Math 145
Chaos Theory

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- Business Cycles from m02b
  - 2. Business Cycles of Kaldor
- New: Gardini's historic paper of 1992

Application 2
Business Cycles
Kaldor

Following H-W Lorenz, 1993

Kaldor Model, 1940

As difference equations,
Y = income, K = capital stock

$$Y^+ = Y + \Delta Y$$
$$K^+ = K + \Delta K$$

Business Cycles
Kaldor Model, 1940

General Kaldor model,
I = investment, S = savings

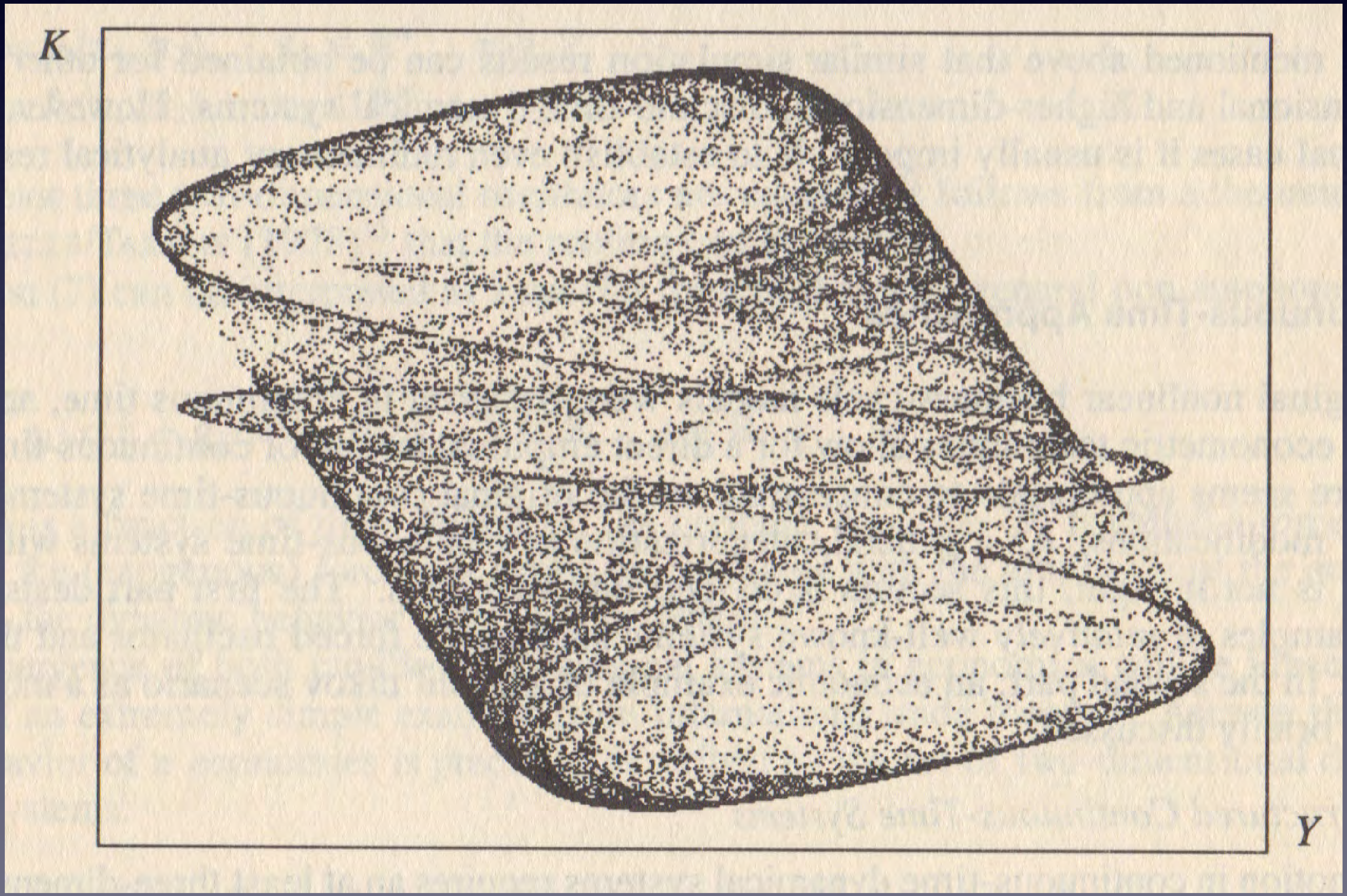
$$\begin{aligned}\Delta Y &= \alpha(I - S) \\ \Delta K &= I - \delta K\end{aligned}$$

Business Cycles
Kaldor Model, 1940

Herrmann form, 1985

$$
\begin{aligned}
\Delta Y &= \alpha(\beta(kY - K) + \delta K + C(Y) - Y) \\
\Delta K &= \beta(kY - K)
\end{aligned}
$$

Business Chaos
Herrmann Model, 1985



New Application Business Cycles

Following Gardini, 1992

Gallegati Model, 1992

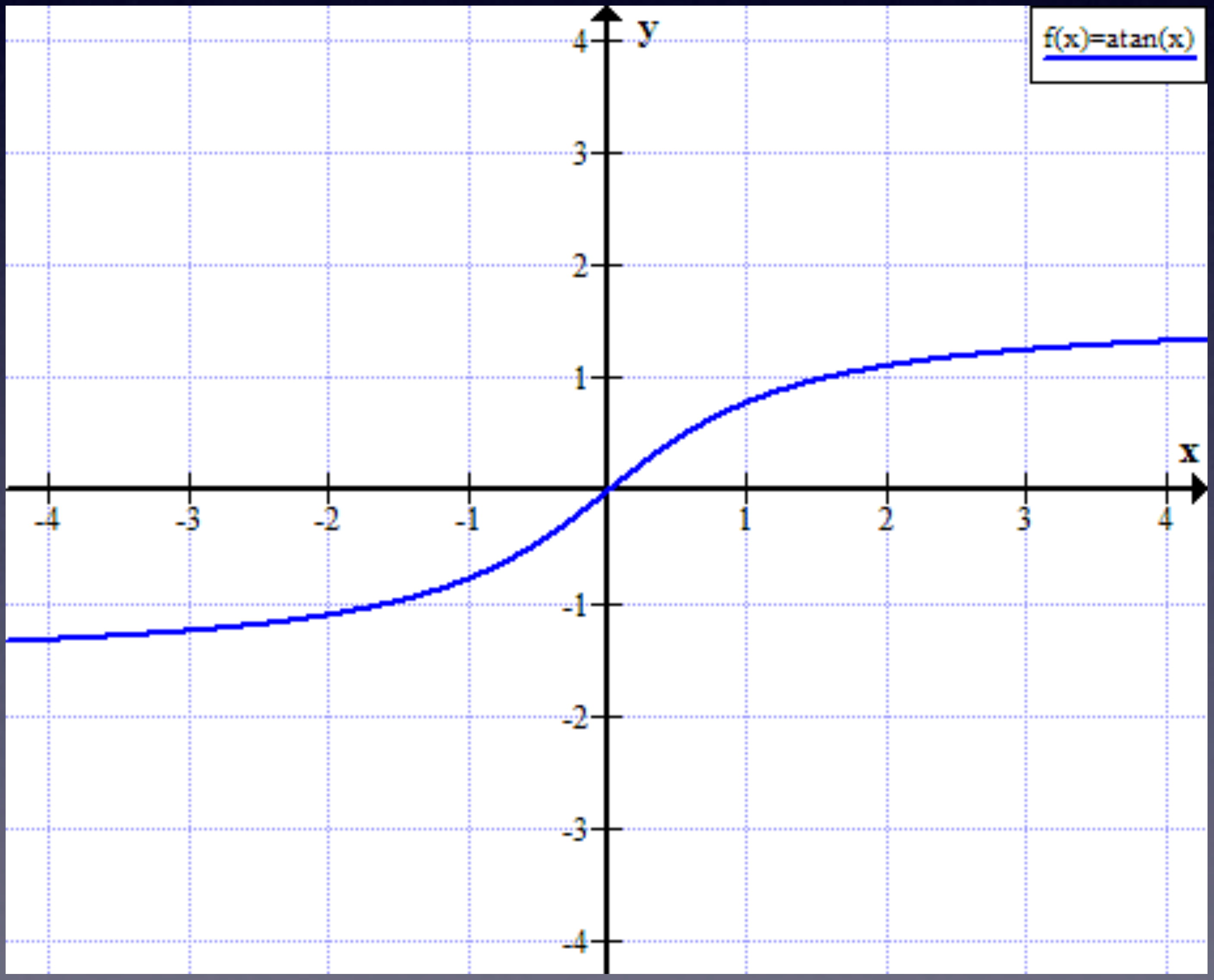
As difference equations,
Y = income, D = debt

$$
\begin{aligned}
Y_t &= \phi_0 + \phi_1 Y_{t-1} + \phi_2 D_{t-1} + \phi_3(\theta\eta Y_{t-1} - r D_{t-1})b \\
D_t &= (D_{t-1} + \beta\theta\eta Y_{t-1})/\alpha
\end{aligned}
$$

All coefficients are constant except for b, the "investment confidence."

B is a sigmoid function

$$b = b_1 arctan(Y_{t-1})$$



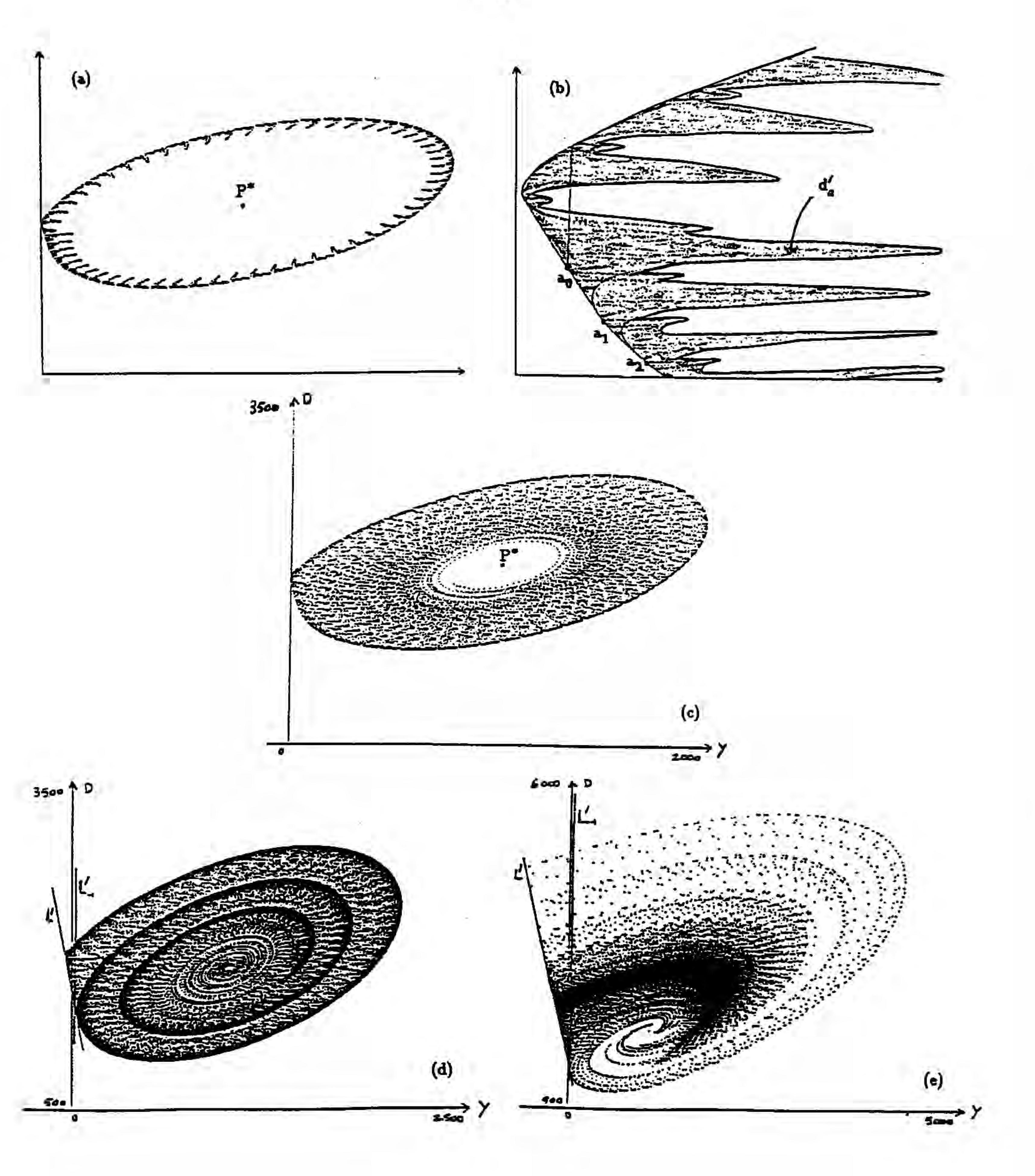
Chosen Control Parameters

$$
\begin{aligned}
u^+ &= 191 + 0.33u + 0.253v + (66.9u - 15.3v)b_1 arctan(u) \\
v^+ &= 0.94v + 0.12u
\end{aligned}
$$

Bifurcations

As b1 increases from 1.0,
there is a Hopf bifurcation,
then chaotic behavior.

Attractors with various values of b1 near 2



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On to 2D Experiments