Math 145
Chaos Theory

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Meeting #2T, April 11

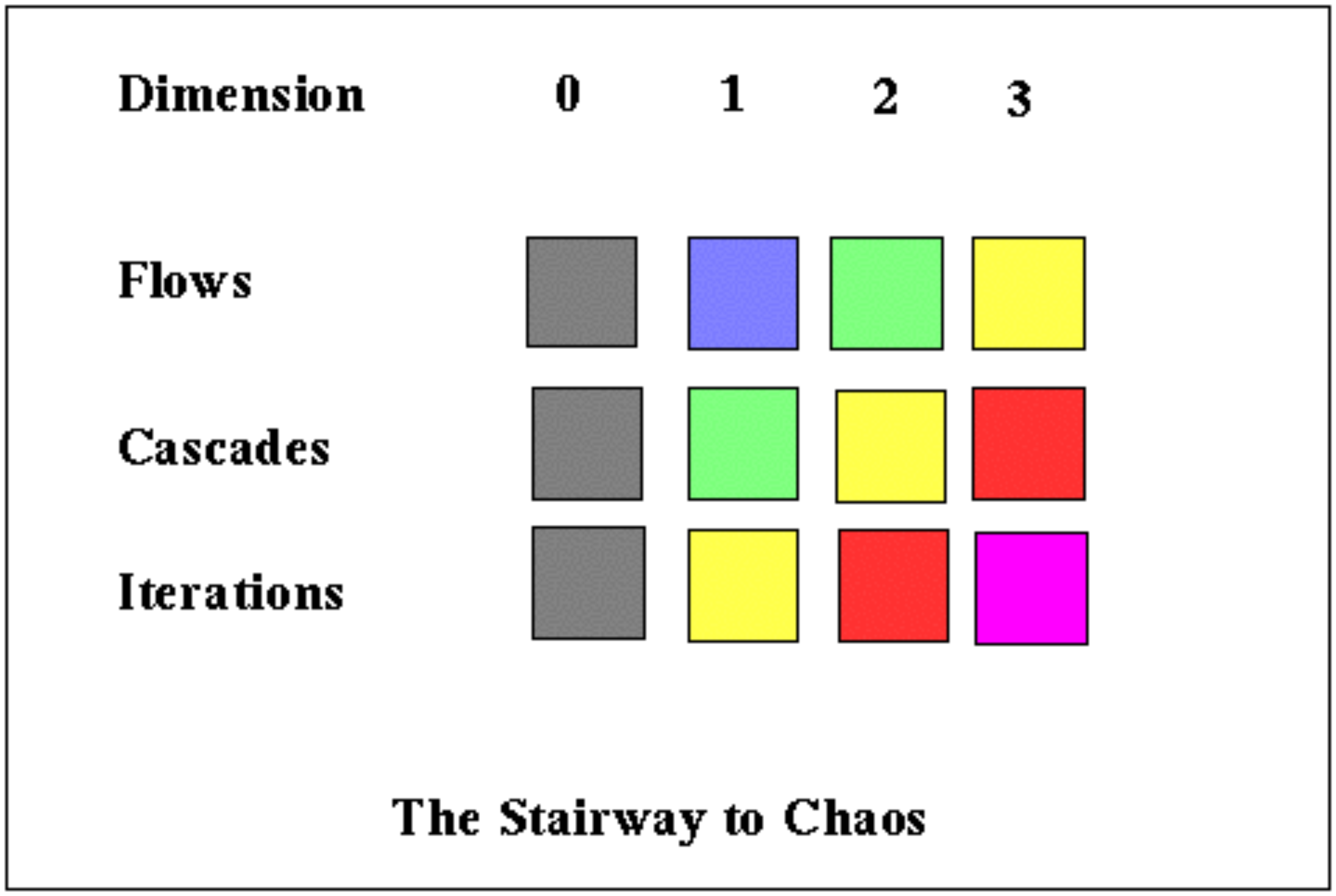
- Notes
- 2D Iterations
- The Mira scheme
- The Dorband scheme

Notes

- Set theory and logic: in Math 100 (FWS)
- Point set topology: in Math 124 (F)
- Euclid: in Math 125A (F)
- Final project (rather than exam)

2D Iterations

- Definitions
  - State spaces
  - Generators


Dimension
0
1
2
3
Flows
Cascades
Iterations
The Stairway to Chaos

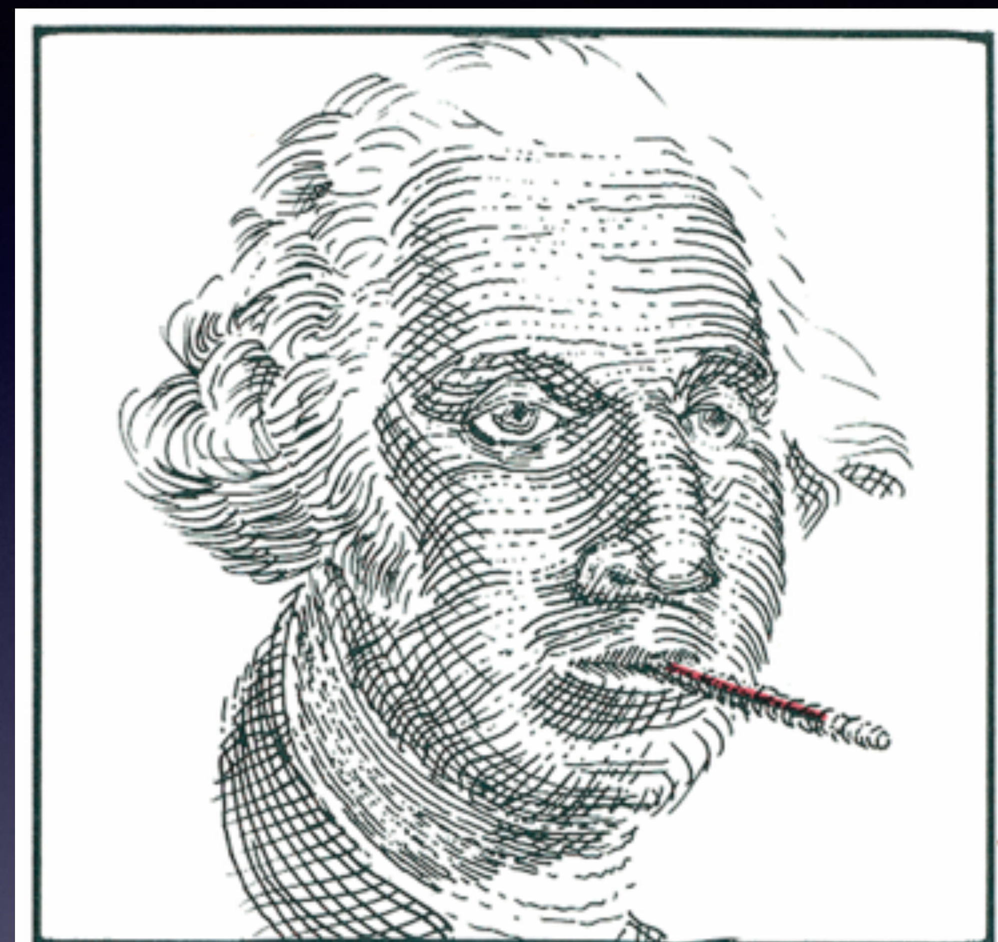
State Spaces

- Differentiable manifolds
- Dimension 1
- Dimension 2

Differentiable Manifolds

- Approximate definition:
  - Locally Euclidean with smooth glue
  - Each component has a unique dimension

Dimension One

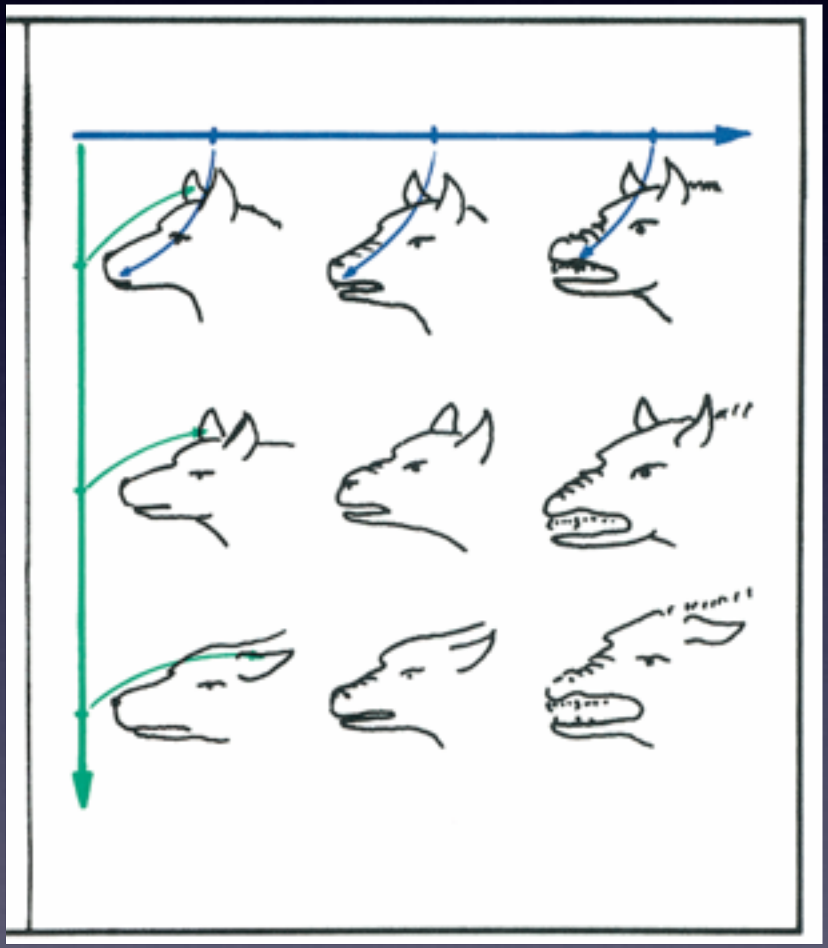


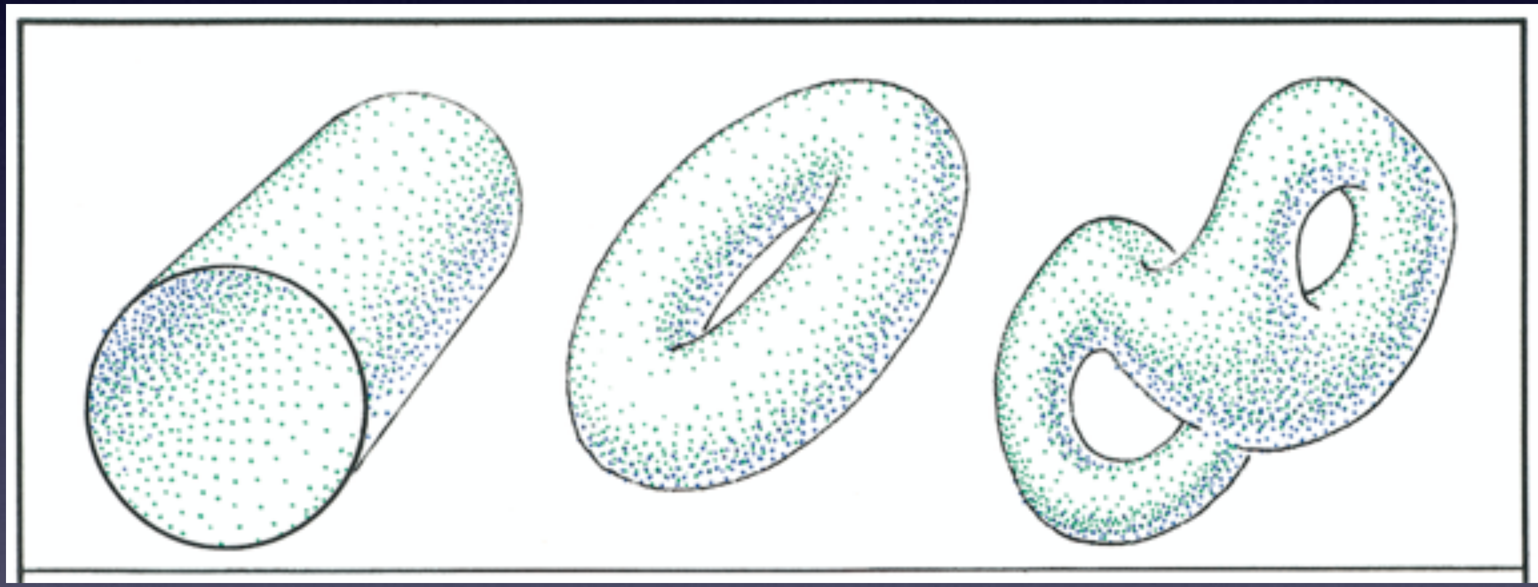
TEMPERATURE

1.1.3.

In these examples,the geometric model for the set of all (mathematically idealized) states is the real number line. This is one of the simplest state spaces.

Dimension Two





Generators

- Three cases (vectorfield, diffeo, endo=map)
- Map, dimension 1 (eg, logistic, beverton)
- Map, dimension 2 (eg, Mira, Dorband)

Planar Endomorphisms

- Given by expressions such as:
  - $(x, y) \longrightarrow (u(x, y), v(x, y))$
- For example, the Mira scheme:
  - $u = ax + y$ (eg, $a = -0.7$)
  - $v = b + x^2$ (eg, $b = 1.0$)
- Or, the Dorband scheme:
  - $u = (1 - c)x + 4cy(1 - y)$
  - $v = (1 - c)y + 4cx(1 - x)$ (eg, $c = 0.6$)

Mira, 4-16

Fractal Notes

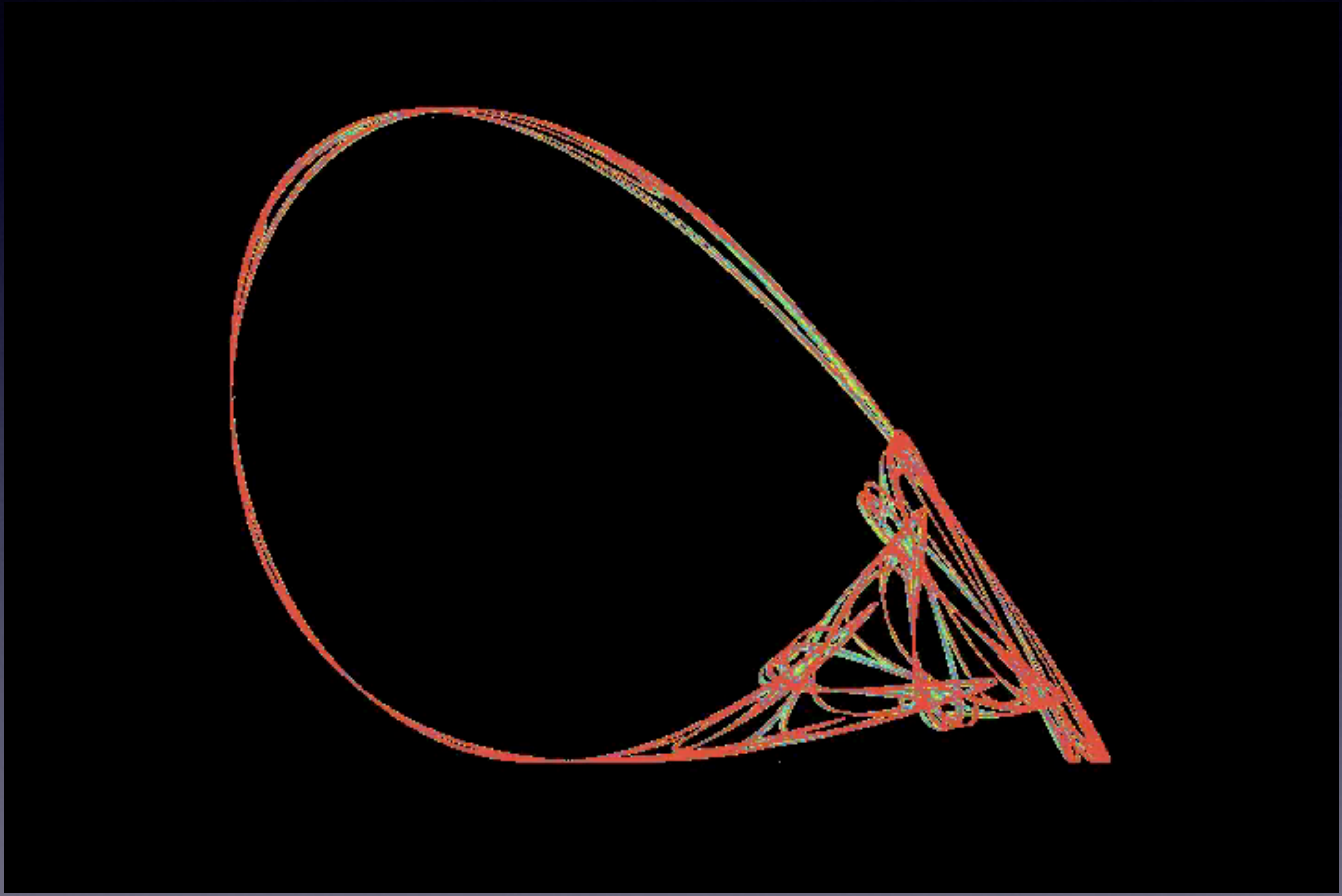
Chaotic Attractors in 2D and 3D
A Project of the Visual Math Institute

Color Graphics by Ralph Abraham and Hiroko Tojo
http://www.vismath.org/fractal-notes/

Recreation of a chaotic attractor defined by the iteration of a polynomial endomorphism.
From the book -- Chaos in Discrete Dynamical Systems -- by Abraham, Gardini & Mira -- 1997.
See: http://www.visual-chaos.org/jpx/book/

Kawakami-Kobayashi-Mira Family (Long Series, MD)
Ch. 4, Bifurcations, a = 0.7, b down from -0.4 to -1.0
in 600 steps, see JPX pp. 50-58

MIra, 5-06



Mira, 6-28

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Kawakami-Kobayashi-Mira Family (Long Series, MD)
Ch. 6 Bifurcations, a = -1.5, b down from -1.98 to -2.0
in 600 steps, see JPX pp. 104-111

Dorband, 7-29

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Dorband-Gardini Family (Long Series, MD)
Ch. 7, Bifurcations, b from 0.6 to 1
in 400 steps, see JPX p. 137

Dorband, 3D


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On to 2D Experiments