Complex Dynamical Systems

A Series of Ten Lectures to the Ross Institute, 2011-2012

by

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Series Outline

1., Nov 2, Intro: Epiphany, What, Why, How

2. Nov 9, Grs. K, 1: Dynamical Systems (DS)

3. Nov 16, Grs 2, 3, 4: Complex Dyn Sys (CDS)

4. Nov 21, Grs 5, 6, 7, 8: NetLogo CDS Models

5. Dec 7, Grs 9, 10, 11: NetLogo CDS Models

Lec. 1: Introduction

- A. My epiphany of the miracle year, 1972.
- B. WHAT: Systems thinking, General systems theory, cybernetics, system dynamics, and complex dynamical systems (CDS).
- C. WHY: The Spiral and World Cultural History as systems, systems thinking to understand the future.
- D. HOW: Foregrounding the systems of each grade with NetLogo (needs participation).

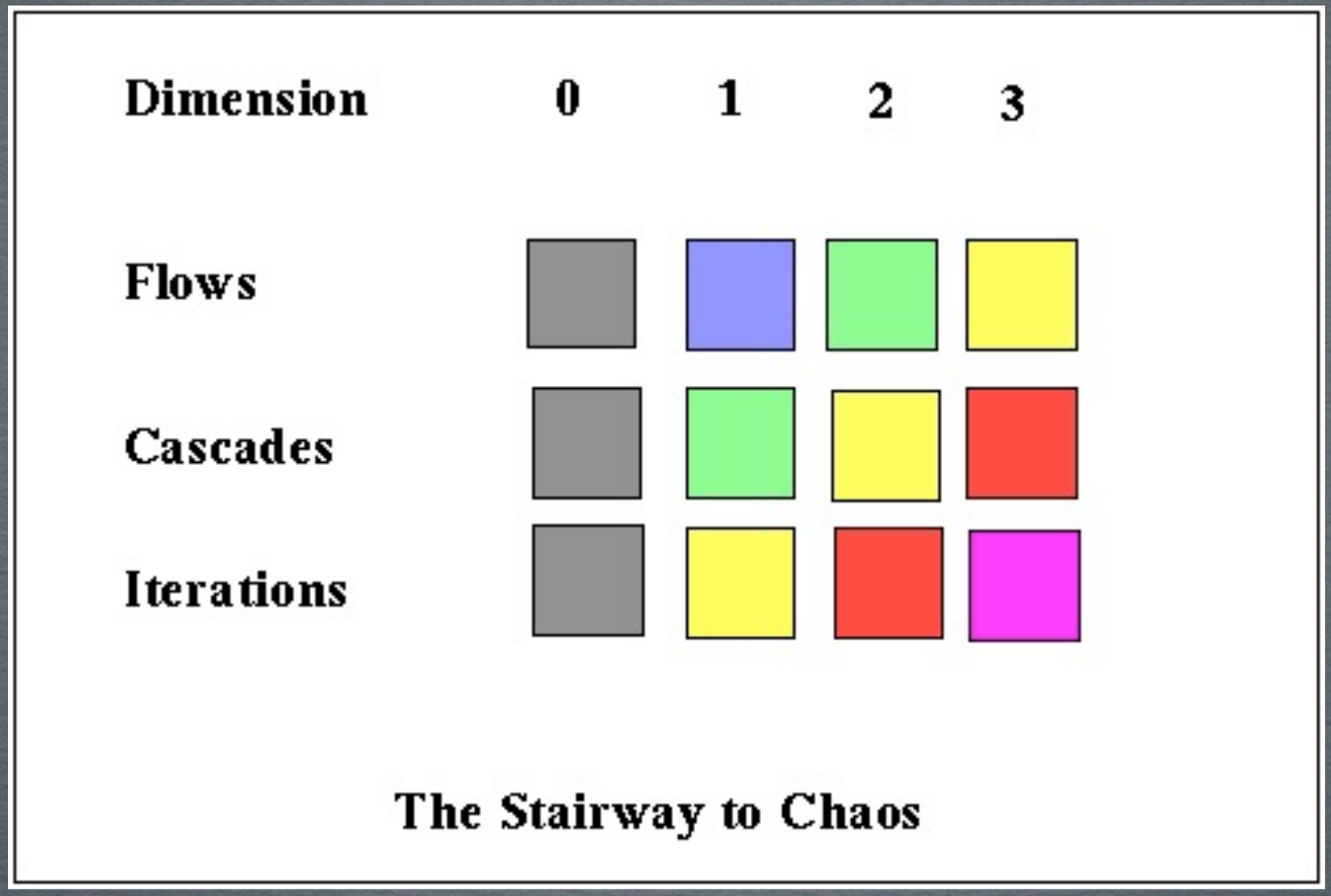
Discussion Board

- Systems vs Complex Dynamical System (CDS)
- Target (Territory) vs Model (Map)

Lec. 2: Dynamical Sys.

- A. Stairway 2 Chaos
- B. Attractors, Basins, and Separatrices
- C. Schemes and Bifurcations
- D. Animated Examples

A. Stairway to Chaos



Types of Dyn Sys

Types of Dyn Sys

- Flows advance with continuous time
- Cascades with discrete time (reversible)
- Iterations with discrete time (irreversible)
- We will consider flows and iterations with dimensions 1 and 2 (and sometimes 3)

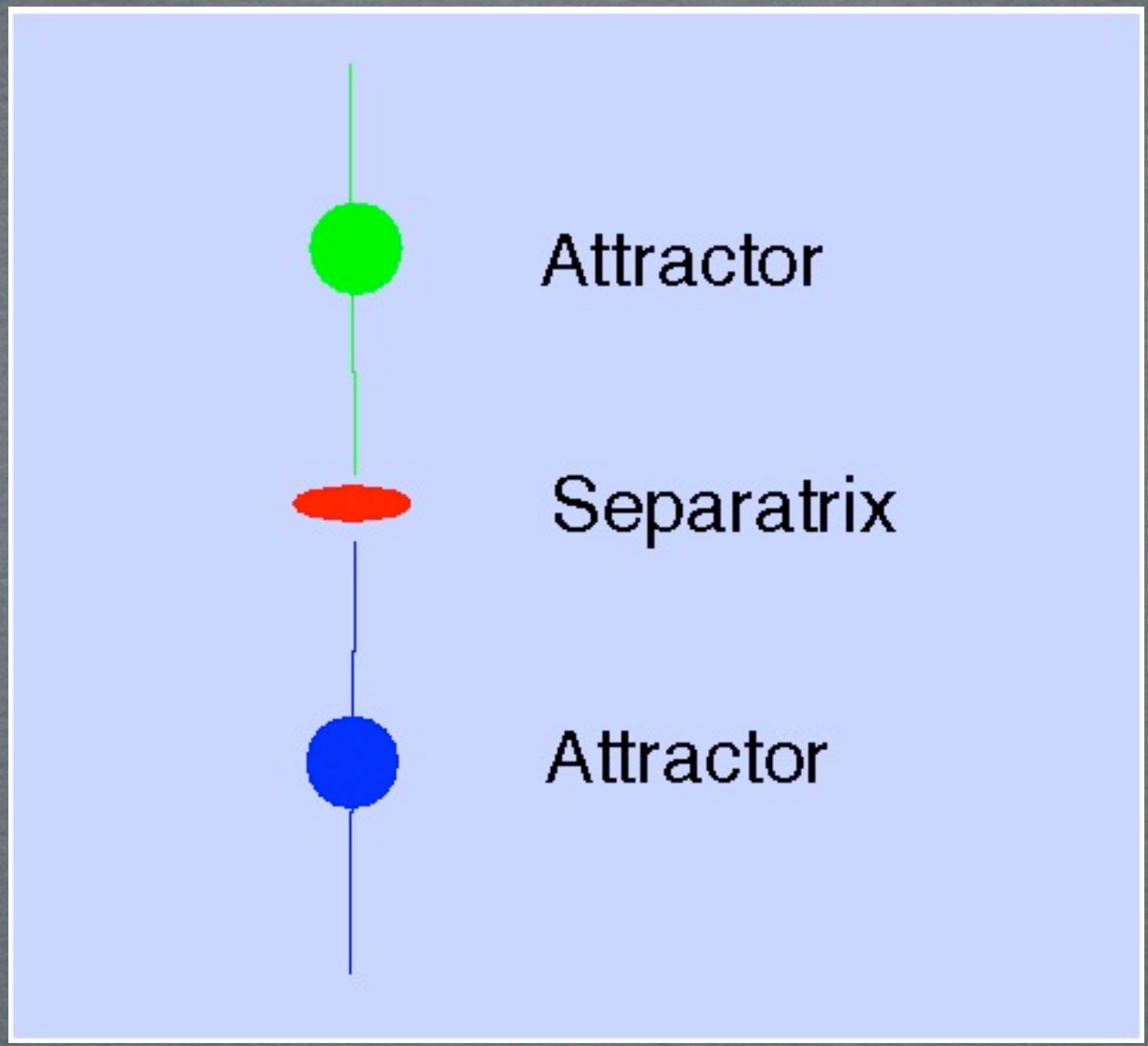
B. Attractors, Basins, and Separatrices

ATTRACTORS

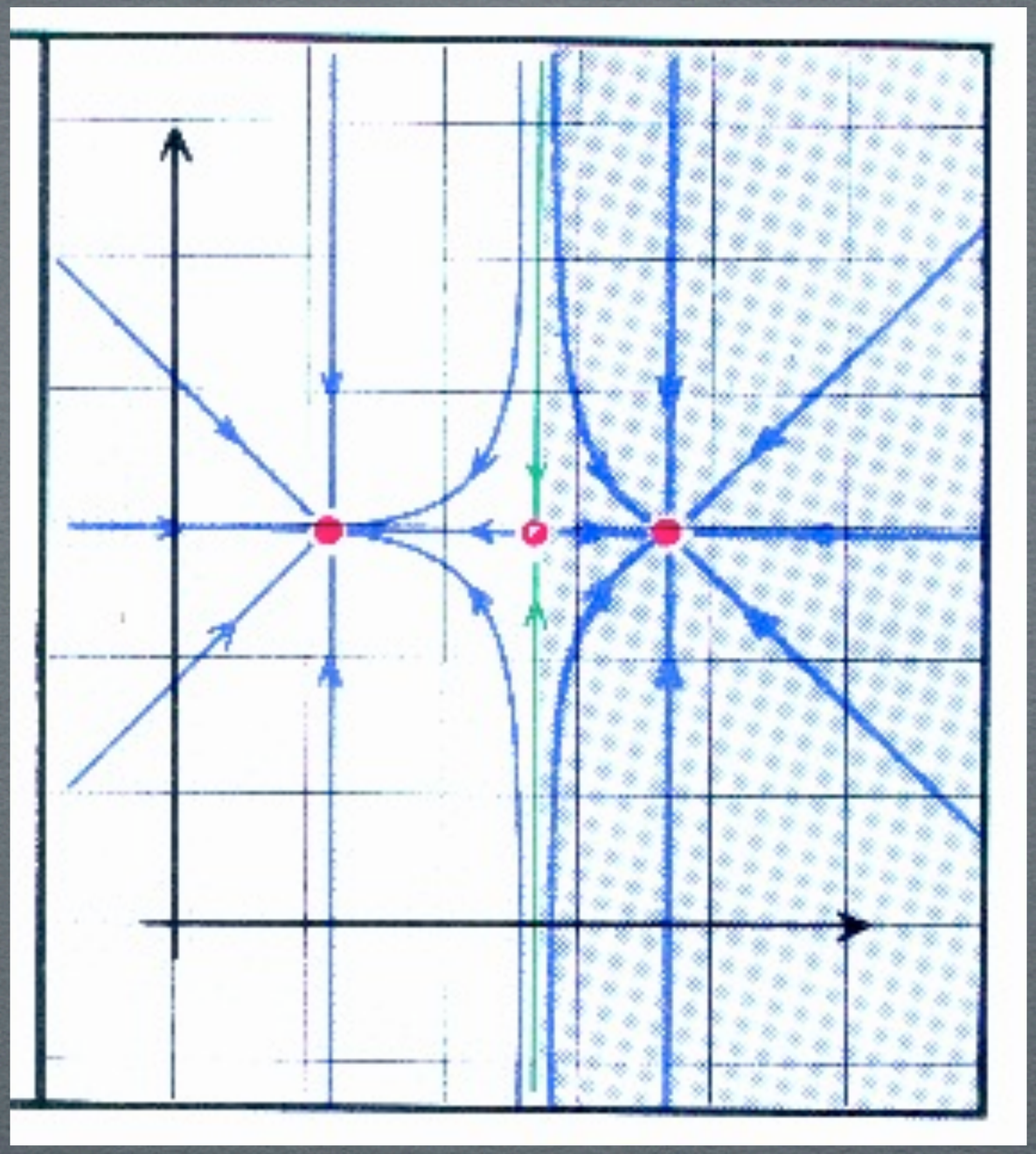
- All points tend to an attractor, 3 types:
  - Point attractors
  - Periodic attractors
  - Chaotic attractors

Basins

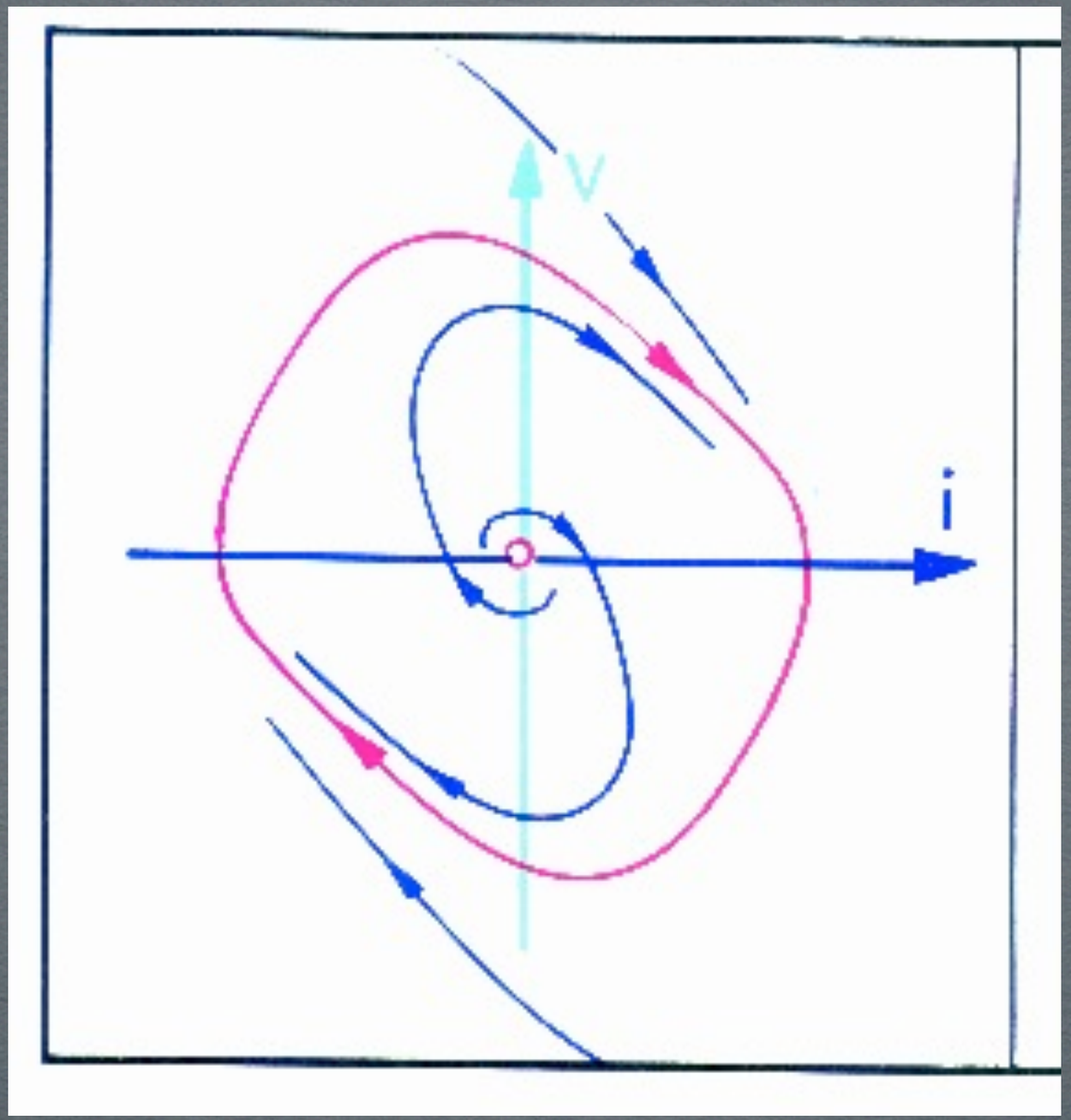
- The domain of the Dyn Sys consists of basins
- Each basin contains one attractor
- Each attractor is contained in one basin
- The basin of an attractor consists of all points that tend to that attractor
- Basins are divided by separatrices



1D Flow Example: Two Point Attractors



2D Flow Example: Two Point Attractors

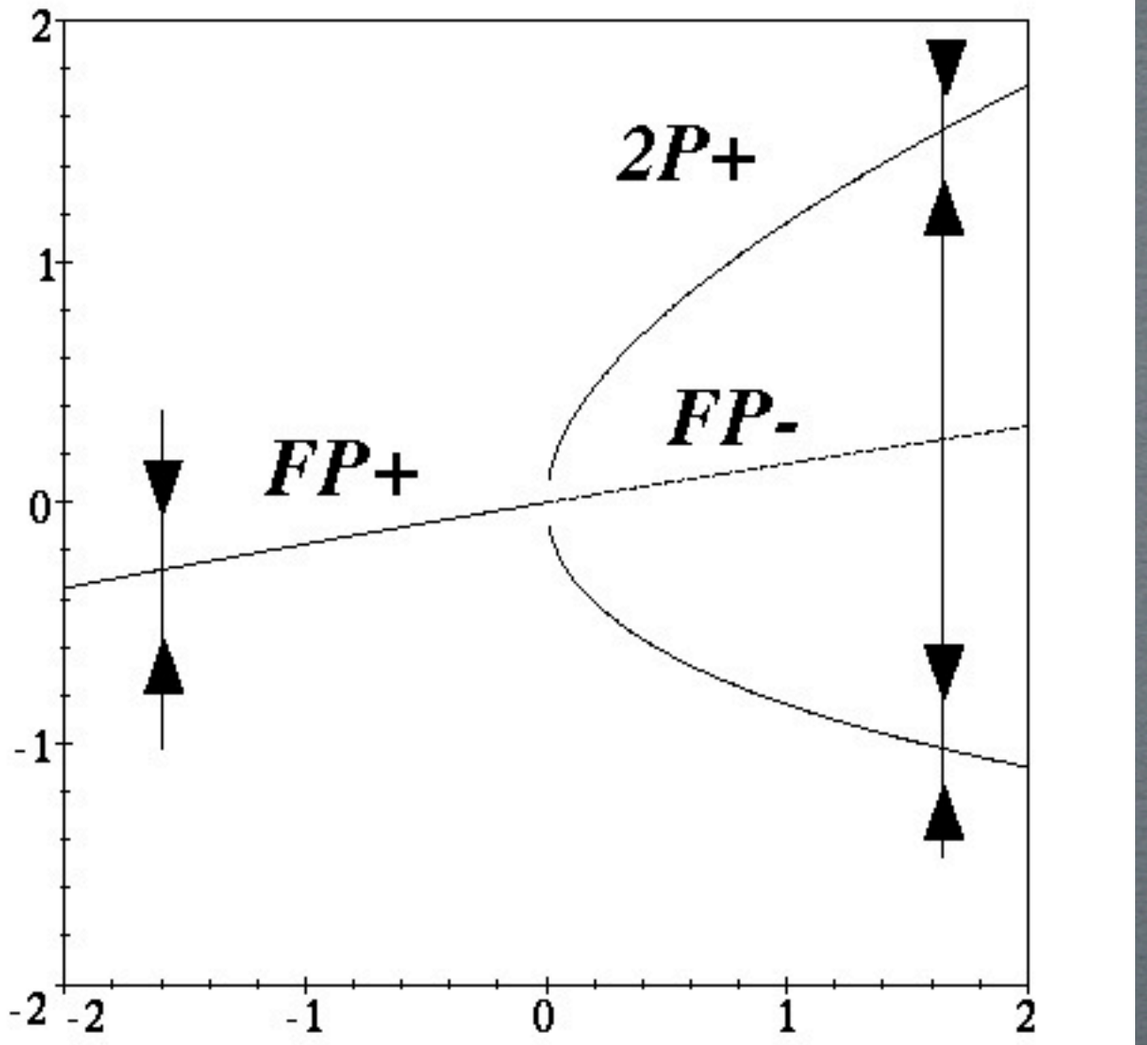


2D Flow with one periodic attractor

C. Schemes and Bifurcations

Schemes

- A scheme is a dynamical system with controls
- A bifurcation is a qualitative change in behavior of a scheme due to changing a control
- Types of bifurcation: catastrophic, explosive, and subtle
- A scheme is visualized by its bifurcation diagram



1D Iter Scheme with Period Doublling

D. Animated Examples

1D and 2D Iterations

www.visual-chaos.org/lab/NetLogo

- Bifurcation Demo with lab1, lab 2 (1D iteration)
- Basin Bifurcation Demo with lab 3
- Basin Demo with lab 4 (2D iteration)

References

- MS#97, Stairway to Chaos (10 pages)
- MS#98, CDS (4 pages, for next week)

Homework #2

- Due 12 hours before next lecture.
- Choose one of your systems from HW#1 and identify its nodes and links.
- Experiment with Lab 2, locate a five-periodic window.

End of Lecture Two of Ten