Lab 2

Introduction Last week we investigated 1-D continuous flows using a population model. This week we will use population models as examples of 2-D continuous flows. In order to get a two dimensional model we must study two populations at once and to get some interesting behavior we will assume that the two populations interact in some way (otherwise we would just get two unrelated populations each of which would behave in exactly the same way as the example we studied last week). In Exercises 1 and 2 we will use both STELLA and MAPLE to study the dynamics of two populations that are competing for the same food supply. For example, a population of rabbits and a population of sheep competing for grass in the same meadow. In Exercise 3 we will study two populations that have a predator-prey relation, i.e. lions and gazelles.

Exercise 1. Rabbits and Sheep We first need to set up a model in STELLA that represents the situation of the two populations competing over a fixed food supply. If they didn't compete we would just draw two separate versions of last weeks model, one for the rabbits and one for the sheep. This is where we start, see Fig.1. Now we need to draw connections that reflect the fact that they are competing. It stands to reason that if there are too many rabbits that the sheeps food supply will be threatened and their death rate will increase. The same would be true if the roles were reversed. Accordingly, we connect Rabbits to death rate 2 and Sheep to death rate 1 as shown in Fig.2.

Set the initial populations of Rabbits and Sheep to similar values, i.e. both somewhere near 1. Make a graph of Rabbits and Sheep on the same plot and with the same scale. Print this graph for your portfolio. Change your parameter values and see if the shapes of the graphs change. Do both populations survive as $t \to \infty$? Can you find values for the initial populations such that (i) the rabbits survive and the sheep do not (ii) the sheep survive and the rabbits do not.

Exercise 2. The following is a set of differential equations describing the dynamics of our two competing populations where $x(t)$ stands for the number of rabbits at time t and $y(t)$ stands for the number of sheep.

$$\dot{x} = x(3 - x - 2y) \tag{1}$$
$$\dot{y} = y(2 - x - y) \tag{2}$$

We will use MAPLE to plot the corresponding vector field. Open MAPLE in our class folder and enter the work sheet labelled "Competing Populations" exactly as it appears in Fig.3. Graph one of your vector fields for your portfolio. By looking at the vector field and plotting different initial conditions see if you can find answers to the questions posed at the end of Exercise 1.

The following set of differential equations provides a simple model of two populations with a predator-prey relationship like Lions $L(t)$ and Gazelles $G(t)$.

$$\dot{G} = G(1 - L) \quad (3)$$
$$\dot{L} = L(G - 1) \quad (4)$$

As in Exercise 2, enter the work sheet labelled "Predator and Prey" exactly as it appears in Fig.4. Graph one of your vector fields for your portfolio. What kind of behavior do you see? Does this reasonably model the situation at hand?