Equations for Flows

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Oscillators in 2D

Rayleigh, 1883

From Dyn (Dynamics the Geometry of Behavior) exm 4a, p. 212.
Current, x, and voltage, y.

$$\begin{aligned} x' &= y \\ y' &= -C(x + By^3 - Ay) \end{aligned}$$

Note, equivalently,

$$y' = ay - by^3 - Cx$$

Van der Pol, 1922

From Thompson and Stewart, p. 84. Current, x, and voltage, y.

$$\begin{aligned} x' &= y \\ y' &= a(1 - x^2)y - bx \end{aligned}$$

OK to set b = 1. Hence,

$$\begin{aligned} x' &= y \\ y' &= a(1 - x^2)y - x \end{aligned}$$

There is a Hopf bifurcation when a passes zero.

Duffing, 1908

From Dyn 1 p. 213 p. 84. See also Ueda in The Chaos Avant-garde, p. 41.

Spring displacement, x, and its rate, y.

$$\begin{aligned} x' &= y \\ y' &= -ay - x^3 \end{aligned}$$

Volterra-Lotka, 1924

From Dyn combined p. 628.

$$\begin{aligned} x' &= (A - By)x \\ y' &= (Cx - D)y \end{aligned}$$

$A, B, C, D > 0$

Other oscillators

For ZBT reaction, se Strogatz p. 256. For glycolysis, see Strogatz, p. 205.

Chaotic Flows in 3D

Ueda, 1961

From XAG (The Chaos Avant-garde) p. 25.

$$\begin{aligned} x' &= y \\ y' &= \mu(1 - \gamma x^2)y - x^3 + Bcos(\nu t) \end{aligned}$$

$\mu = 0.2, \gamma = 8, B = 0.35.$

Forced Van der Pol, 1922

From Thompson and Stewart, p. 87. Current, x, and voltage, y.

$$\begin{aligned} x' &= y \\ y' &= \alpha(1 - x^2)y - \omega^2 x + Asin(\omega_0 t) \end{aligned}$$

Shaw variant: put forcing term on the velocity.

Lorenz, 1963

From Thompson and Stewart, p. 212.

$$
\begin{aligned}
X' &= -\sigma(X - Y) \\
Y' &= -RX - Y - XZ \\
Z' &= XY - bZ
\end{aligned}
$$

Rossler, 1976

From Thompson and Stewart, p. 235.

$$
\begin{aligned}
X' &= -Y - Z \\
Y' &= X + aY \\
Z' &= b + Z(X - c)
\end{aligned}
$$

Try $a = 0.398, b = 2, c = 4$.