function basic to chaos theory, and the fractals of Julia and Fatou as first shown in computer graphics by Mandelbrot. Appolonaire's review of the ballet coined the word *surrealism* to describe the revolutionary ballet.

The team worked together again to produce the ballet *Mercure*, first preformed in Paris in 1924, after the Dada movement had been replaced by Surrealism. This ballet included twelve principal scenes, one of which is actually called *Chaos*. Satie's score for this scene combined the themes of two earlier (and very disparate) scenes to suggest chaos, which is surprisingly similar to the discovery of chaotic behavior in electronic circuits by Van der Pol, working in Holland at about this same time, with the coupling of two electronic oscillators. (Templier, pp. 36-41; Harding, p. 155; Myers, pp. 49, 102-105)

Conclusion

We may now sit back and compare the homoclinic tangle, Figure 1 of 1936, the Julia set of Mandelbrot, Figure 3 of 1979, the Amorpha painting of Kupka, Figure 4, of 1912, and the score of Satie of 1916. Many more such comparisons could be made.

Setting aside paranormal explanations, séances, and the like, we may grasp at straws in seeking connections from the Poincaré line to Kupka and Satie. All we have found are:

- Poincaré’s first two popular books appeared in the original French editions in 1902 and 1904, and were extensively read by Parisian intellectuals, including the painter Duchamps.
- Kupka spent most of 1905 reading philosophy books, and attending lectures and working at the Sorbonne, where Poincaré was then lecturing.
- Cocteau and Satie were intimate with the literary and artistic elite of Paris from 1905.

Connections tenuous at best. Truly a bolt from the blue!

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