

# Math 145

# Chaos Theory

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# Complex Dynamical Systems

Most mathematical models in the sciences.

Chief characteristic features:

Chaotic behavior

Bifurcations

Indeterminacy

# Meeting #IT, April 4

- Chaos and indeterminacy
- Course organization
- Begin with ID iterations

# Chaos and Indeterminancy

- Two butterfly effects
  - Sensitivity
  - Bifurcations

# Steve Smale, 1996

Chaos is a new science which establishes the omnipresence of unpredictability as a fundamental feature of common experience.

A belief in determinism, that the present state of the world determines the future precisely, dominated scientific thinking for two centuries. This credo was based on certain laws of physics, Newton's equations of motion, which describe the trajectories in time of states of nature. These equations have the mathematical property that the initial condition determines the solution for all time. Thus lies the mathematical and physical foundation for deterministic philosophy. One manifestation of determinism was the rejection of free will and hence even of human responsibility.



# Steve Smale, 1996

In the 1970s the scientific community recognized another revolution, called the theory of chaos, which deals a death blow to the Newtonian picture of determinism. As a consequence, the world now knows that one must deal with unpredictability in understanding common experience. The coin-flipping syndrome is pervasive. "Sensitive dependence on initial conditions" has become a catchword of modern science.

# Course organization

- Class times
- Lab assignments
- Grading

# Class Times

- Meetings will begin at 9:50 am, and end by 11:25 am
- Meetings will usually begin with calling the roll or sometimes a quiz



# Lab assignments

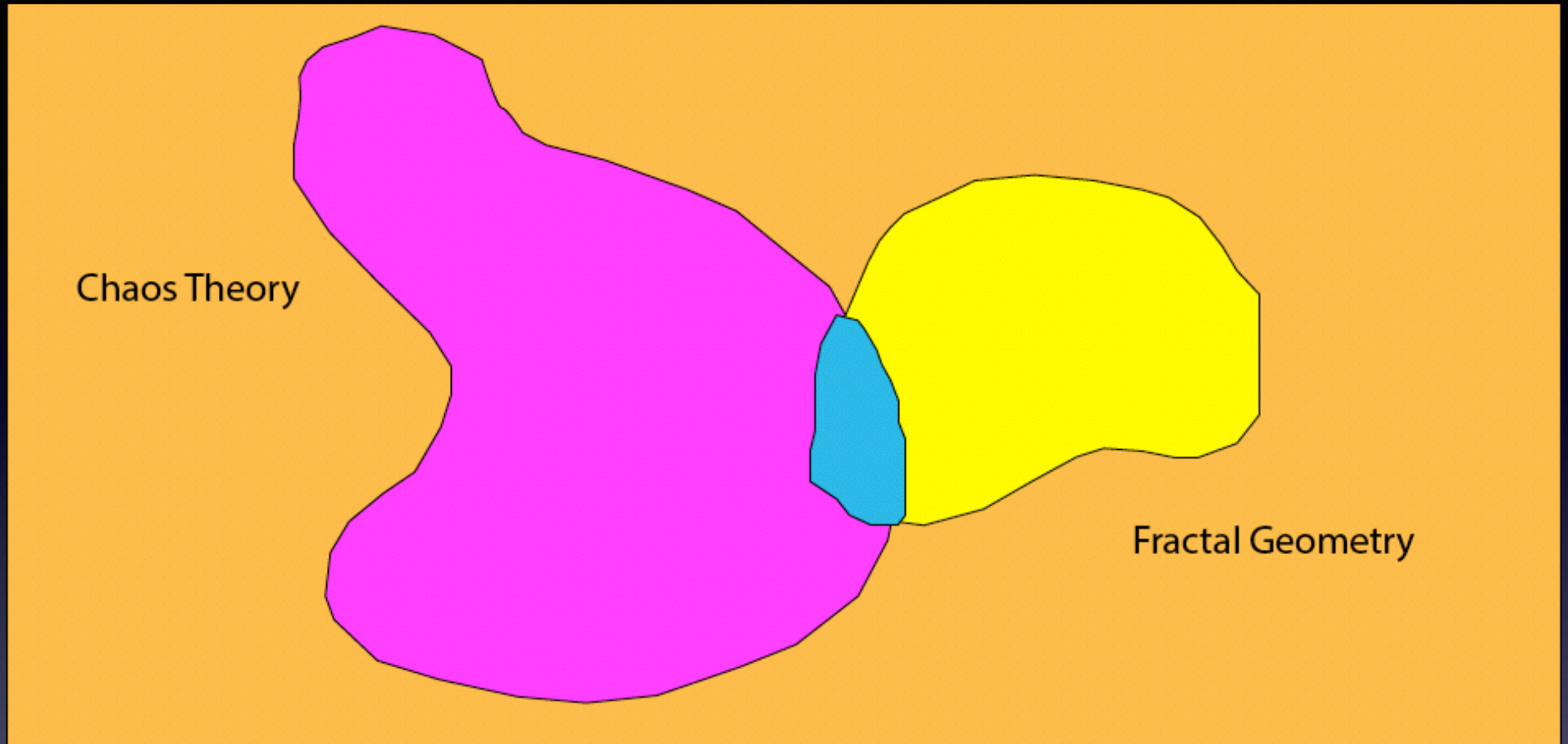
- Weekly lab assignments will be posted online every Friday morning, and due the following Friday at 9 am.
- Lab results must be posted online on-time to be scored.
- Math software: NetLogo and p5.js
- Web software: HTML, CSS, JavaScript

# Grading

- Pop quizzes
- Lab/homework posted on personal websites
- Final exam

# Chaos Theory

- Chaos and fractals
- Stairway to chaos
- 1D experiments



# Chaos and Fractals

**Dimension**

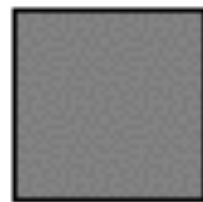
**0**

**1**

**2**

**3**

**Flows**



**Cascades**



**Iterations**



**The Stairway to Chaos**



# Chronology

- 1946, Macy Conferences (cybernetics)
- 1957, Beverton and Holt (1D iterations)
- 1958, Myrberg (2D iterations)
- 1960, Smale horseshoe (2D cascades)
- 1961, Ueda attractor (3D flows)
- 1966, Thom catastrophe theory (1D flows)
- 1972, IIASA (complex dynamical systems)
- 1974, Li and Yorke, May (the chaos revolution)

Math 145 Spring 2017 Meeting #17

On to ID Experiments