

On Maurício Matos Peixoto and his Mathematical Work

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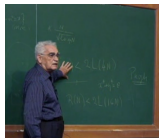
An abridged mathematical biography.

Maurício Matos Peixoto (Fortaleza, April 15, 1921 – Rio de Janeiro, April 28, 2019) studied at Escola Nacional de Engenharia, Universidade do Brasil, Rio de Janeiro, graduating as Civil Engineer in 1943.

His interest in Mathematics consolidated there, but started while he was at Pedro II School, a traditional High School in Rio de Janeiro, where he received his secondary education. At the School of

Engineering he became best friends with his classmates Leopoldo Nachbin (1922-1993), that later became a very distinguished Brazilian mathematician, and Marília Chaves (1921-1961), one of the pioneers among Brazilian women in mathematics research. In 1946, he married Marília. They also worked in mathematical collaboration.

Timeline of Peixoto's Career with labeled landmarks: 1955 and after. Highlight at 1987, TWAS Mathematics Award. Dates of photos: upper, 1936; bottom, 2010.



Landmarks, chronologically increasing

- 1955: Reading of H. F. DeBaggis, Dynamical Systems with stable structure, Contr. Theory of Nonlin. Oscillations II, 1952, 37-59. Translation into Russian, with Foreward and notes by M. I. Minkevich, Uspekhi Mat. Nauk, 1955, Vol 10, 4(66), 101 - 126.
- 1958-59: The golden years in Princeton, evocated in A.S. Communication at the Edingburgh ICM. Contact with Smale.
- 1960 - 61: Smale in Rio, first semester 1960, Generalized Poincaré Conjecture and the Horseshoes in the Copacabana beaches. Contact with Thom. Lecture on Structural Stability at the Third C.B.M.
- 1962 - 64: Publication of Structural Stability on two-dimensional manifolds, Topology.
Seminar on the Qualitative Theory of Diff. Equat. The evocative essay On a List of Open Problems on Differential Equations, SJMS, 2019.
- 1987: TWAS Mathematics Award.

For his contribution to Dynamical Systems, in 1986, he was awarded the World Academy of Science (TWAS) Prize for Mathematics, 1987.



“For his fundamental and pioneering study of structural stability of dynamical systems, in particular for proving that flows on surfaces are generically structurally stable.”

The text above is engraved in the medal and it was read in the ceremony by the eminent mathematician S. S. Chern.

The medal was delivered by Abdus Salam, President of TWAS.

Sitting, to the left, with dark glasses, is M. G. K. Menon, Physicist founder of the Tata I. F. R.

The Meeting of the TWAS in Beijing was the occasion for China to launch his opening to the world in Science matters.

The Acceptation Speech (A.S.) was published in the Proceedings of the Second Conference Organized by the TWAS, World Scientific, 1989.

This speech conveys an idea of the mathematical motivation that led the author to produce the work for which he received the award.

A history whose starting point, for expository reasons, we placed pictorially in 1955.

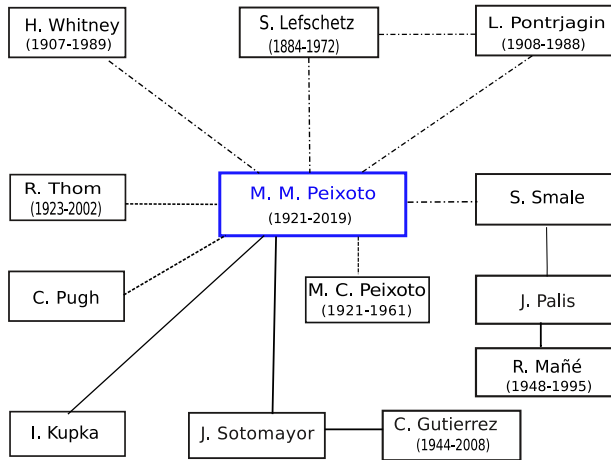
We will also consider the 1955 landmark, and before, to complement our glimpse into Peixoto's mathematical career.

The belief that the charming speech (A. S.) will help the reader to grasp a crucial part of the present day diversified domain of Dynamical Systems, prompted us to collaborate in a new Portuguese translation of the speech, directly from the original, adding mathematical comments.

A first translation of A.S. was published in Notícias, Matemática Universitária N. 8, December 1988, 1–25.

In his speech Peixoto refers to Cantor, Poincaré and Hilbert and links them with Lefschetz, Anosov, Thom and Smale, not forgetting of Andronov and Pontrjagin, the Russian pioneers of Structural Stability.

An outline of the of mathematical influences and interactions of M. Peixoto, cited in A.S.



In 1958 - 59 M. Peixoto visited Lefschetz at Princeton



P. Seibert, A.P. Stokes, M. Peixoto and S. Lefschetz, Princeton, 1959.

Interactions and influences in A. S., an outline.

Lefschetz had a key role in the transition of Structural Stability from Russia to Brazil.

First step: He directed the translation of Andronov - Chaikin, 1949, in which the name Structural Stability, which he coined to replace the Russian name *robustness* appears several times referring to the systems whose characterization was formulated by Andronov and Pontrjagin in 1937, and was reproduced also as an appendix.

Second step: He requested the production of a proof of the theorem stated by Andronov and Pontrjagin to H. F. DeBaggis. In 1952 he published the outcome of the work of DeBaggis in Contributions to Nonlinear Oscillations, which Peixoto read in 1955.

Before taking this executive move, Lefschetz had addressed a letter to Andronov suggesting that a proof of such an important theorem should be published.

More on Lefschetz

In 1951 Peixoto read parts of Andronov - Chaikin since it is cited in Stability Chapter the Thesis he presented in the competition for a Chair (a Cathedra) in the University of Brazil (later UFRJ). But in the Thesis no mention is made to Structural Stability.

The fact is that when, in 1955, Peixoto read and presented in a Seminar the work of DeBaggis, he became strongly interested in Structural Stability and soon reformulated it and contributed with new ideas and results, which, along the years, led him to the TWAS award.

One may ask: How took place the contact between Lefschetz (1884 - 1972) and DeBaggis (1916 - 2002)? How and why did Lefschetz select DeBaggis?

This seems to be an intriguing matter, for which very little written information can be found. Based on sparse chronological data available, only some probing conjectures may be proposed, at the end.

On a self taught mathematician in 1946 - 1955.

Let's comment some fragments of the Article - Obituary of Peixoto published in June in the Magazine Piauí.

“Surpreendentemente, o homem que mudou a história dos sistemas dinâmicos **não só não era formado em matemática** como nunca concluiu um doutorado na área. Peixoto chegou a **iniciar o curso** na Universidade de Chicago, em 1949, mas **desistiu da formação** ao prestar um concurso para professor na Escola de Engenharia – e ser aprovado.

Começaria a dar aulas na instituição em 1952.”

“Nunca me arrependi de ter largado o doutorado, porque uma cátedra na Universidade do Brasil era muito difícil de aparecer, pois era vitalícia; não era oportunidade que se desperdiçasse.”

Peixoto, 1946 - 1955. Comments

In 1955, when Peixoto read the article of DeBaggis and got interested in Structural Stability, he was already Full Professor (Catedrático).

He had published an interesting collection of research papers covering a wide spectrum of subjects, listed below in decreasing chronologic order:

MR0044608 Monteiro, A. A.; Peixoto, M. M.

Le nombre de Lebesgue et la continuité uniforme. (French)

Portugaliae Math. 10, (1951). 105 - 113. (Rev. : T. A. Botts) 27.2X

MR0033081 Matos Peixoto, Mauricio

On convexity. Anais Acad. Brasil. Ci. 21, (1949). 291 - 302. (Rev.: E. F. Beckenbach) 52.0X

MR0029949 Matos Peixoto, Mauricio

Generalized convex functions and second order differential inequalities.
Bull. Amer. Math. Soc. 55 (1949), 563 - 572. (Rev. : E. F.
Beckenbach) 27.0X

MR0029948 Matos Peixoto, Mauricio, Convexity of curves.
(Portuguese)

Notas de Matemática, no. 6. Livraria Boffoni, Rio de Janeiro, 1948.
i+66 pp. (Rev. : E. F. Beckenbach) 27.0X

MR0028360 Matos Peixoto, Maurício An inequality among positive
numbers. (Portuguese) Gaz. Mat., Lisboa 9, (1948). no. 37-38, 1948.
(Rev. : R. P. Agnew) 27.0X

MR0027038 Matos Peixoto, Mauricio

On the existence of derivatives of generalized convex functions.

Summa Brasil. Math. 2, (1948). no. 3, 35-42. (Rev. : W. Gustin)

27.2X

MR0015602 Matos Peixoto, Mauricio

On the solutions of the equation $yy'' = \varphi(y')$ which pass through two points of the half-plane $y > 0$. (Spanish)

Revista Unión Mat. Argentina 11, (1946). 84-91. (Rev. : W. Kaplan)

36.0X

Peixoto, 1946-1955.

He had also written the following Academic Dissertations:

Thesis prepared for Livre Docência diploma that did not happen:

M.M. Peixoto, Sistemas não holônomos, 68 páginas, Rio de Janeiro, 1947.

Two Livre Docência Theses:

E. N. E., 1947,

M.M. Peixoto, Princípios variacionais de Hamilton e da menor ação, 55 páginas, Rio de Janeiro, 1947.

Escola de Agricultura, 1948,

M.M. Peixoto, Convexidade das curvas, 66 páginas, Rio de Janeiro, 1948.

Thesis for the position of Catedrático E.N.E., 1951

M.M. Peixoto, Equações gerais da dinâmica, 110 páginas, Rio de Janeiro, 1951.

This work cites Andronov - Chaikin.

In September 1955, year of the reading of the paper of DeBaggis, Peixoto published a reformulation and new results on Structural Stability:

NOTE ON STRUCTURALLY STABLE SYSTEMS — M. M. PEIXOTO — *Instituto de Matemática Pura e Aplicada, Conselho Nacional de Pesquisas, Rio de Janeiro, D.F.* — Let \mathbf{F} be the set of two dimensional systems $\dot{x}=P(x,y)$, $\dot{y}=Q(x,y)$ where P, Q are defined and of class $C^{(1)}$ on a certain domain G .

If $X \equiv (P(x,y), Q(x,y))$, $Y \equiv (P_1(x,y), Q_1(x,y))$ are elements of \mathbf{F} define a distance between them by

$$d(x, y) = \sup_{(x, y) \in G} \{ |P - P_1| + |Q - Q_1| + |P_x - P_{1x}| + |P_y - P_{1y}| + |Q_x - Q_{1x}| + |Q_y - Q_{1y}| \}$$

Then \mathbf{F} becomes in a natural way a real Banach space. Call Σ the subset corresponding to all structurally stable systems [See: *Annals of Mathematics Studies*, Study 29, p.37—59].

Then Σ is an open disconnected cone. Its connected components are denumerable and when two systems X, Y belong to the same connected component there is a homeomorphism of G on to itself transforming trajectories of X on to trajectories of Y — (27 de setembro de 1955).

Back to Piauí

The above ambiguous expression **desistiu da formação** does not apply to Peixoto:

The diploma of Livre Docente, which involved a Thesis, was the only formal requisite to have access to a competition for a Chair, a Cathedra, which was the highest university academic position. It amounted to have a permanent position, “a place under the sun”. He liked this expression.

In 1949 the career of Researcher did not exist in Brazil.

A selftaught mathematician has already a “formação”.

More on Piauí Obituary Article: Symbolic or Pradigmatic

“O pesquisador brasileiro que havia inspirado Smale, contudo, logo **perderia o bonde do desenvolvimento da área.**”

“o papel do Mauricio em sistemas dinâmicos tornou-se sobretudo **simbólico** ”

The presentation in his A.S., TWAS, reveals that Peixoto followed very neatly the developments in Dynamical Systems in higher dimensions. He was on the “bonde” still for a long additional ride.

The expression “paradigmatic” instead of “symbolic” applies better to the role of Peixoto’s work in stimulating research in Dynamical Systems. For instance:

In 1981 it was crucial guide for developments in the extension of Structural Stability to the Differential Equations of Classical Geometry, Gutierrez and Sotomayor. Later it appeared in the regularization of Discontinuous Dynamical Systems, Teixeira and Sotomayor 1996.

DeBaggis, a link with Lefschetz.

DeBaggis migrated from the Axiomatic Hyperbolic Geometry, field in which got his M.Sc. and PhD. with Karl Menger at Notre Dame Univ., 1945-47, to the Qualitative Theory of Differential Equations, subject in which he had to gain competence in order to work in association with Lefschetz in Princeton, 1950-52.

The work that DeBaggis did on Differential Equations, the only one he published outside his Master and PhD. dissertations, on Hyperbolic Geometry, attracted the interest of M. Peixoto, the first Brazilian mathematician to attain worldwide recognition for his achievement developing new ideas improving the results in Andronov - Pontrjagin, 1937, and DeBaggis, 1952, works.

Peixoto 1962-64 Landmark Seminar. Evocative essay in SPJMS, 2019.

Solving Problems.

The Problem of Bifurcations.

A Proof of a Genericity Theorem of M. M. Peixoto and M. C. Peixoto in the Plane:

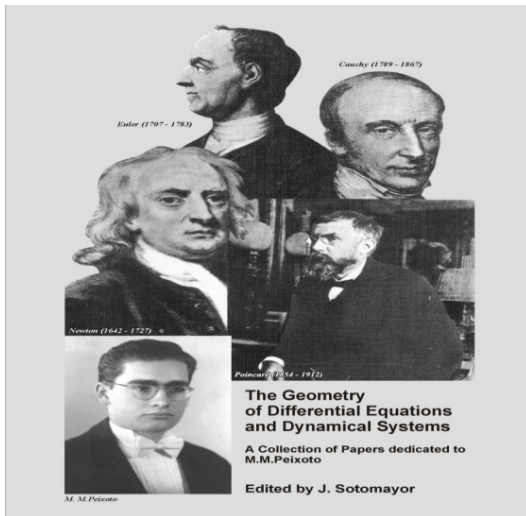
Σ , the set of C^r , $r \geq 1$, Structurally Stable Vector Fields in a plane compact region with regular boundary, is open and dense, moreover it has full Lebesgue Measure.

J. Sotomayor, Course delivered at the CBM 1981.

Another event in 1981: The emergence of Structural Stability in Classical Differential Geometry.

The 80th anniversary meeting at IME-USP.

Published in Comp. and Appl. Math. 2001



The 80th anniversary of M. Peixoto at IME-USP: Nourishment for mind and body.



Epigraph in Peixoto's 1951 Full Professor Thesis, from a speech of Maxwell, 1870.

There are . . . some minds which can go on contemplating with satisfaction pure quantities presented to the eye by symbols, and to the mind in a form which none but mathematicians can conceive.

There are others who feel more enjoyment in following geometrical forms, which they draw on paper, or build up in the empty space before them.

Others, again, are not content unless they can project their whole physical energies into the scene which they conjure up. They learn at what a rate the planets rush through space, and they experience a delightful feeling of exhilaration. They calculate the forces with which the heavenly bodies pull at one another, and they feel their own muscles straining with the effort.

Epigraph, speech of Maxwell, 1870. Cont.

To such men momentum, energy, mass are not mere abstract expressions of the results of scientific inquiry. They are words of power, which stir their souls like the memories of childhood.

For the sake of persons of these different types, scientific truth should be presented in different forms, and should be regarded as equally scientific, whether it appears in the robust form and the vivid colouring of a physical illustration, or in the tenuity and paleness of a symbolical expression.