

Math 145

Chaos Theory

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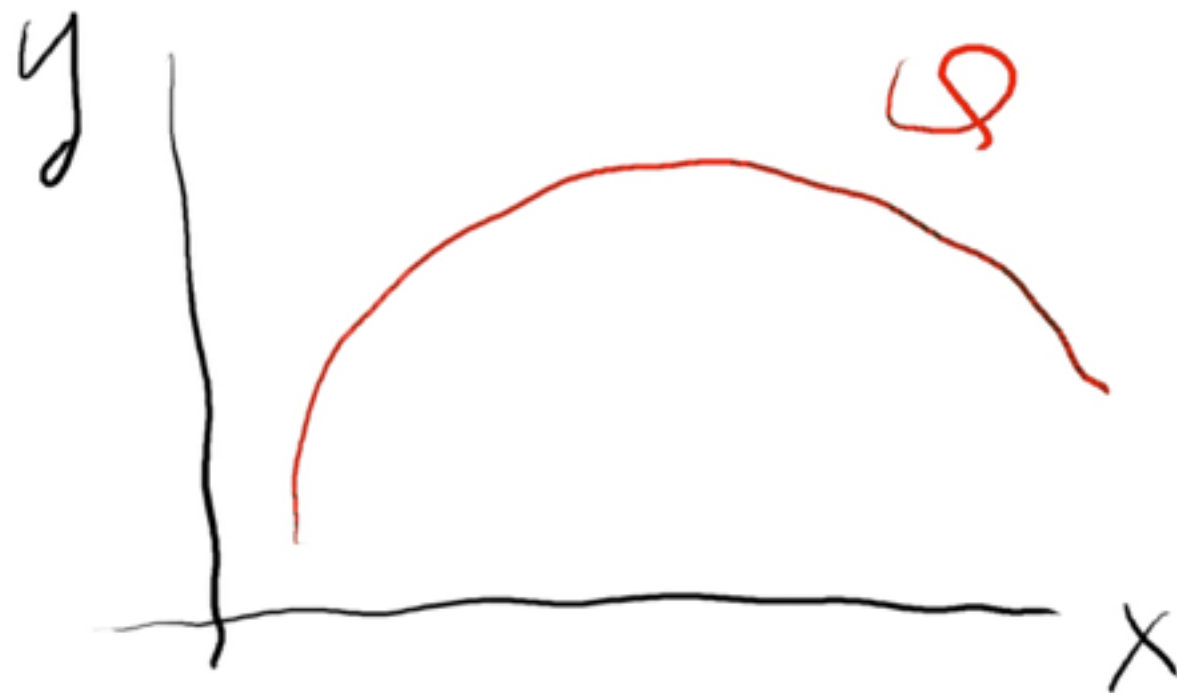
Meeting #3T, April 18

- 1D Flows
 - A. Gradient systems
 - B. The fold bifurcation
 - C. The cusp bifurcation
- p5.js

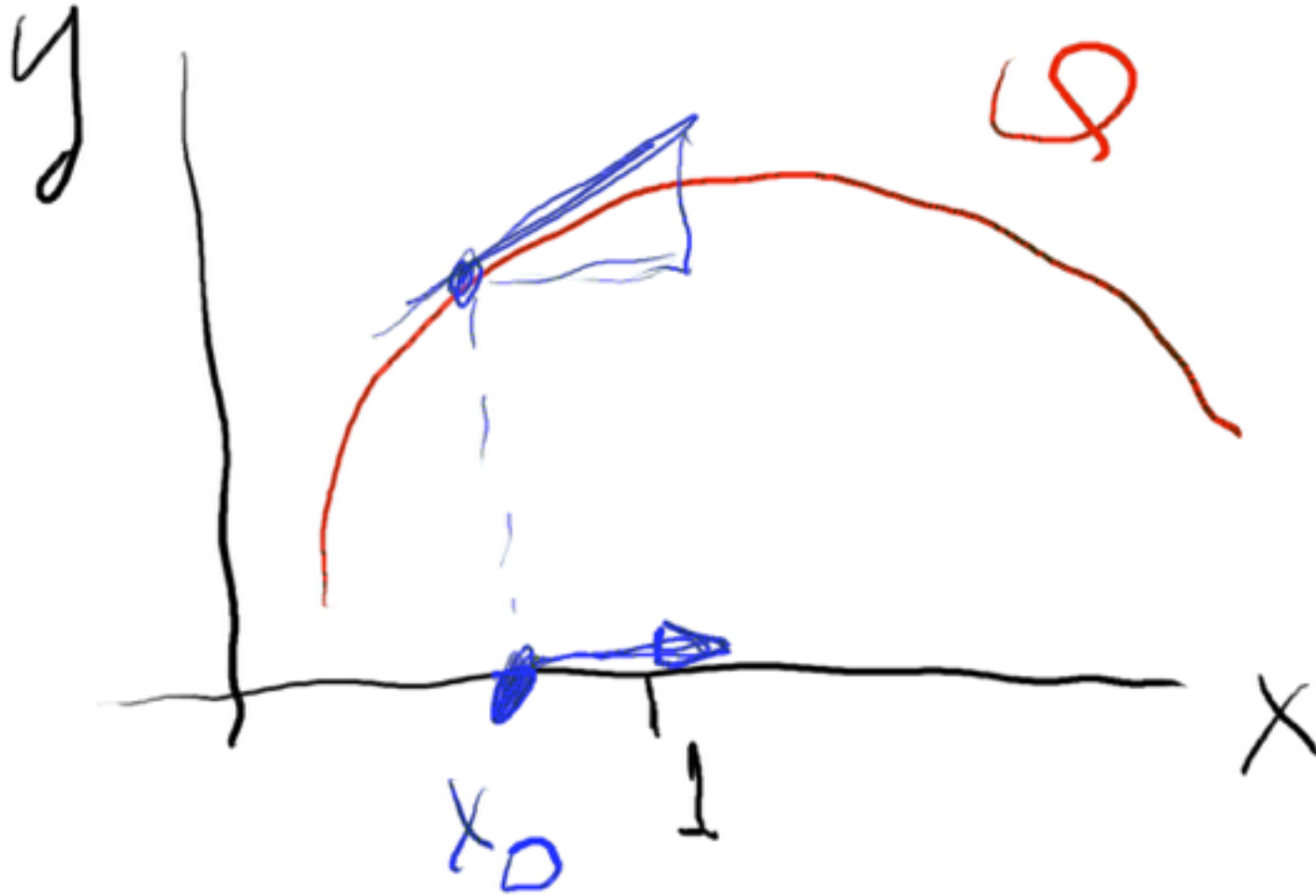
A. Gradient Systems

- Quadratic potential
- Cubic potential
- Quartic potential

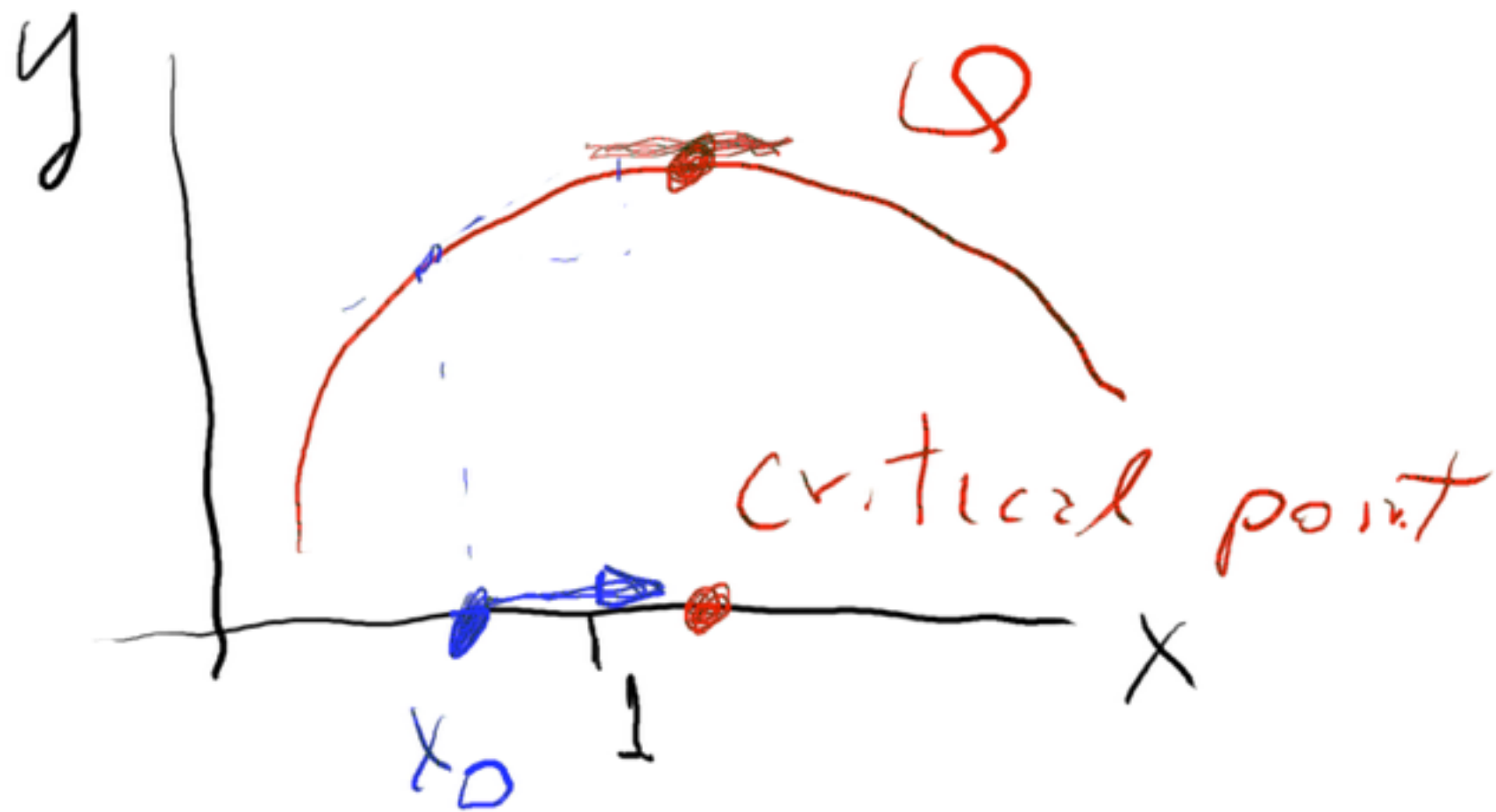
Quadratic potential



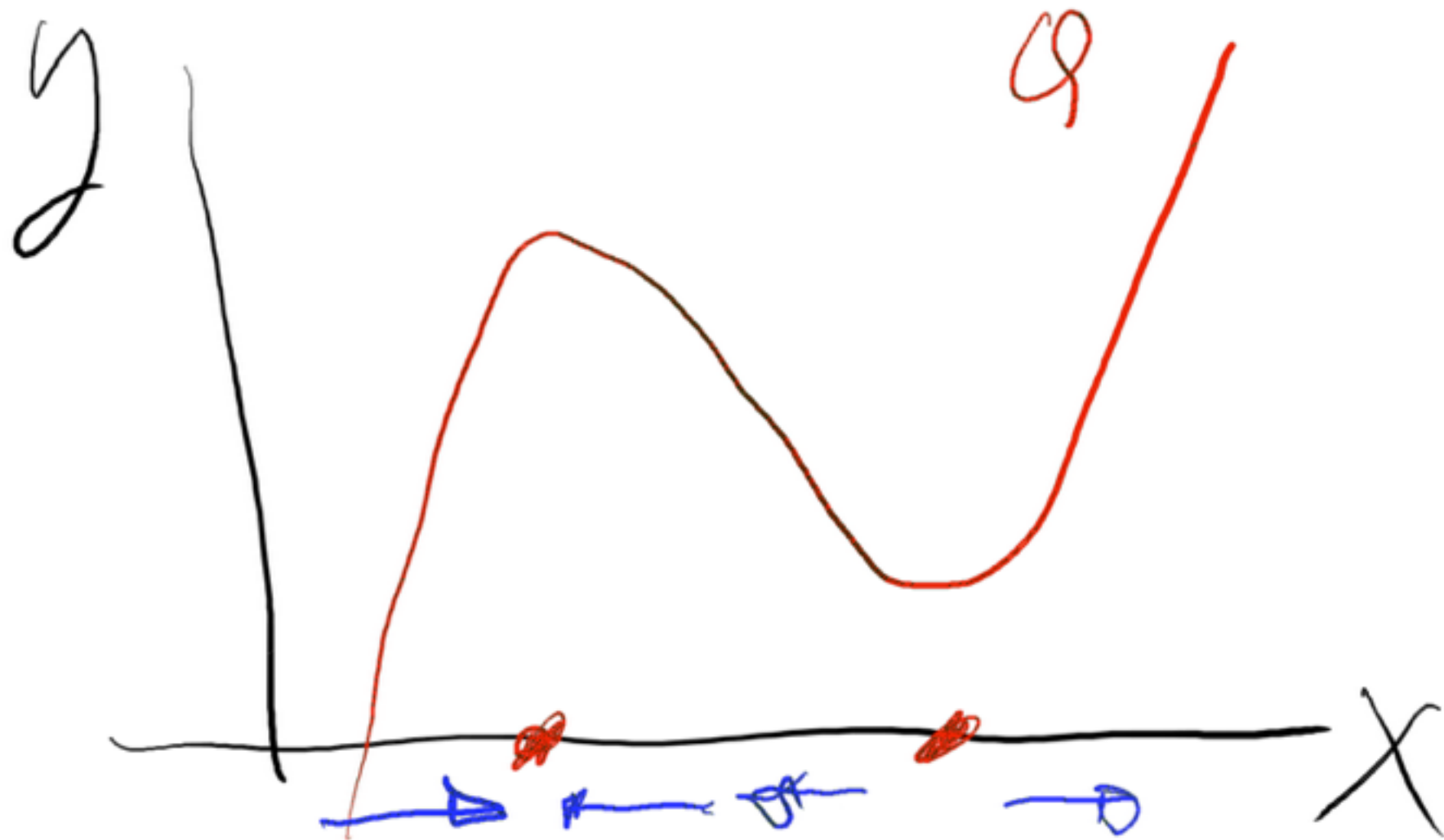
Gradient vector



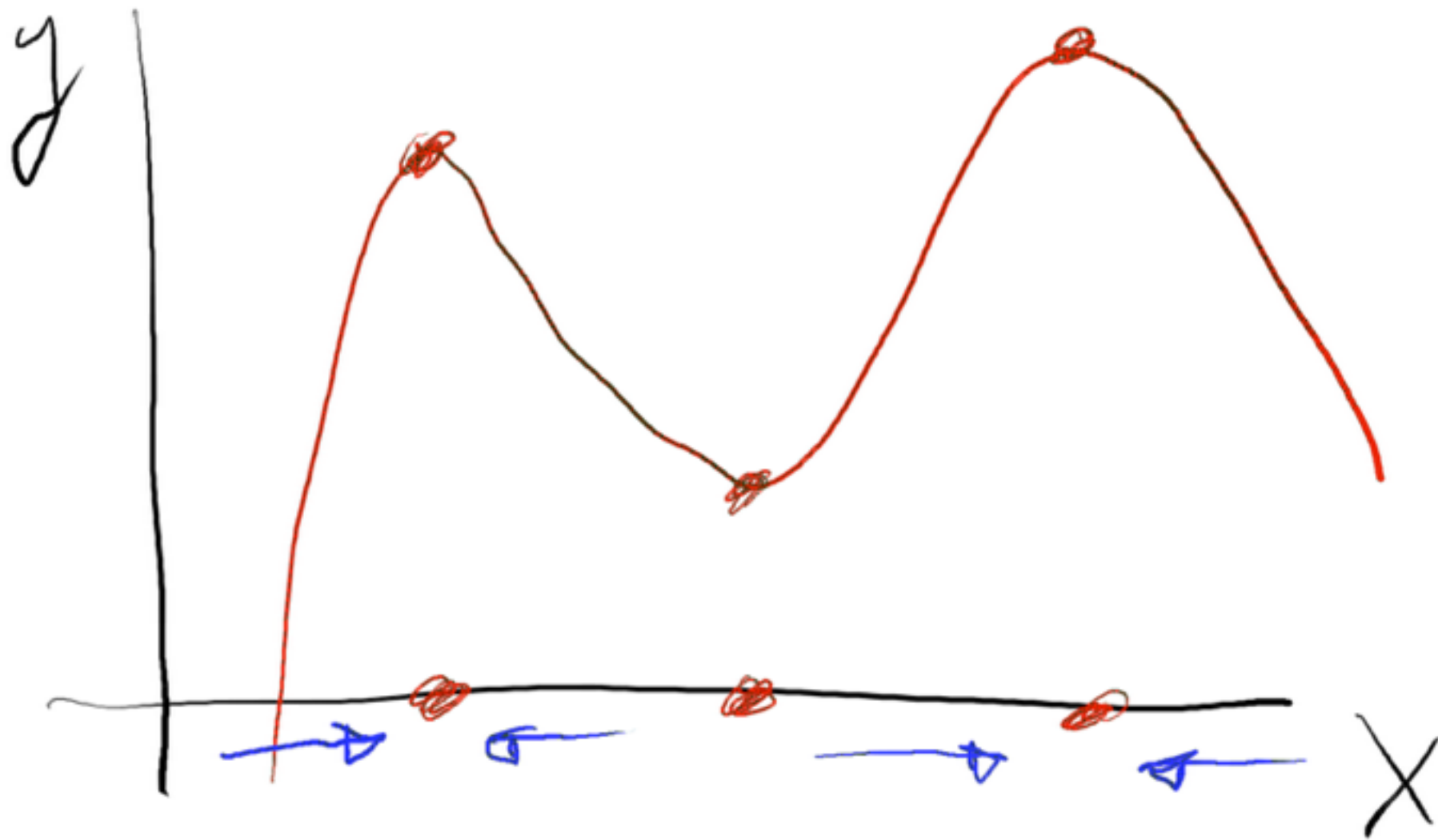
Critical point



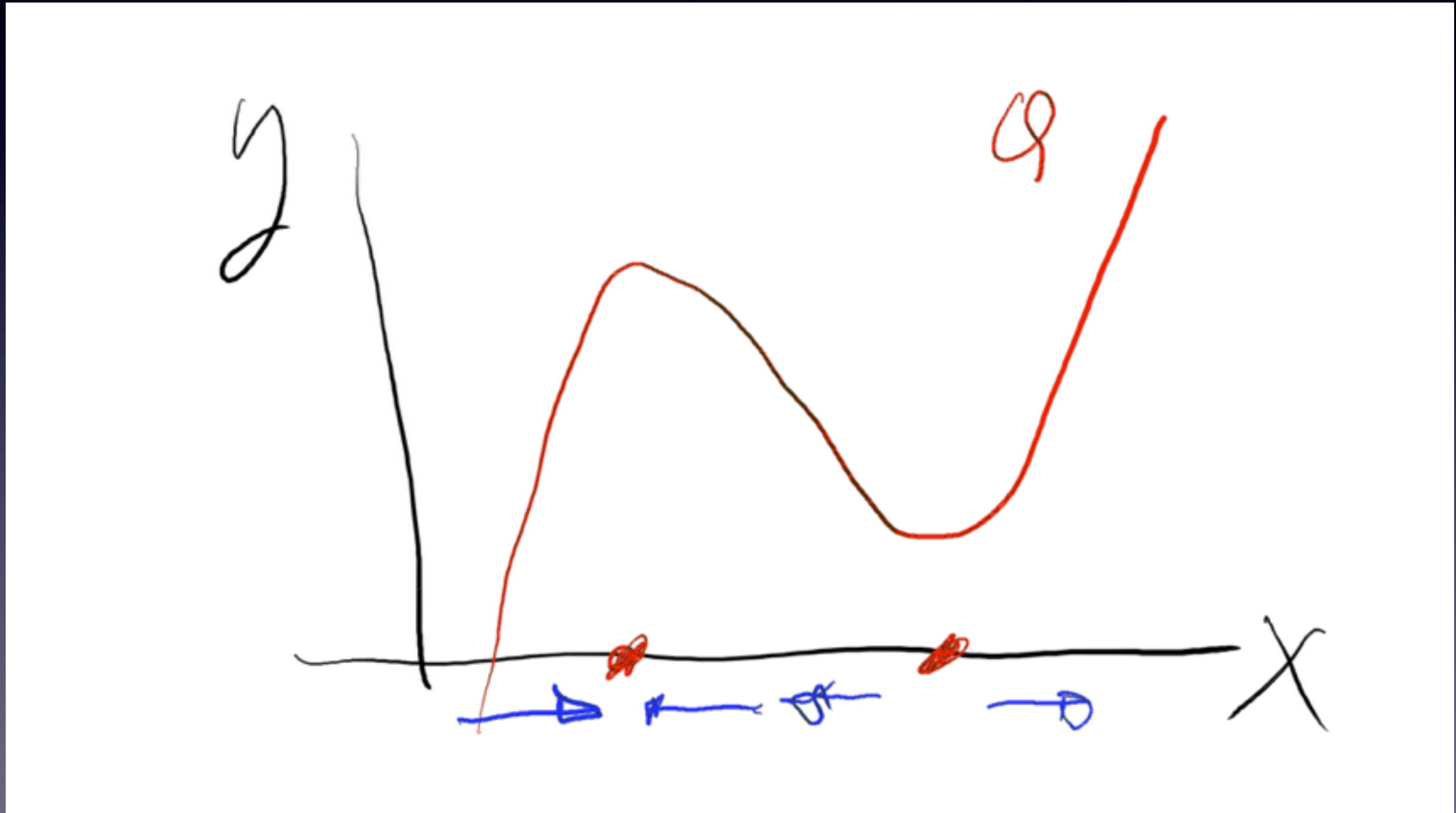
Cubic potential



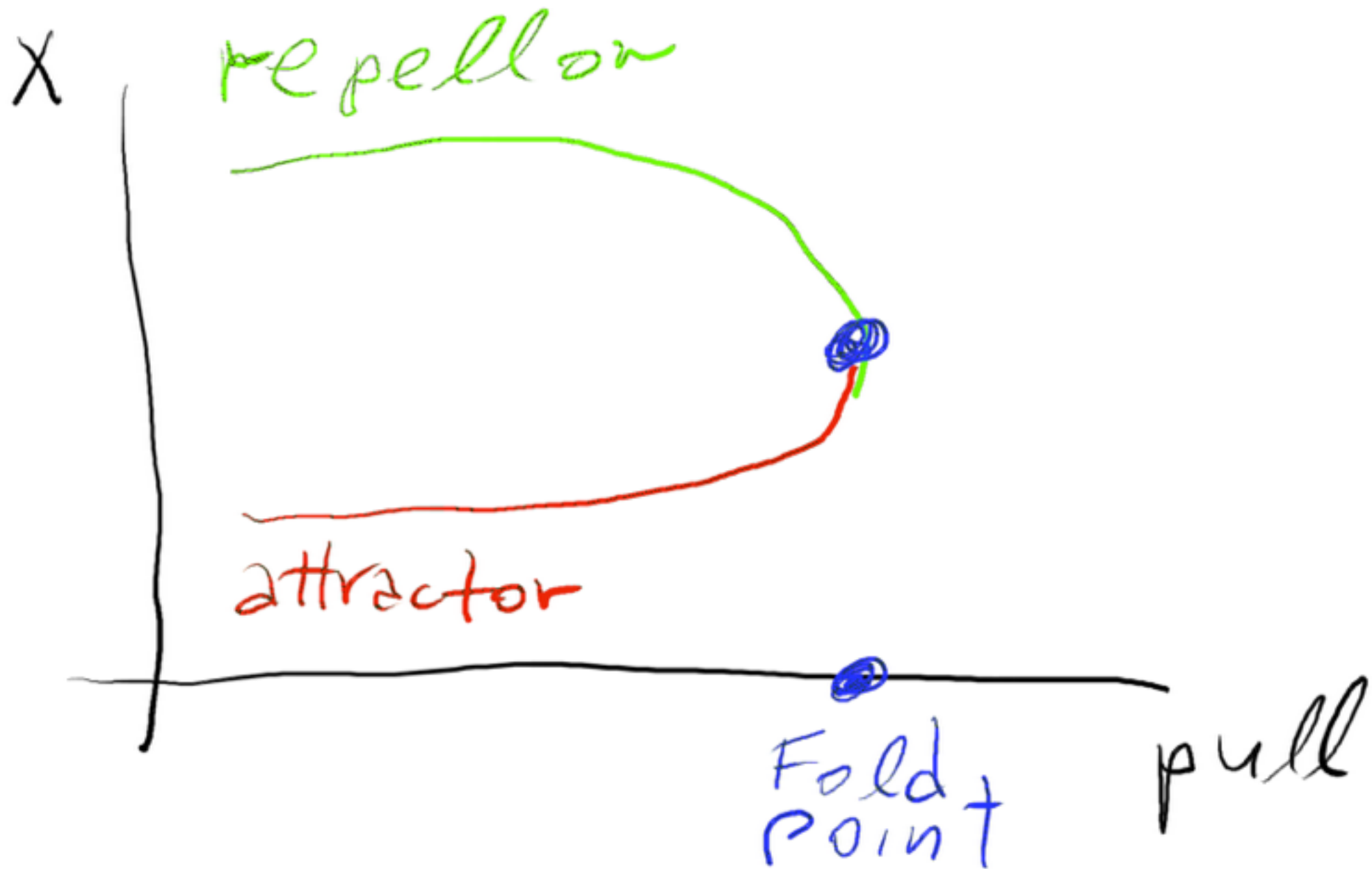
Quartic potential



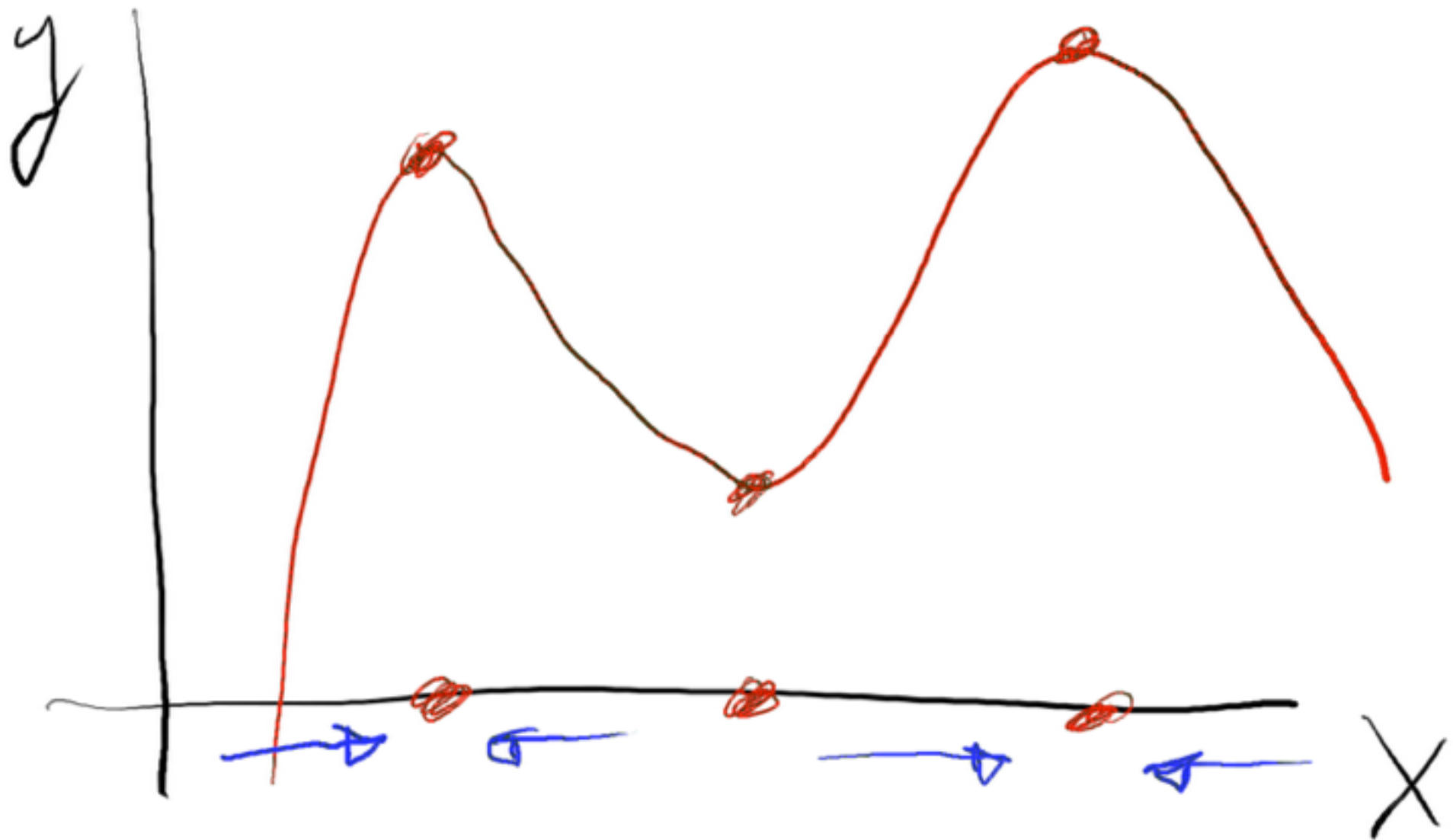
B. The fold bifurcation



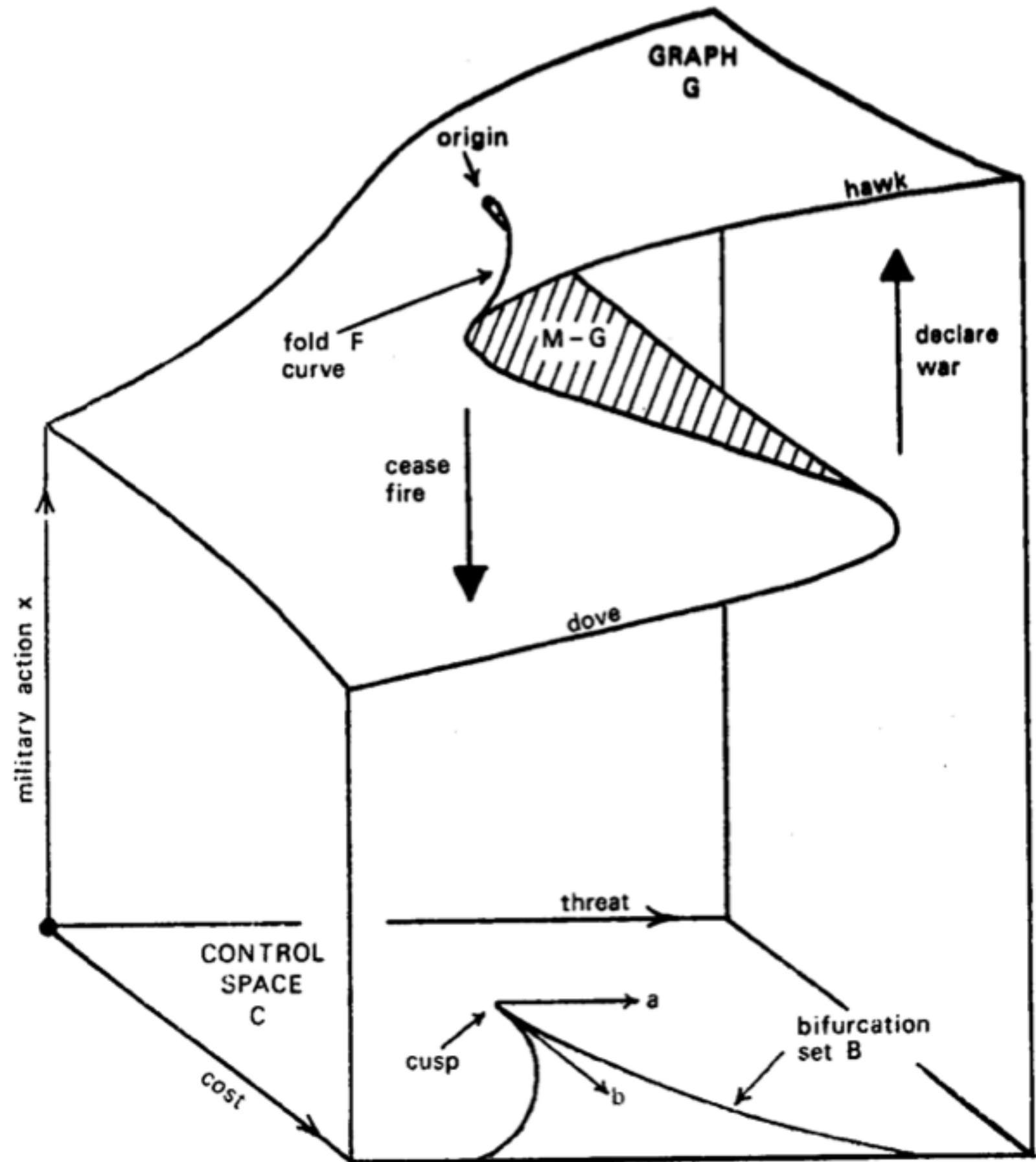
Response diagram of the fold



B. The cusp bifurcation



Response diagram of the cusp



The equation for the bifurcation set, B in the (x, y) plane

With $a, b \in R$, let

$$\phi(x) = -\frac{1}{4}x^4 + ax + \frac{1}{2}bx^2$$

Then the critical points of ϕ are the roots of, $\phi_x = 0$. or

$$x^3 = a + bx$$

Applications of the cusp

- Aggression, war
- Stock market crashes
- Anorexia nervosa
- Heartbeat, nerve firing
- Opinion formation
- Prison riots

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On to p5.js