Lab 1

Introduction This week we will investigate 1-D continuous flows using both STELLA and MAPLE. In one dimension the types of behaviors that occur are limited in nature, i.e. there are no oscillations and there is certainly no chaos. We do however see fixed points that either attract or repel nearby points and, in some degenerate cases, do both.

The system we will look at today is the population model from last week (a.k.a. the logistic equation). It corresponds to the flow given by the ordinary differential equation

$$\dot{N}(t) = rN(t)(1 - \frac{N(t)}{K}). \quad (1)$$

Exercise 1 In STELLA, set up the flow corresponding to Fig.1 and Fig.2. Experiment with graphs of the population for a wide range of birth rates, death rates and initial populations to see what kinds of behavior you can produce. Then fix the birth rate and death rate so that they are equal and somewhere between 0.1 and 1. Do a seneitivity analysis for initial population that takes 7 values between 0 and 2. Print out this plot. Where are the fixed points? Are they attracting, repelling or degenerate?

Exercise 2 (a quick glance at chaotic behavior) As we previously noted, continuous 1-D flows can not exhibit chaotic behavior. However, computers do not plot things continuously: that would be impossible (why?). They "cheat" and chop up intervals into extremely small pieces (like a Riemann sum, for those who have taken calculus). What we usually see is a good discrete-time approximation to a continuous flow. We shall see later that discrete-time dynamics can exhibit chaotic behavior even in one dimension. To take advantage of the cheating computer, set up the flow as described in Fig.3 and Fig.4. Plot population with a sensitivity analysis for r using 3 values ranging from 7 to 12.1. Print out this graph and note that the behavior exhibited is not allowed by continuous 1-D flows.

Exercise 3 (an introduction to MAPLE) Download the maple worksheet from the webpage. This worksheet plots the vector field corrsponding to (1). Experiment with different values of r, K, and the initial conditions.