Math 145
Chaos Theory

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Meeting #6T, May 9

- Neuroscience applications
- Cybernetics
- McCulloch and Pitts
- Neuron models
- Phase synchrony

Cybernetics

- 1 Macy Cerebral Inhibition Conference
  - Planning, May, 1942
- 10 Macy Cybernetics Conferences
  - March and October, 1946, 1947
  - March, 1948 — 1953

Planning Meeting, May 1942

- Participants
  - Warren McCulloch, Neuro, Chair
  - Gregory Bateson, Social Sciences
  - Margaret Mead, Anthropology
  - Arturo Rosenblueth, Physiology
  - Lawrence Kubie, Psychoanalysis
  - [Frank and Fremont-Smith, Macy]

References

- Norbert Wiener, Cybernetics, 1948
- Steve Heims, The Cybernetic Group, 1991

Warren McCulloch, 1898-1969
Walter Pitts, 1923-1969

- 1943, "A logical calculus of the ideas immanent in nervous activity,"
- The MCP Neuron (the beginning of Artificial Neural Network theory)
- Basis of artificial intelligence, cybernetics

McCulloch and Pitts

- 1941, Pitts (age 18) was a student (along with Jerome Lettvin) of McCulloch at Univ Illinois Neuropsychiatric Institute
- 1943, Showed an ANN could emulate a Turing machine (universal computer, 1937)
- 1952, joined Wiener at MIT
- Students included Mandelbrot, Minsky and Papert, Kaufmann

Neuron Models

- Hodgkin — Huxley, 1952, 4D flow
- Fitzhugh — Nagumo, 1961, 2D flow
- Hindmarsh — Rose, 1984, 3D flow

Hodgkin — Huxley

- 1938, Giant squid axon (1 mm diameter)
- 1939, First recording of action potential
- 1952, Electronic circuit model
- 1963, Nobel Prize

Fitzhugh — Nagumo

The equations for this dynamical system read

$$\dot{v} = v - \frac{v^3}{3} - w + I_{\text{ext}}$$

$$\tau \dot{w} = v + a - bw.$$

v = membrane voltage
w = gate voltage

Hindmarsh — Rose

$$\frac{dx}{dt} = y + 3x^2 - x^3 - z + I$$

$$\frac{dy}{dt} = 1 - 5x^2 - y$$

$$\frac{dz}{dt} = r \cdot (4(x + \tfrac{8}{5}) - z)$$

x = membrane voltage

y =fast ion transport

z = slow ion transport

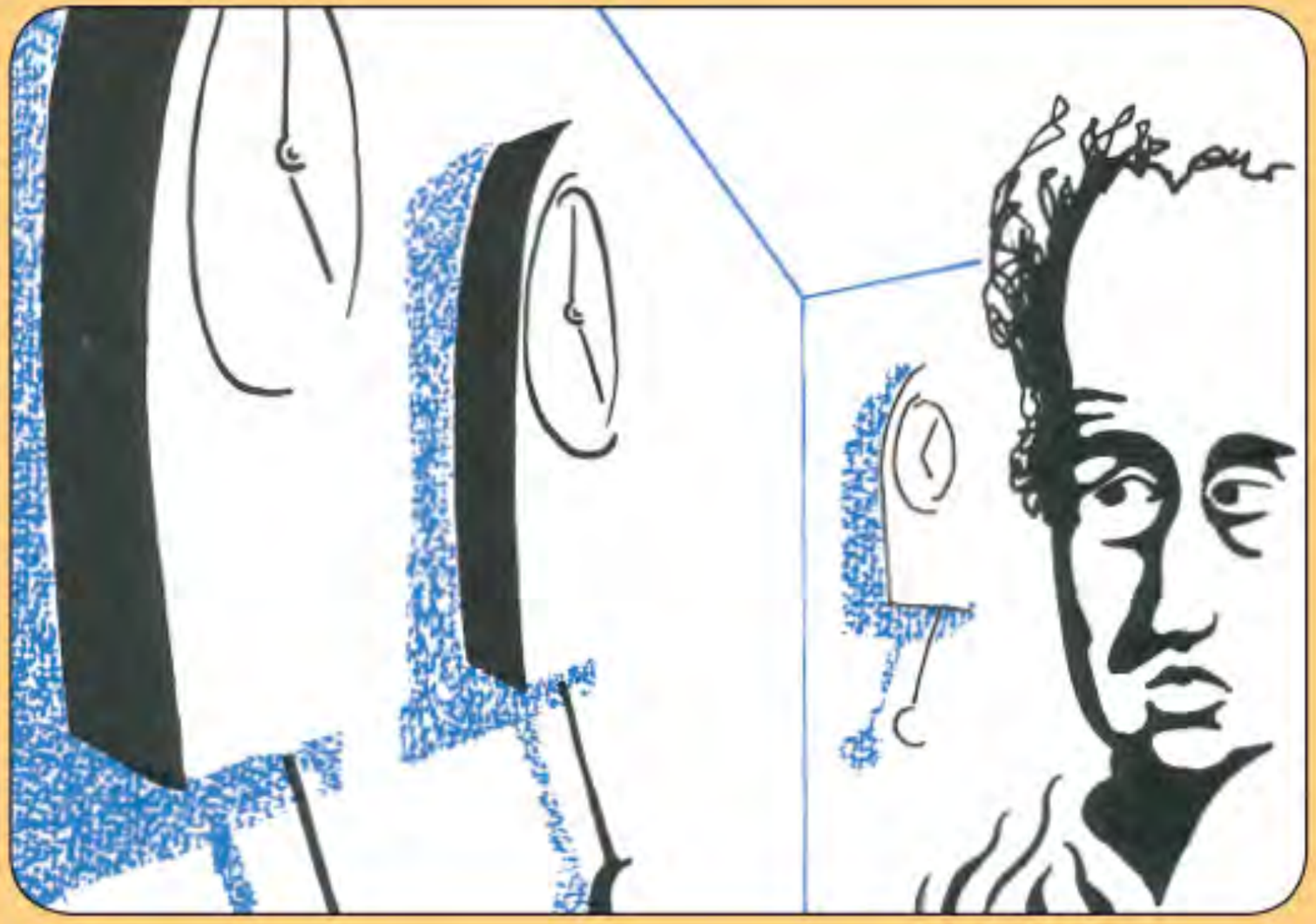
REFS

- Wikipedia
- MS## 28-29, 35-38, 40a, 48, 57, 111, 140, 141

Sympathy (Entrainment)

- 1612, Galileo
- 1658, Huyghens
- 1981, Vassalo-Pereira
- 1984, Hayashi

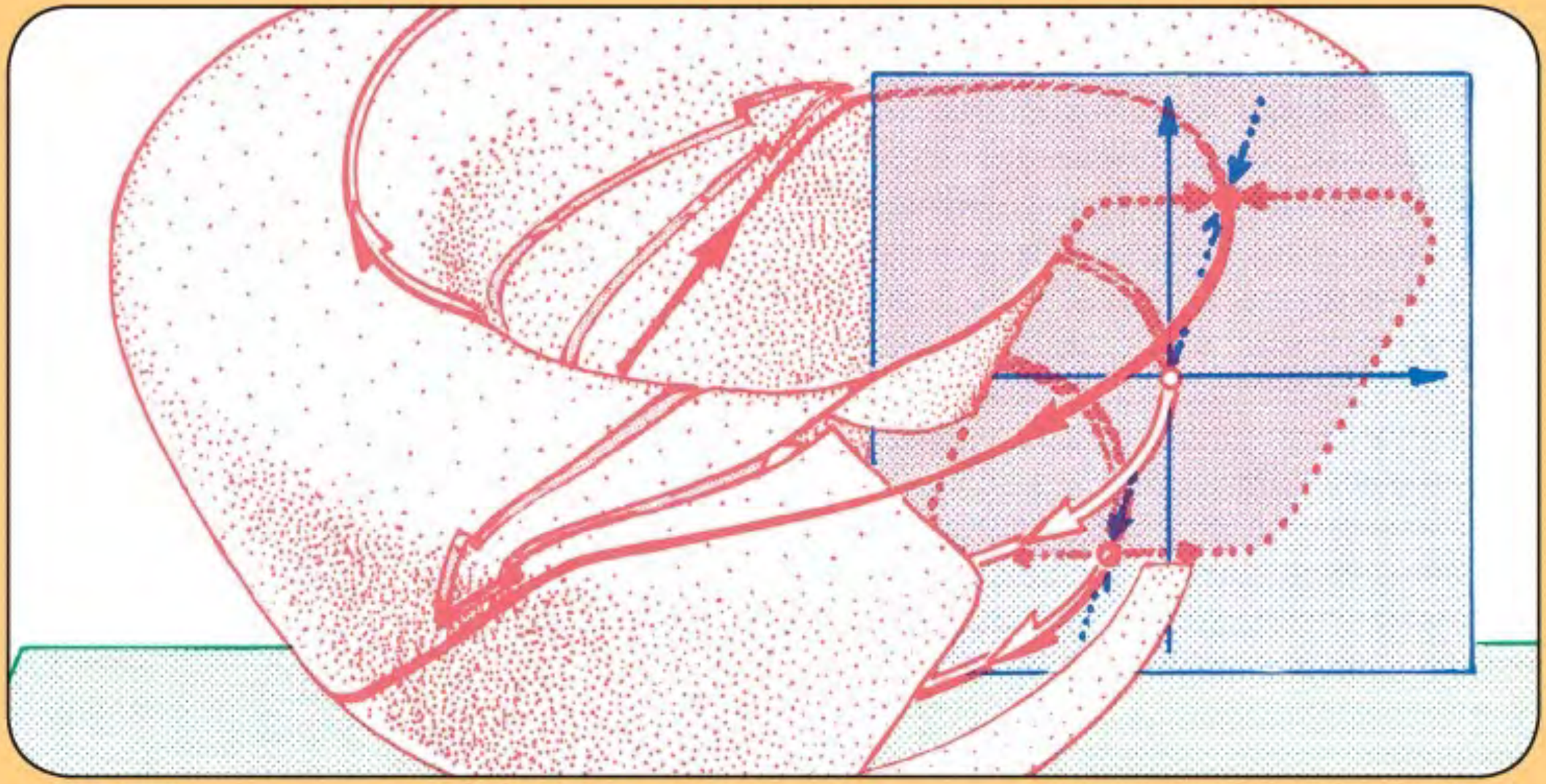
Huyghens



5.2.1.
An example is provided by coupling two clocks. This particular system was observed by Christiaan Huyghens, an outstanding dynamicist of the seventeenth century. He noticed that two clocks hung on the same wall tend to synchronize and suspected that this *entrainment* phenomenon was caused by nonlinear coupling through the elasticity of the wall. The full explanation of entraininent, a recent result in dynamical systems theory, is due to Peixoto and is described in Part Two.

Huyghens
(recreation)

Structural Stability

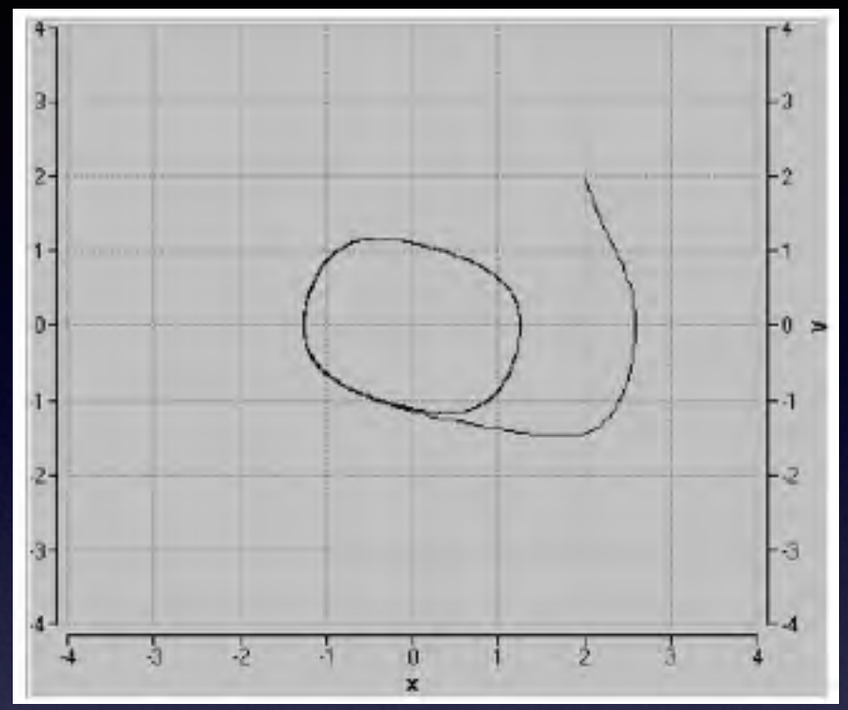


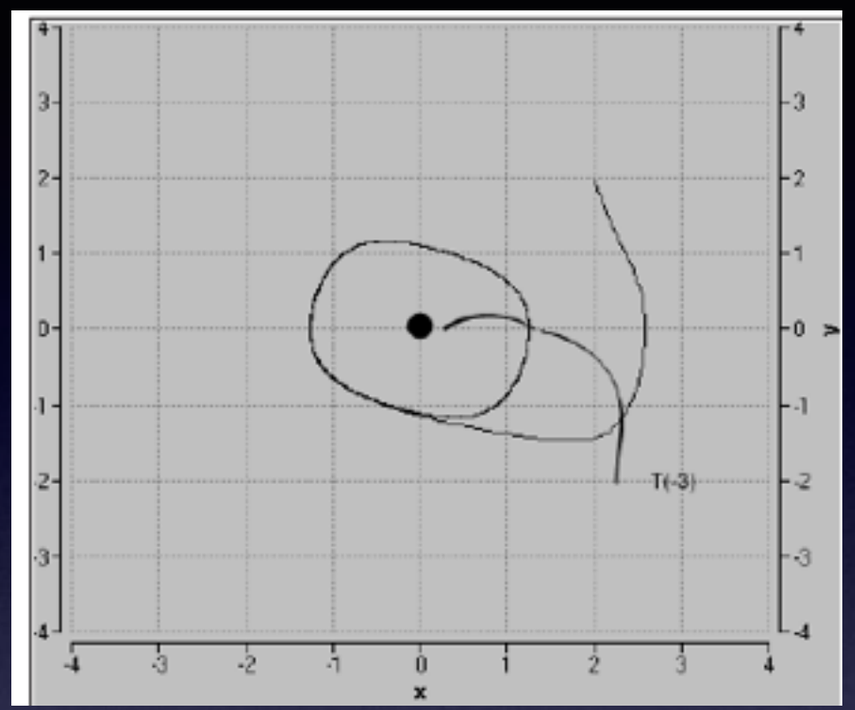
5.6.12.
But we expect this, after the explanations of the preceding section. The *structural stability* of the coupled phase portrait implies *frequency entrainment* of the coupled oscillators.

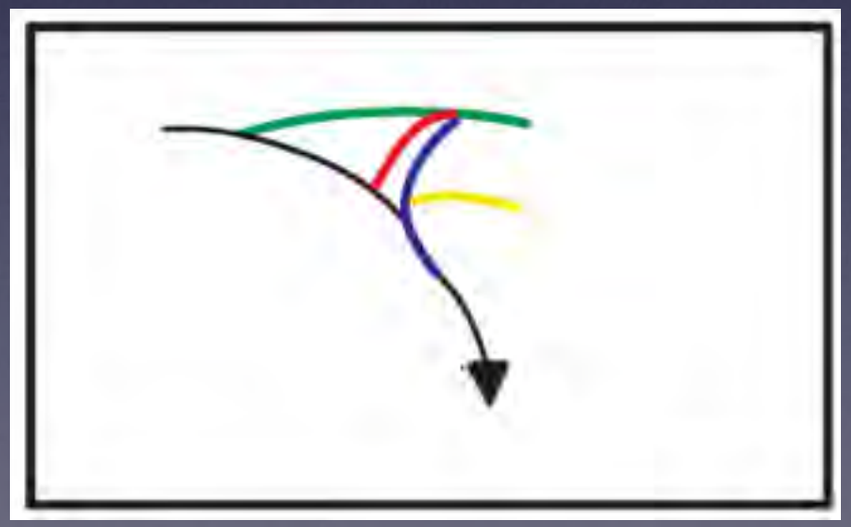
Pulsatile Forcing

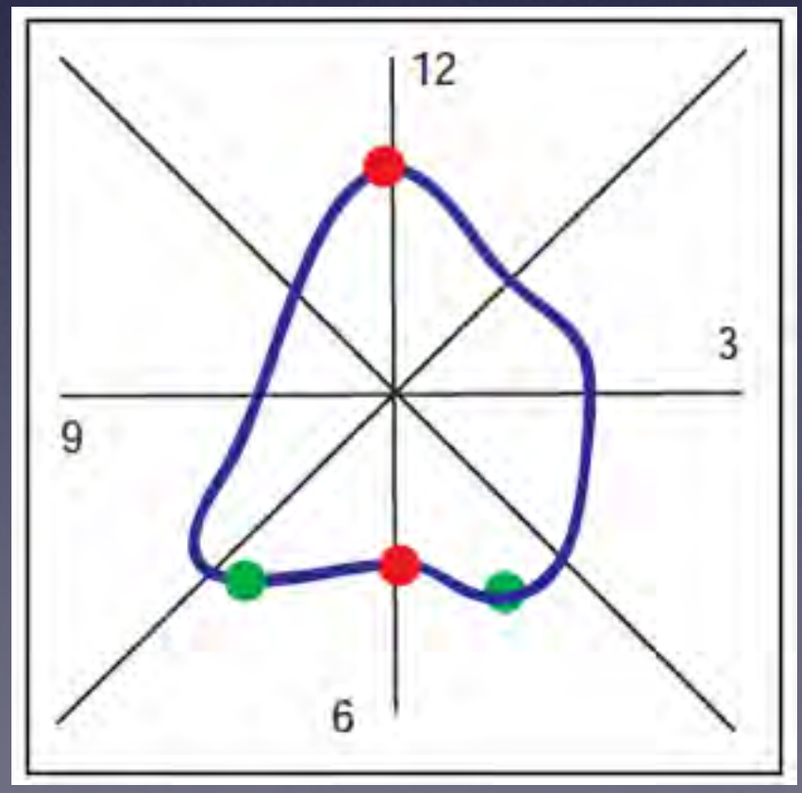
- A generic oscillator in 2D
- The pendulum clock (Vassalo-Pereira)
- The Hodgkin-Huxley neuron (MS#111)

Generic Oscillator in 2D

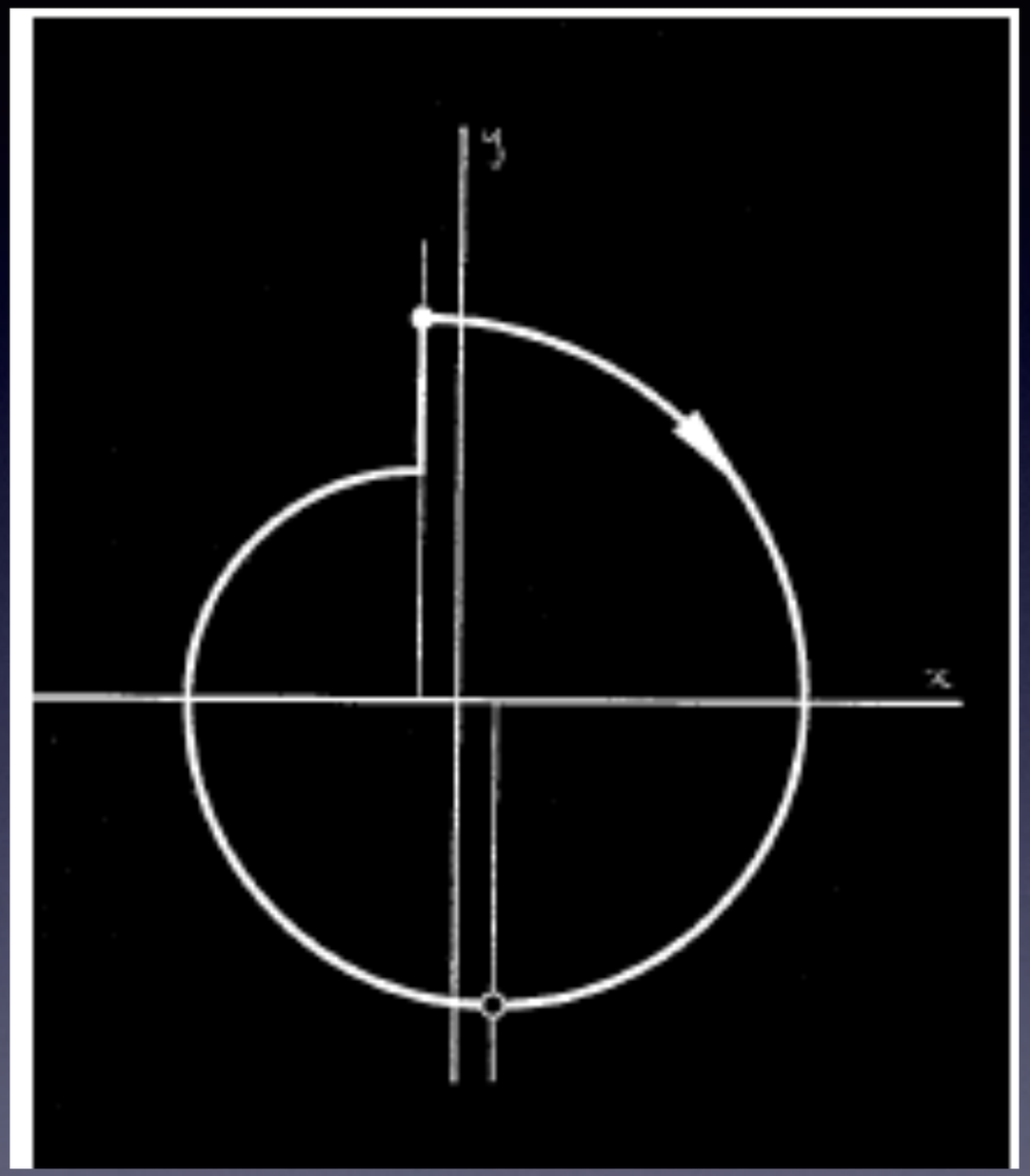




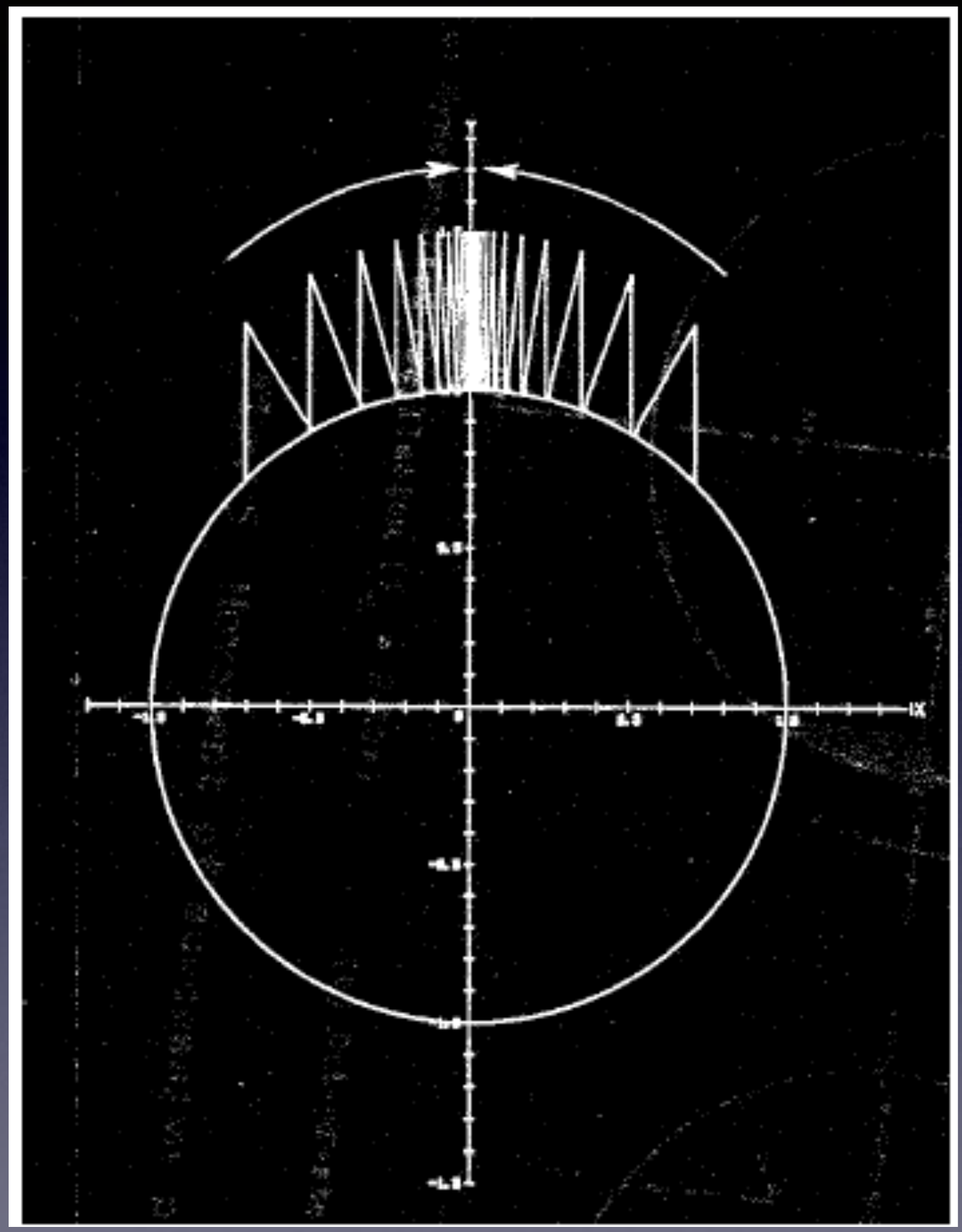




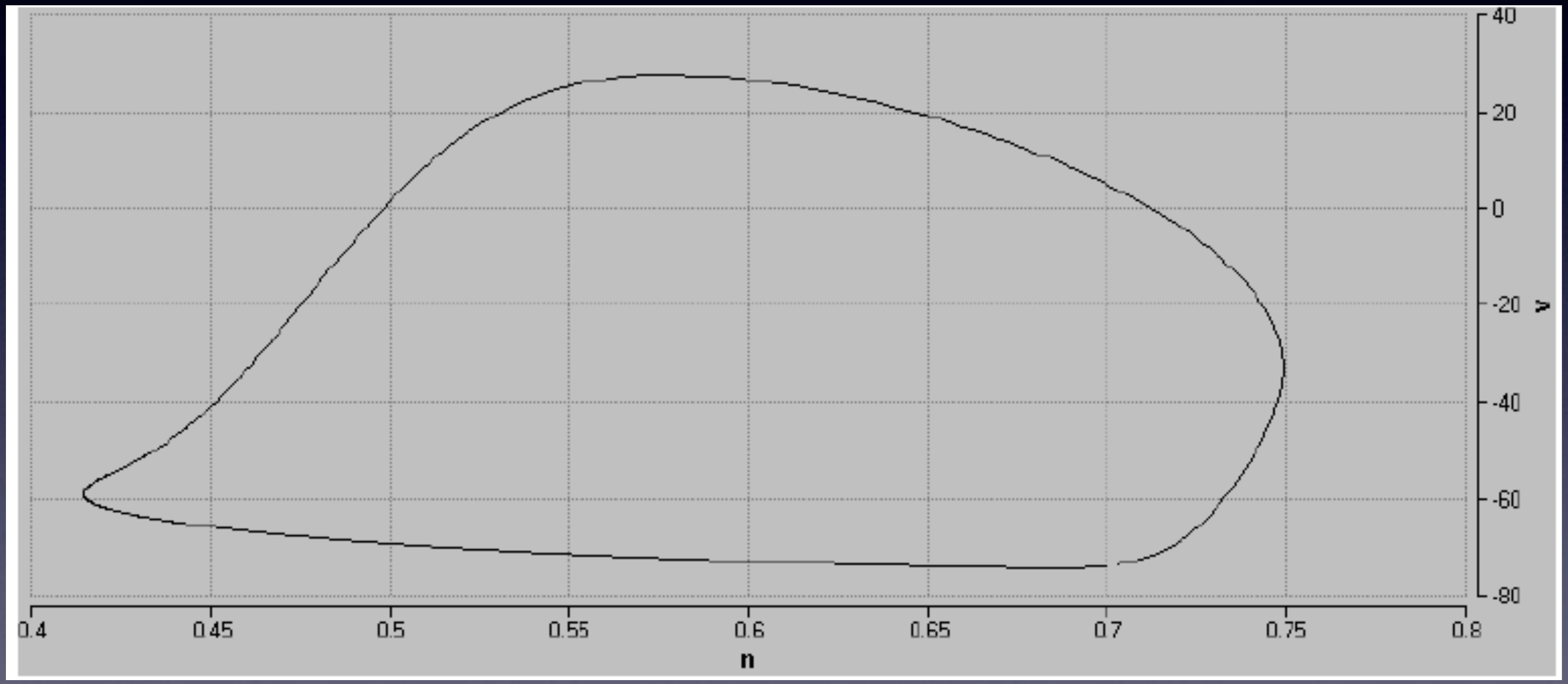
Vassalo Perreira



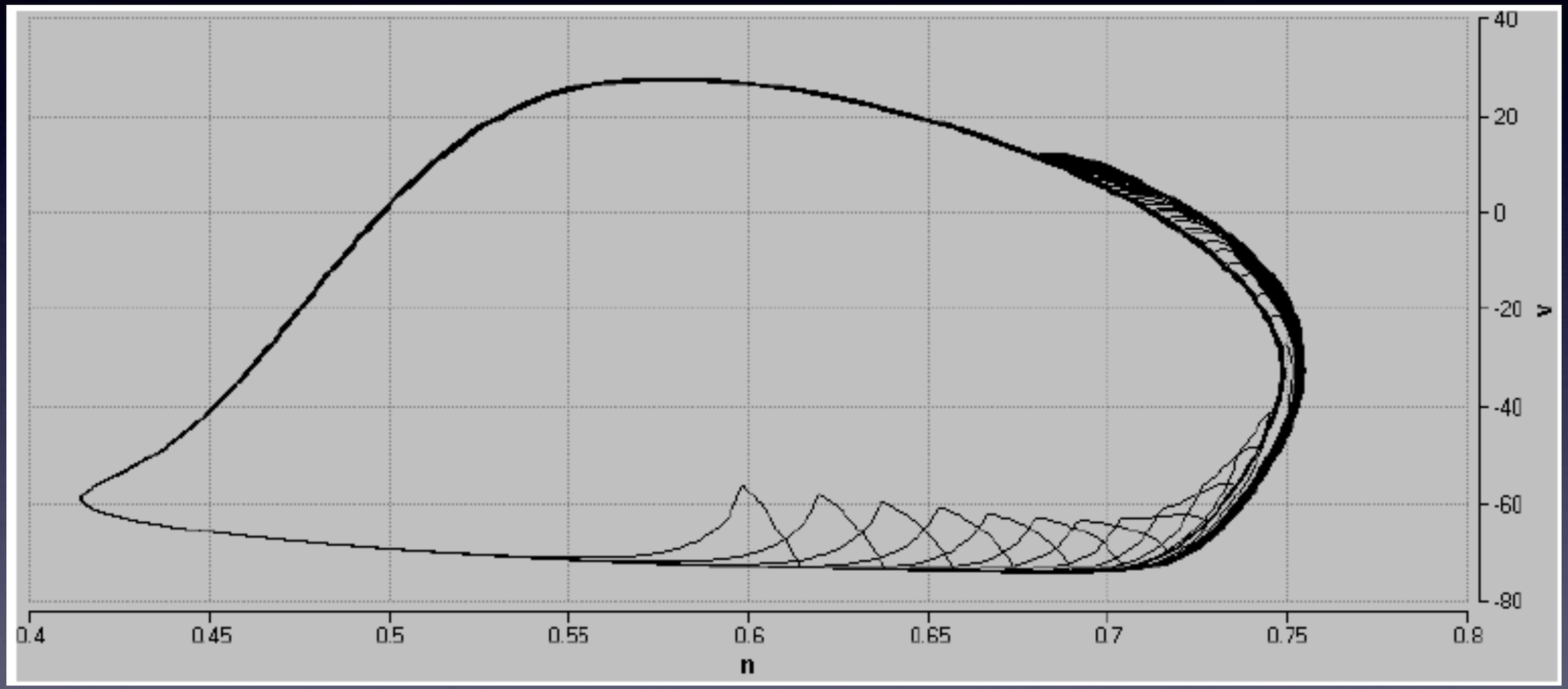
Hayashi's Ideal Case



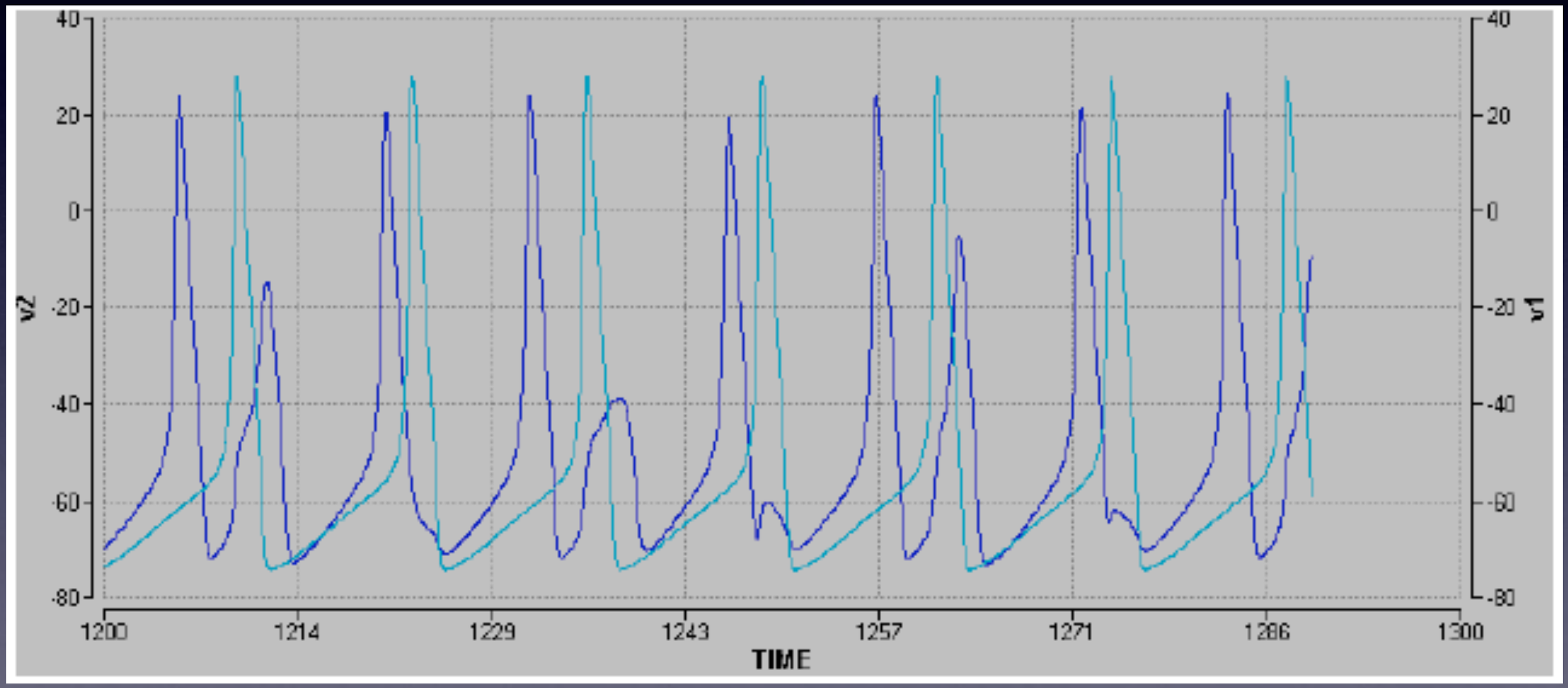
The Hodgkin-Huxley Neuron



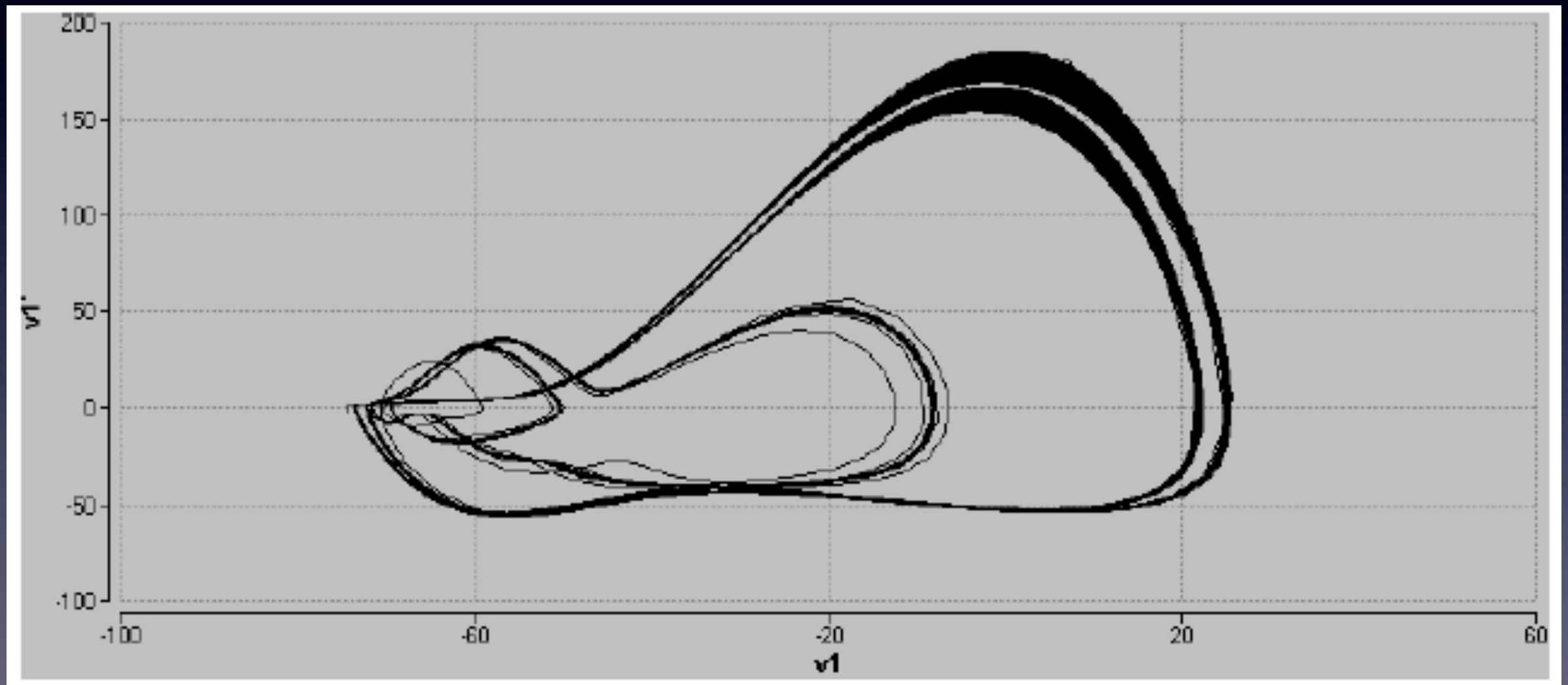
Hayashi Plot for NN Neuron



Two Coupled HH neurons



Stronger Coupling



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