Program note 2: About the images

Chaos theory, the math

Chaos theory is a pop name for what is known in the math world as dynamical system theory. It has several branches, the two most prominent are continuous and discrete systems. The distinction involves the model for the time variable. In continuous systems time is continuous, the state of such a system advances along a continuous curve in the state space, usually two or three-dimensional space. In a discrete dynamical system, time changes in uniform steps, and the state of the system jumps around from one point in the state space to another.

The mathematical theory of dynamical systems advanced initially in the continuous case. Chaotic behavior was discovered in this context in 1961. The understanding of these systems involves the division of the state space into disjoint regions called basins, in each of which a unique attractor is found. As the system evolves in continuous time, every point in a basin moves along a continuous curve called its trajectory that moves ever closer to the attractor. The attractor might be a single point, a closed curve or loop, or, in the chaotic case, a sort of tangle like a bowl spaghetti.

The discrete branch is especially important in applications to economics, where the observed data tends to be discrete in time. The theory of chaos in discrete systems came to the foreground in 1974, and it is quite different from the theory of continuous dynamical systems. Here the analysis involves attractors and basins, but also an additional feature called the critical variety.

The Sprott collection

In 1993, an eminent chaos theorist, Julian Clinton Sprott, published a definitive list of discrete dynamical systems exhibiting chaotic behavior. He sought to discover all such systems defined by polynomial functions of degree 2, 3, 4, and 5 in dimensions 2, 3, and 4. He simulated these systems, recording the details of all such systems, millions of cases, running a desktop computer for months. His results were published in a book including still monochrome images of all cases, about 200 in all.

The Fractal Note Gallery.

In the period 2013-2016, I undertook an extension of Sprott's work, by recomputing almost all of his cases using the NetLogo simulation language, recording color images in the two-dimensional cases, and short videos in the three-dimensional cases. The colors, pseudo-colors actually as the attractors are just clouds of points in space, are chosen to indicate the density of points in the cloud.

The videos show the chaotic attractor, a thick 3D fractoid, rotating slowly around a vertical axis through its center. All these results are posted in a website at www.visual-chaos.org, along with explanatory notes.

These 3D rotating videos are the raw source of the primary animations shown in our performance in Santa Cruz, June 16, 2023. In the performance, the videos from the website are modulated by the audio stream from Ben's Mutantrumpet, as well as controls manipulated by faders in my hands.

New works

In addition to the works posted in 2013-2016 on the website, I am currently busy with a new project, to add to the attractor portraits new images of the critical varieties, never before visualized. Preliminary results from this new project have been utilized as a pointilistic overlay in our recent performance, and will be featured in future performances. Thus math research and art creation proceed hand-in-hand, as forecast in our shared dreams since 2014.

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