BOLTS from the BLUE

Also by Ralph Herman Abraham

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BOLTS from the BLUE

Art, Mathematics, and Cultural Evolution

Ralph Herman Abraham

Dedicated to the late:

Andra Akers

Terence McKenna

Nina Graboi

FOREWORD
by
William Irwin Thompson

The End of the Age of Religion and the Birth of Symbiotic Consciousness[1]

Through my collaboration with the chaos mathematician Ralph Abraham in designing an evolution of consciousness curriculum for the Ross School in East Hampton, New York, I began to understand that the shift from the linear causation of Galilean dynamics in the early modern era to the complex dynamical systems of our era also expressed a shift from linear modernist ideologies and religions to planetary ecologies of consciousness in which diversity was affirmed. In the evolution of the catastrophe theory of the 1960s—with their images of saddles and butterfly folds—to the images of fractals and Lorenz attractors in the chaos dynamics of the 1980s, our cultural Imaginary was given a gift of a new alphabet of symbols. Dynamical systems were given geometrical portraits of their behavior, and these were therefore called phase portraits.[2] The linearity of left-brain thinking was now to be balanced with a right-brain activation. This emergence of a new visual mathematics expressed, in effect, a return on a higher turn of the spiral to hieroglyphic thinking.

It all started with Poincaré in Paris in 1889 when he showed that the clean and consistent system of Kepler in which the planets rotated around the sun in neat ellipses was not correct, that the solar system was actually a chaotic system. You can date the birth of complex dynamical systems with Poincaré and say that the new era begins with his mathematical revisioning of the geometry of behavior of the solar system. At about this time the premodernist esoteric cosmologies began to experience what Marshall McLuhan called

1   Reprinted from *Seven Pillars House of Wisdom*.
2   (Abraham, 1999)

"cultural retrieval," and thinkers like Rudolf Steiner, Hazrat Inayat Khan and William Butler Yeats began their visionary careers. The linear reductionism of modernism was going to be challenged by a cultural retrieval of animism on one side and higher mathematics on the other. The composer Satie was a Rosicrucian, and the painters Kandinsky and Mondrian were Theosophists. Clearly, complex dynamical systems began to impact on the cultural evolution of human spirituality.

What could this new planetary culture possibly look like? First, egocentric monumentality and the extensive clutter of industrial civilization could be eliminated. We could shift from industrial object to ecological process—as foreshadowed in the "Living Machines" of John Todd.[3] Some buildings through the effectiveness of nanotechnologies could become ephemeral and evanescent; enduring structures could be more ecologically embedded in their setting—like the Shire of the Hobbits in the writings of Tolkien. We could become electronic nomads who pitch their tent, and then pack up and move on. Buildings could become appliances that we turn on with a switch, and then turn off to make them disappear in the forest or meadow, and this would enable the human and animal domains to coexist more peaceably. Think of this as an electronic version of the Arthurian Lady of the Lake who used enchantment to keep her settlement hidden to mortals so that it appeared to the local inhabitants only as a lake.

Our machines could become intimate and ensouled by the elemental beings whose presence we might rediscover in the coming period of intense volcanic and tectonic activity. We will all have a chance to become animists again—like the present population of Iceland or the kahunas of Polynesia—those people who have been living with volcanoes for some time. The kind of being that once was envisioned as ensouling a sacred mountain could now be seen to ensoul the noetic lattice of crystals in electronic and quantum computers in a new cultural Imaginary. As these computers are worn on and in our bodies and our body-politic, our sense of "in here" and "out there" would be transformed as a cube became a tesseract or a sphere became a spiraling hypertorus in which the inside and outside

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3   (Todd, 1994)

surface are continuous through the spiraling axis.

Our consciousness could become symbiotic, as elementals become to us what mitochondria are to nucleated cells. But this symbiotic consciousness need not simply be restricted to human and elemental or animal realms, it could also be extended to involve the celestial intelligences. To imagine these "software" beings that are made out of music and mathematics, we need once again to go back to the end of the film 2001: A Space Odyssey (which I discussed with Arthur C. Clarke over breakfast in New York long ago in 1971). When the astronaut approaches the monolith in orbit over Jupiter, he sees coming toward him rotating crystals of light and complex pulsing topologies. These are Stanley Kubrick's technological envisioning of what esoteric initiates would recognize to be the Neoplatonic "celestial intelligences"—the Jinn of the Moon and the Angels and Archangels of the planets and stars. For Kubrick's and Clarke's vision, however, it is these high tech cosmic beings who serve as midwives to the astronaut's rebirth as he moves out of history into myth.

To draw a circle one moves from the point to the line; to draw a sphere, one pulls the circle up into the third dimension; to create a hypersphere, one rotates the sphere into the fourth dimension. Our physical body, or what the yogis call our food sheath, has three dimensions, but our other bodies or sheaths have more dimensions, and it is in the facets of the topology of these dimensions that the celestial intelligences can interface with us and participate in the field of our consciousness. The brain may be a three-dimensional volume, but neurons in separate parts of the brain can fire together in the neuronal synchrony of the range of 40 Hertz. The geometry of the synchronies engage as facets of the higher-dimensional geometries of the subtle bodies—where both the Dalai Lama and Rudolf Steiner say memory is stored—so the play of consciousness should not simply be reduced to a section of the brain.[4] Cultures have in the past called this process of consciousness, imagination or intuition, but whatever one calls it, it is basic to the creative process in art, science and spiritual contemplative practice.

Mediating between the elemental and human on one polarity and the celestial intelligences on the other is the realm of the

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4   (Varela, 1997; p. 174)

soul, which for men is often figured as a feminine being—a Tara or Beatrice. (Jung maintained that for women this contrasexual "animus" was male—the Christ of Saint Teresa of Avila or the Krishna of Mirabai.) This being appears regularly in the intermediate life of our dreams—and here one needs to understand that dreams, as Sri Aurobindo pointed out for his generation, are very muddled memories of higher spiritual experiences blended with the proprioperceptions of the physical body and the brain's return to waking consciousness. As the spirit returns to the confinements of incarnation, it can start to dream it is in a conference, or a crowded airport, and as it becomes aware of the body's full bladder, it will begin to dream that it is looking for the restroom in the airport. To interpret these dreams with Freudian or Jungian symbolic systems is, at this level, a category mistake. The imagery is taken from personal memory and is being used as metaphors for the reactivation of specific brain modules that are operative in the cognitive functions of the waking mind.

Shamanism was the form of spirituality that evolved in the oral culture of preliterate societies. Religion was the form of spirituality that emerged with literate societies and their new temple-based readings of the stars and sacred texts. Though traditionalists may wish time to stop, it does go on, and now in our global electronic society, a new transreligious form of spirituality is emerging, one that will not replace religions, anymore than the nucleus of the cell replaced the mitochondria, but will envelop them in a much vaster form of consciousness. In this futuristic ontology, we are already beginning to glimpse an evolutionary Entelechy—a symbiotic consciousness of human, elemental, psychic and celestial intelligences. In the smuggled esoterism of children's literature, comic books and science fiction, an archetypal group of four becoming one is being foreshadowed. For example, we see this grouping expressed in Fantastic Four and The Wizard of Oz.

In preindustrial animist cultures, the human would establish a place for the elementals to cohabit or participate in its human life through the intermediary use of a magical object—a magical ring or stone, an Aladdin's lamp or an ensouled sword such as Roland's Durendal or Charlemagne's Montjoie. Like the needles used in acupuncture, this numinous object can interact with its possessor at the

subtle-physical or etheric level—the level of qi or prana. In shaman-istic cultures, the individual would project out of his or her body and travel in a spiritual or astral world. If one is able through meditation to remain watchfully awake in the state of deep dreamless sleep, then one experiences a vast magenta sea of cognitive bliss in which one hears the music of everything—every existent being in the universe sounding its presence. This is the world of the Holy Spirit that is "above" or "below"—remember these terms express a merely Euclidean geometry—the psychic realm of dreams or astral out-of-the-body travel. In the astral, one travels, but in the realm of celestial music, the center is everywhere and the circumference nowhere, so there is no need to move. One simply joins in this Hallelujah Chorus to the nth power by listening and sounding one's ontic note. In this sense, music is not a representational art of mimesis, but an ontological performance.

The human is the ordinary ego in time, but in the completion of our emergent evolutionary spiritual process the ego becomes transhuman, "anointed" or Christic. For esoteric Christians, the prophet Jesus became the Christ at the time of the baptism by John, and this narrative of the Son of Man describes the process of enlightened individuation. For Buddhists, this process is seen more like a wave than a particle, one in which egohood is transformed in dependent co-origination (pratityasamutpadha) of enlightened Buddha Mind.[5]

Now you may have noticed that one thing that results from this ontology of symbiotic consciousness is a non-locality in which "out there" is "in here" so that it is no longer necessary to put three-dimensional bodies in expensive tin cans and space suits and try to propel them to the stars. We may be able to go to the Moon, and my Lindisfarne colleague James Lovelock's proposed atmospheric and bacterial "greening of Mars" would certainly be a worthy project for sublimating the nations' defense industries into a transnational technological project with Europe, Russia, Japan and China, but I think it is highly unlikely humans could travel in physical bodies to the stars. Indeed, that is precisely what the astronaut of 2001 discovers in his transformation from technological man to star child. So the Christian

5   (Varela, 1997; p. 174)

fundamentalist notion that we can trash the Earth and move on, and that whatever mess we make here is permitted because Jesus will play the role of a suburban mom coming in at the end of the day to clean up our room for us and then take us away in a vacation rapture to some theme park heaven is an expression of folk superstition and the limited three-dimensional thinking of religion.

Jean Gebser taught us that when a new evolutionary form becomes efficient, the old becomes deficient.[6] When religion emerged, shamanism decayed into sorcery and black magic. Now that a new planetary spirituality is emerging, religion has become a toxic dump, as witnessed in the recent terrorist attacks in Mumbai.

But the religionists are right in one way; it is the end of their world, but that also means the end of the age of religion and the beginning of a unique/universal self-similar architecture of consciousness that is based upon individual experience and not upon priestcraft, rigid dogma and collective forceful indoctrination. If we can avoid the dark age of religion that now stares us in the face, we may discover a more surprising and delightful politics of Being behind the mask.

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6   (Gebser, 1984; p. 93ff)

PREFACE

Andra Akers was the stimulus and cheerleader for much revolutionary thought during the 1980s and 1990s in Los Angeles and New York until her premature death, and we miss her. William Irwin Thompson and I have collaborated in sharing ideas since our meeting in 1985 at Andra Akers' home in West Hollywood.

In the context of our cooperation over these years, our particular ways of looking at the development of art and mathematics in world cultural history converged into a narrative with a mathematical frame.This book owes a huge debt to Bill Thompson, whose ideas sparked four of the six chapters, and added much to the glue and integrity of the whole project.

For crucial editorial help and all kinds of advice and encouragement over four decades, I am especially grateful to my friend, philosopher Paul A. Lee.

For keen and useful feedback on this book, many thanks to Claudia l'Amoreaux, and Kevin Cashen. For Chapter 1, thanks to Ernest McClain and Edmund Carpenter for their comments on an early draft. And for publication assistance, many thanks to Paul Cohen and his team at Epigraph Books.

Ralph Herman Abraham
Santa Cruz, California
October 26, 2010

Our title refers to a singular spark of inspiration in the creative synergy of the arts and mathematics. For example:

> *The invention of logarithms came on the world as a bolt from the blue. No previous work had led up to it, foreshadowed it or heralded its arrival. It stands isolated, breaking in upon human thought abruptly without borrowing from the work of other intellects or following known lines of mathematical thought.[1]*

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1   Inaugural address by Lord Moulton, on the 300th anniversary of the invention of logarithms (a radically new way to multiply numbers) by John Napier, Edinburgh, 1914. Published as "The invention of logarithms," in *Napier Tercentenary Memorial Volume*, p. 3. Also quoted by Eli Maor, in: *The Story of a Number*, Princeton, 1994, p. 13.

Contents

INTRODUCTION

This book is about the role of mathematics in the evolution of culture, and the evolution of mathematics itself. It aims to clarify the question: What is mathematics, and who is a mathematician? We believe that everyone has a native mathematical talent, before it is tainted in school by math anxiety. In this book we give six examples of bolts from the blue, in which an artist and self-taught mathematician brings forth an important new mathematical idea in an intuitive revelation, an innovation that triggers a major transformation within mathematics, and in cultural history as well. The book is organized around three main themes:

• *Dynamical historiography*, that is, world cultural history regarded as a complex dynamical system, a network of cultural ecologies, a history evolving through epochs (plateaus) segmented by *bifurcations* (generalized paradigm shifts). *Chaos theory* — comprising the new computer-based mathematical theories of nonlinear dynamics, fractals, chaos, bifurcations, complexity, neural networks, and so on — provides a new way of looking at any complex dynamical system. The first of our three main themes is based on this new way of looking at one of the largest complex systems of all: world cultural history[1]

• *Mathematical mentality*, that is, the dominant style of mathematical cognition exhibited by individuals in a cultural ecology. We will describe five mentalities from the viewpoint of dynamical historiography, cultural ecology, and bifurcation theory. This theme is a math-centric kind of cognitive psychology, in which we characterize an entire cultural ecology by its mathematical style. This is analogous to art history being divided into Ancient, Medieval, and Modern, for example.

• *Bolts from the blue*, that is, the reception of a new mathematical strategy by an artist or intellectual, as if by telepathy from

[1]   See (Abraham, 1994) for an early account of this view. One of the first historians to adopt the chaos theoretic view is William Irwin Thompson.

the stars, leading to a bifurcation in a cultural-historical system.

Our six chapters are exemplary of our third theme: bolts from the blue. But taken together, they also illustrate a subtheme: *Anyone can be an important mathematician, if not handicapped in school by math anxiety.* This leads us to an urgent problem: how to design a school math curriculum that avoids math anxiety. We now explain these themes, one at a time.

Dynamical historiography

The new mathematics of complex dynamical systems, that we call chaos theory for short, emerging into the mainstream since 1970 or so, has provided a new cognitive style that we call the *chaos dynamical mentality*. The main concepts here, derived from the new mathematical theories, include attractors, basins of attraction, bifurcations, the emergency of new attractors, self organization, pattern formation, and many more. For details of these new concepts one may refer to other books.

But the new view of world cultural history, called dynamical historiography, is fully explored in my book, *Chaos, Gaia, Eros.*[2] The word bifurcation, a technical term from chaos theory, provides more precision to the ideas of paradigm shift (from Ludwik Fleck[3] and Thomas Kuhn[4]) and cultural mutation (of Jean Gebser[5]).

In chaos theory, there are three kinds of bifurcation: catastrophic bifurcation (such as a quantum leap), subtle bifurcation (such as the gradual onset of vibration in a machine), and explosive bifurcation (in which a chaotic state suddenly expands in magnitude, like an earthquake).

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2   (Abraham, 1994)
3   Fleck, a holocaust survivor, introduced the paradigm shift idea in his book, Entstehung und Entwicklung einer wissenschafilichen Tatsache; Einfuhrung in die Lehre von Denkstil und Denkkollectiv, of 1935. See the English edn., (Fleck, 1979).
4   Kuhn, inspired by Fleck, based his paradigm theory on the example of the Copernican revolution.(Kuhn, l962)
5   Gebser presented a complete theory of the evolution of consciousness in his book, Ursprung und Gegenwart, of 1966. For the English, see (Gebser, 1984).

The sequence of five stages

In the l960s and 70s, William Irwin Thompson joined anthropology, artistic studies, and political history into a unique approach to cultural history.[6] He intuitively made use of the ideas of dynamical historiography to parse our whole history into plateaus punctuated by major shifts.

In his *Pacific Shift* of 1985, he described four major stages in the history of the West: the Riverine, Mediterranean, Atlantic, and Pacific-Space stages.[7] The approximate dates for the major bifurcations separating them (and their chief characteristic features) are:

- Tl. Beginning of Riverine: 4000 BCE[8] (writing)
- T2. Riverine to Mediterranean: 2000 BCE (alphabet)
- T3. Mediterranean to Atlantic: 1500 CE (printing)
- T4. Atlantic to Pacific-Space: 2000 CE (computer)

The Riverine refers to the cultural ecology of the Indus, Nile, and Mesopotamian valleys. The Mediterranean is the cultural ecology all around the Mediterranean Sea. The Atlantic includes Western Europe and Eastern North America, in particular. And the Pacific cultural ecology is that currently emerging around the Pacific Rim. Hence his title, *Pacific Shift*. We are now in the midst of a major bifurcation (a tipping point) from the Atlantic to the Pacific cultural ecology.

Independently, in my *Chaos, Gaia, and Eros* of 1994, I presented world cultural history in four epochs, divided by three major bifurcations:

- Al. Paleolithic to Neolithic: 10,000 BCE (agriculture)
- A2. History: 4000 BCE (writing)
- A3. Chaos: 2000 CE (computational math)

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6   See (Thompson, 1967) and (Thompson, l97l).
7   Developed in a talk on February 13, l983. See (Thompson, 1985, esp. Ch. 4).
8   ln this book we use BCE and CE in place of BC and AD for dates before and after the time of the Christ.

In subsequent joint work, we settled on the sequence of five epochs (and mathematical mentalities) described below, which frame the structure of this book. Ignoring the Neolithic bifurcation, Al, the Thompson sequence is a refinement of my sequence. A2 coincides with Tl, and A3 with T4. Our joint sequence, includes: Al, A2=Tl, T3, A3=T4, and replaces T2 with a new shift between T1 and T3, the Islamic (see list below).

The main motivation for our special emphasis on bifurcations is to understand our current transformation. To participate wisely in the creation of the next cultural ecology, we must study the major bifurcations of the past.

Between any two consecutive large bifurcations there may be several medium bifurcations, and between them, many small ones. Two medium bifurcations within the Mediterranean epoch figure in this book:

- the enlightenment of early Islam, 800 CE, and
- the Italian Renaissance, 1400 CE.

To some extent our periodization into five stages is arbitrary. We could have had a longer list, regarding finer structure in the evolution of mathematics and cognitive styles. But we regard our five stage sequence as the shortest that is consistent with the history of mathematics, as normally understood by mathematicians. This is the sequence of periods basic to this book, with very approximate starting dates:

- Prehistoric, from 100,000 BCE
- Historic, 4,000 BCE
- Islamic, 800 CE
- Dynamic, 1400
- Chaotic, 2000

Cultural ecologies

Our concept of cultural ecology is derived from *Gaia theory*. Developed by Earth scientist James Lovelock in a series of talks, ar-

ticles, and books since 1968, this is a massively integrated theory of the complex ecosystem comprising the Earth's oceans, land masses, biosphere, atmosphere, energy balance with the sun and space, and so on.[9] Building on earlier work by Vernadsky in Russia, Lovelock taught the principles of complex dynamical systems in the context of our living Earth, as he called this massive ecosystem. In this book we are taking Gaia one step further, to consider world cultural history as an integral part of the big picture.

The concept of cultural ecology is an application of complex dynamics to cultural history, that is, an example of dynamical historiography. Historical and ecological ideas are combined, and applied to world cultural history. Cultural ecology regards a culture as an ecosystem. Its parts — for example, literature, visual arts, musical creations and performance, mathematical developments and applications, scientific discoveries, economics, etc.— are interconnected like the flora, fauna, and environment in a biospheric ecosystem.[10] Local cultural microsystems are interconnected in a global cultural macrosystem, linked by trade, cultural diffusion, and so on. Cultural and biospheric systems are interconnected in Gaian superphysiology. This giant web evolved, and continues to evolve, as a complex dynamical system, with emergent properties, bifurcations, etc.

Each cultural ecology is characterized by attributes, including a dominant mathematical mentality.[11]

Mathematical Mentalities

By mathematical mentality we mean more than specific mathematical knowledge. We mean a characteristic cognitive style in approaching perception, analysis, reasoning, magical operations, and so on. It is characteristic of the whole intellectual approach of a cultural ecology. Here is the list of the five cultural ecologies, their mathematical mentalities, and their mnemonic codes, basic to this

9   For Gaia theory, see all books by James Lovelock, for example, (Lovelock, 1995). Also, (Abraham, 1994).

10   For the science of biospherics originally due to Vemadsky, see (Snyder, 1885).

11   See (Thompson, 2004).

book:

- Paleolithic cultural ecology, aRithmetic mentality (R)
- Riverine, Geometric (G)
- Islamic, Algebraic (A)
- Renaissance, Galilean Dynamical (D)
- Modernist, Chaos Dynamical (X)

We use RGADX as a mnemonic for this sequence. Each cultural ecology and math mentality is separated from its sequel by a cultural bifurcation. Our chapters will clarify these systems with examples.

How we came to this picture

The sequence of cultures and mentalities, RGADX, is just one of many that might occur to an amateur historian. Why and how did we come to this? We must now confess that it has evolved through extensive efforts to revise the school curriculum so that mathematics and cultural history might be integrated, and math anxiety avoided. We intend that this sequence be used as the outline of a school curriculum in middle and high schools around the world. Used how? This book aims to give an indication of integrated teaching units, taught in sequence, along with the related math skills motivated by the stories.

Bolts from the blue

The six chapters of this book are intended as exemplary, self-standing units, defining our basic concepts, and leading to the full integration of math and cultural history. In addition, each is the story of a bolt from the blue, precipitating a cultural bifurcation. They are presented here in historical order.

The first is devoted to the *Venus of Lespugue*. This is a small paleolithic sculpture, one of a number of similar objects found all over Old Europe. It belongs to the paleolithic, prehistoric past, an

represents the essence of the aRithmetic Mentality around 23,000 BCE. Its dimensions conform to a precise set of ratios, derived from the musical intervals of the Greek Doric scale. The Venus was created thousands of years before the beginning of the Riverine ecosystem in Mesopotamia, with its Geometric Mentality. Advanced arithmetic was developed by paleolithic musicians, and carved in stone for the future.

The second is devoted to the *Bethels of Scotland*. These are small sculptures the size of a baseball, carved in one of the hardest stones of Europe. Many bethels, dating from the time of the megalithic monuments such as Stonehenge, have been found all over Scotland. Their shapes include the cosmic figures, also known as Platonic solids much later in ancient Greece. They demonstrate the solid geometry of the megalithic phase of the Riverine cultural system, as it approaches the bifurcation from the aRithmetic to the Geometric Mentality. Sublime geometrical knowledge was intuited by megalithic sculptors, and again, saved for posterity in stone, around 3000 BCE.

The third, on the origins of algebra and the Algebraic Mentality, discloses a smaller but thrilling bifurcation within the Mediterranean cultural ecology, in which the aRithmetic Mentality is revived, along with its association with writing, and extended: a Riverine renaissance, preserved in manuscript form.

The fourth, following the revival of perspective by artists of the Renaissance, demonstrates the anticipation by the painter Fra Angelico, in his geometry of angels, of an advanced result in the branch of mathematics called topology, which emerged 300 years later in academic circles in the 20th century, at the beginning of the Pacific Shift. This is an extraordinary case of mathematical clairvoyance, preserved as a painting: an archetypal bolt from the blue.

The fifth, on the birth of the Dynamical Mentality, gives us a step-by-step analysis of a bifurcation, seen from the complex dynamical point-of-view characteristic of our new Chaos Dynamical Mentality.

The sixth and final chapter reveals the mathematical precognition by a modem painter of *fractal geometry*, fundamental to the Chaos Dynamical Mentality, 60 years before fractal images were

created for the first time by computer graphics, in the work of Benoit Mandelbrot. A bolt from the blue, again sent forward to us as a painting.

In each case, a *bolt* — an intuitive leap, major advance for mathematics, and trigger of a cultural bifurcation — has been received and recorded from the *blue,* the intuition of an artist or intellectual.

Math anxiety

This book is motivated to show the role of artists and intellectuals in the evolution of mathematics, and to promote Thompson's scheme for world cultural history. But its chief concern is the perilous situation of mathematics in our contemporary society. For there is now a pandemic of math anxiety. And believing as I do that our future is doomed without the intellectual and cognitive support of a healthy and vigorous mathematical culture, this situation demands attention.

After more than fifty years of teaching math in universities in several countries, I have developed the conviction that every person is born with a substantial talent for math which is subsequently destroyed in our schools by a faulty pedagogy that has become traditional during the last century or so. Over the years, I have identified three major flaws. This book is intended to help remedy them.

Flaw #1: No graphics.

The first flaw came to my attention around 1974, when computer graphics first arrived at my university. After creating computer graphic software for research in chaos theory, then a new branch of mathematics, we adapted the hardware and software to support our lower division math courses: calculus, linear algebra, differential equations, and so on. With support from the State of California, these efforts evolved into a major program called the Visual Math Project. Computer graphic illustrations and animations were piped into classrooms using television cables.

We discovered that many students were saved from math

anxiety and became successful students of mathematics. Some were so enthusiastic that they became programmers in our project, developing software to teach others what they had learned.

Mathematicians communicate among themselves by coordinating multiple intelligences: verbal, graphic, and symbolic. For example, one cannot learn math without graphics. As our school math programs present math without adequate graphics, learning is handicapped. Students fail to learn, and then are persuaded that it is their own fault, which it is not.

Flaw #2: No history

The second flaw came to me around 1987. A book on chaos theory, in which I was quoted extensively, became very popular. Journalists called me to ask what the fuss was all about, leading me to write a book, *Chaos, Gaia, Eros,* on the historical context and philosophical significance of chaos theory. Meanwhile, a new course on the history of mathematics was instituted at my university. My colleagues, knowing that was writing a book on the history of chaos theory, offered me this course, and I taught it annually for a decade or more. A friend, Rupert Sheldrake, persuaded me that people learn things better if they are presented in historical order. In the case of mathematics, this is the opposite of the usual, logical order. Armed with this idea, and with my new knowledge of the history of math, I gradually changed all my teaching to a historically based style.

In a historically based program, topics are presented in historical order, so that cognitive prerequisites are available when needed. This avoids the most common obstacles that prevent students from grasping new mathematical concepts.

Flaw#3: No integration.

The historical sequence is crucial, yet not enough. The whole historical sequence should be integrated with the cultural context in which it evolved, providing meaning and motivation for students.

I am convinced that these three flaws are major causes of difficulty that students have in learning math in our school system

today. I found fewer failures in my courses after adopting all three remedies — visual representation, historical sequence, and cultural integration — in my courses.

The big test.

The weaknesses in our school math programs today are commonplace, and widely proclaimed. The remedies usually proposed — standardized multiple-choice testing and coaching, word problems, short-question drill and kill, and so on — will do more harm than good. Rather, we advocate graphics, history and integration.

The program of this book

The six chapters of this book are exemplary of the graphical, historical, and integral approach to math. In addition, they illustrate the stages of math as they occurred in the evolution and history of our culture, according to the theory developed by William Irwin Thompson and myself on the five mathematical mentalities: aRithmetic, Geometric, Algebraic, Dynamical, and Xaotic (RGADX).

For example, we advocate teaching geometry before algebra in school. This conforms to historical order, as G precedes A in RGADX. In the history of mathematics, Babylonian geometry evolved into Greek geometric algebra,[12] an essential prerequisite for the Islamic development of rhetorical algebra. Breaking this sequence may be a major cause for math anxiety in our schools.

The chapters of this book are devoted, one each and in order, to the five mentalities, RGADX, and one extra, on Fra Angelico, who wins the prize for the biggest bolt of all.

12   (Katz, 1993; p. 64)